

أسلوب جديد في حل المسائل ذات أهداف متعددة  
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المستخلص

هنالك العديد من البحوث التي تتعامل مع أستنباط ما يسمى بمجموعة الحلول الكفوءة (*Efficient Solutions*)، لمشاكل حقيقية ذات أهداف متعددة ومتضاربة (*confronted*) بعضها مع البعض. ففي هذا البحث تم أستنباط قرار متعدد الأهداف متأسسا على بناء نموذج رياضي وذلك بتكوين دالة هدف واحدة من خلال ربط الدوال ذات أهداف مختلفة. كذلك تم عرض بعض النظريات المتعلقة بالموضوع، وأعطاء بعض المؤشرات لكفاءة أداء النموذج الرياضي والاسلوب المتبع من خلال عرض مسألة تطبيقية حقيقية وكذلك للنتائج المستحصلة من حل عدد من المسائل التي تم توليدها عشوائيا.

## New Approach for Solving Multi – Objective Problems

### Abstract

There are many researches deals with constructing an efficient solutions for real problem having Multi - objective confronted with each others. In this paper we construct a decision for Multi – objectives based on building a mathematical model formulating a unique objective function by combining the confronted objectives functions. Also we are presented some theories concerning this problem. Areal application problem has been presented to show the efficiency of the performance of our model and the method. Finally we obtained some results by randomly generating some problems.

**Keywords:** Multi – objective, optimal solution, decision making, efficient solution, linear combination.



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### Introduction



In many real life – problems, one is usually confronted with several objectives, which are in mutual conflict. Multi – objective programming, and their optimization methods are difficult to use because human subjectivity in an integral part of them. We cannot just formulate a model and leave it to an optimization expert to calculate an optimal solution.

Many algorithms appeared in the literature have been designed to obtain solutions to decision problems which must accomplish with multiple objectives. Each algorithm has its own claim of power. Decision maker may not be able to select an appropriate procedure to support the decision making ( DM ). The lack of guidelines in the selection of Multi – Objectives Decision Making ( MOMD ) algorithm is partially reflected in the fact that not many. Empirical tests have been reported in the literature various models and methodologies are frequently developed in the theoretical sense without addressing the practicality of applying them in a real – world setting. Applications which use only illustrative data may also mislead the practitioner to believe that the model may be practical in a wider setting. From the managerial point of view, there is a need to investigate which method would be better in what situations.

Many of the recent works deals with the determination of efficient solutions set, and with their utilization in solving problems. An enormous researches effort in an area known as "Efficient Solution" is constructed "local efficiency" sets. A motivated some works are discussed in the context of "proper efficiency" ,see [ 2 ] & [ 6 ].

In this paper, a new approach for solving multi – objective problems is constructed, by interpolating multi – objectives functions with variable coefficients. Some theories and experimental results are presented to point out how efficiency of our model and procedure is good.

### ***The problem***

Multi – objective optimization problem can be stated as follows:

Find  $X = (x_1, x_2, \dots, x_n)^T$ , which;  
 maximize ;  $f_1(X), f_2(X), \dots, f_k(X)$ , (1)

subject to:

$$g_i(X) \leq 0, \quad \text{for } i = 1, 2 \dots m$$

Where any or all of the functions:

$f_k(X)$  ( $k = 1, 2, \dots, K$ ) &  $g_i(X)$  ( $i = 1, 2, \dots, I$ ) may be nonlinear.

***Definition ( 1 )***: A point ( $X \in S$ ) is said to be efficient in  $S$  with respect to  $f_k(X)$ , if  $\nexists Y \in S$  with  $f_k(Y) \geq f_k(X)$ .

The set of all such points ( $X \in S$ ) is denoted by;

$$\mathcal{E}(X, f_k) = \{ X \in S : \nexists Y \in S \text{ with } f_k(Y) \geq f_k(X) \}$$

### ***Model formulation***

Multi – objectives decision making ( MODM ) procedures seek to obtain the "most preferred" of the feasible solutions across all the objectives which the decision maker wishes to optimize. Usually, no solution can be found which

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allows concurrent optimization of all objectives, because of the conflicting nature of the individual objective. For instance, an objective related to reduction the manufacturing costs may conflict to an objective of maintain full employment.

Nearly, all the literatures { see [ 5 ], [ 9 ], [ 7 ] }, who proposed the properties of different types of solution sets, with respect to linear combination of the original objective functions, in which the coefficients are constants, denoted by  $(t_k)$  with  $(0 \leq t_k < 1)$  and  $(\sum_{k=1}^K t_k = 1)$ .

In this paper, in order to construct an efficient solution set, we are finding more suitable values of these coefficients, by considering  $K$  optimum solutions points as the base points in constructing new weight coefficients as variables functions, denoted by  $t_k(X)$  defined as;



$$t_k(X) = \prod_{k \neq 1} \left( \frac{X - X^{k*}}{X^{l*} - X^{k*}} \right) \quad (2)$$

Therefore, a new multi-objective functions problem can be formulated as following:

$$F(X) = \sum_{k=1}^K t_k(X) f_k(X) \quad (3)$$

Subject to:

$$g_i(X) \leq 0, \quad \text{for } i = 1, 2, 3, \dots, m$$

$$t_k^l \leq t_k(X) \leq t_k^u$$

Where  $(t_k^l \& t_k^u)$  are the given lower and upper bounds of the weight function  $\{f_k(X)\}$ , and  $(X^{k*})$  are the optimum points of  $\{f_k(X)\}$ , which is unique vector and in practical problems, such vector is always unfeasible (otherwise there would be no conflicts), but it is conceivable that the nearest feasible solution could be an acceptable compromise for the decision maker.

Let  $S' = \{X \in R^n: l_k \leq X^k \leq u_k\}$ , where ;

$l_k = \min_k \{X^k\}$  , &  $u_k = \max_k \{X^k\}$  , then we would state the following definition;

**Definition (2):** A point  $(X \in S')$  is said to be global efficient in  $(S')$  with respect to  $F(X)$  , if  $\nexists Y \in S'$  , with  $F(Y) \geq F(X)$  ,  
 $G(S', F) = \{X \in S': \nexists Y \in S' , \text{ with } F(Y) \geq F(X)\}$

if we set  $d_j = X_j^e - X_j^{k*}$  ,

Therefore, the problem (1) can be reformulated as following;

$$\text{Minimize } d_\alpha = \sum_k \left( \sum_{j=1}^n |d_j|^\alpha \right)^{\frac{1}{\alpha}} \quad (4)$$

Subject to:

$$g_i(X^e) \leq 0, \quad i = 1, 2, 3, \dots, m$$

$|X_j^e - X_j^{k*}| \leq \epsilon_j, \forall j \& k$  , in which  $X_j^e \& X_j^{k*}$  have the same signs at their objective functions.

$$t_k^l \leq t_k(X^e) \leq t_k^u(x), \quad k = 1, 2, 3, \dots, K$$

$$\sum_{k=1}^K t_k(X^e) = 1$$

Where,  $\epsilon_j$  , are small sufficient positive number,  $1 \leq \alpha \leq \infty$  , designates the norm in the objective space,  $X_j^e$  is the  $j^{\text{th}}$  coordinate of the efficient solution  $X^e$  of  $F(X)$  that minimized  $d_\alpha$  , and  $X_j^{k*}$  is the  $j^{\text{th}}$  coordinate of the optimum solution  $X^{k*}$  of the problem:



$$\max f_k(X),$$

Subject to:

$$g_i(X) \leq 0, \quad i = 1, 2, 3, \dots, m$$

(5)

**Theorem:**  $\varepsilon(S, f_j) \subseteq G(S', F)$

**Proof:** Let  $X \notin G(S', F)$ , that means  $\exists Y \in S'$ , s.t.  $F(Y) \geq F(X)$ . And since  $\{\exists Y \in S'\}$  it means that we have either  $\{Y \in S' \setminus S\}$ , (i.e.  $Y \notin S'$ ) which is of no interest, or  $(Y \in S')$  with  $f_j(Y) \geq f_j(X)$ , and that means  $X \notin \varepsilon(X, f_j)$ .

**Corollary:**  $G(S', F)$  be independent of the ordering of the components of (F).

### **The approach & Computational Results:**

The optimum value for each objective function  $\{f_k(X^{k*})\}$  are calculated, then we solve the problem (5). The percentage of  $\{d_\alpha\}$  (for certain value of  $\alpha$ ) is calculated. This can be demonstrated in the following steps:

- 1- Solve problem (5) for all k.
- 2- Formulate F(X) from (2) & (3).
- 3- Solve problem (4), to find  $X^e$ .

During step (3); the unfeasibility solutions may arise, due to the unfeasibility of one or more constraints of the form  $|X_j^e - X_j^{k*}| \leq \epsilon_j$ . To overcome this problem, the corresponding values  $\epsilon_j$  should be changed into suitable value to get such constraints be feasible.

In order to point out the efficiency of our model, and the proposed approach, several tested experiment are designed to include the following factors:

- a - the number of objective  $f_k(X)$ ,
- b - the number of constrains  $g_i(X) \leq 0$ ,
- c - the number of decision variables  $x_j (j = 1, 2, \dots, n)$

The problem sets generated for this research contain hypothetical situations with the restriction number of objectives, constraints and decision variables. The result is presented in the table below, which show as that an acceptable convergence on  $\{X^e\}$  is obtained, from 6 randomly generated problems.



**Table (1):** Computational Efficiency ( $\bar{d} = (\sum_l |d_l|)/l$ , for all  $l = 1, 2, 3 \dots L$  and  $L \leq n$ , s. t.  $|X_j^e - X_j^{k*}| \neq 0$ ).

No.of obj.func.(j)	Problem size ( $m * n$ )	% $\bar{d}$
2	8 * 12	0.5
3	13 * 17	0.4
4	14 * 19	0.45
5	18 * 25	0.25
6	22 * 30	0.20

### **Application:**

An illustrative application in Watershed Management taken from [2] is conducted to investigate the potential of several watershed management options, deciding whether to continue with present management practices, or to implement new ones. By let;

$x_{1j}$  = square miles of grassland with current management practices in decision period j;

$x_{2j}$  = square miles of grassland with compacted earth (CE) treatment (to increase runoff) in period j;

$x_{3j}$  = square miles of Chihuahuan desert with current management practices in period j;

$x_{4j}$  = square miles of Chihuahuan desert converted to grasslands (to increase runoff and grazing) in decision period j;

$x_{5j}$  = square miles of Chihuahuan desert with compacted earth (CE) treatment (to increase runoff) in period j;

$x_{6j}$  = square miles of riparian stands with current management practices in period j;

$x_{7j}$  = square miles of riparian stands with 75 percent removal and conversion to grasslands (to increase runoff and grazing) in period j;



$x_{8j}$  = square miles of woodland with current management practices in period  $j$ ;

$x_{9j}$  = square miles of woodland stands with 75 percent removal and conversion to grasslands ( to increase runoff and grasing ) in period  $j$ ;

$x_{10j}$  = square miles of pine – mixed conifer with current management practices in period  $j$ ;

$x_{11j}$  = square miles of pine – mixed conifer with 75 percent logging ( to increase runoff and economic benefits ) in decision period  $j$ ;

The time horizon for this nonlinear programming is a 30-years period, intended to correspond with the effect life of the mechanical soil treatments and subperiods was deemed necessary because water-runoff rates, sedimentation rates, and so forth will not remain constant over the entire period.

Five objective functions and 18 constraints (linear and nonlinear) on land, capital, and extent of treatment make up the nonlinear mathematical model used in the analysis. This analysis of the watershed is concerned, specifically, with the extent of application of land treatments of (1) increasing water runoff to the San Pedro River, (2) increasing recreational benefits, (3) maintaining wildlife levels in the area, (4) increasing commercial benefits, and (5) controlling sediment yield while operating with specified capital and land constraints.

### **Objective functions**

The vector maximization problem has the following noncommensurable objective functions:

**Water runoff**

$$z_1(x) = \sum_i \sum_j a_{ij} x_{ij}$$

**Sedimentation**

$$z_2(x) = - \{ \sum_i \sum_j b_{ij} x_{ij} \}$$

**Animal wildlife unbalance**

$$z_2(x) = - \{ \sum_i [(c_{1j} - c_{2j})x_{2j} + (c_{3j} - c_{4j})x_{4j} + (c_{3j} - c_{5j})x_{5j} + (c_{6j} - c_{7j})x_{7j} + (c_{8j} - c_{9j})x_{9j} + (c_{10j} - c_{11j})x_{11j}] \}$$

**Recreation**

$$z_4(x) = \sum_i \sum_j d_{ij} x_{ij}$$

**Commercial**

$$z_5(x) = \sum_i \sum_j e_{ij} x_{ij}$$



In the above functions,  $a_{ij}$  represents the water runoff (in 1000 cu ft / sq mile) associated with pair  $i$  of vegetation type and land treatment through period  $j$ ,  $b_{ij}$  is the sediment rate in acre-feet per square mile,  $c_{ij}$  is the animal biomass in pound-years per square mile,  $d_{ij}$  is the recreational benefit in \$ 1000 per square mile resulting from logging and grazing (after subtraction of the cost of logging operations and seeding). An appropriate constant has been subtracted from each objective function so as to make its value zero when current practices alone are being implemented.

The first three objective functions, with their different nondollar units of measurement, represent project effects whose values are not necessarily fully reflected in individuals' willingness to pay for them. Thus recreation and commercial benefits are both measured in dollars, but recreation might have a social value not fully reflected in individuals willingness to pay for it, in contrast with commercial benefits.

### **Constraints Set**

The above objective functions are to be maximized while operating with specified land and capital constraints.

The land constraints are as follows:

$$x_{1,1} + x_{2,1} = 264.8$$

$$x_{1,2} + x_{2,1} + x_{2,2} = 264.8$$

$$x_{1,3} + x_{2,1} + x_{2,2} + x_{2,3} = 264.8$$

$$x_{3,1} + x_{4,1} + x_{5,1} = 147.0$$

$$x_{3,2} + x_{4,1} + x_{4,2} + x_{5,1} + x_{5,2} = 147.0$$

$$x_{3,3} + x_{4,1} + x_{4,2} + x_{4,3} + x_{5,1} + x_{5,2} + x_{5,3} = 147.0$$

$$x_{6,1} + x_{7,1} = 10.0$$

$$x_{6,2} + x_{7,1} + x_{7,2} = 10.0$$

$$x_{6,3} + x_{7,1} + x_{7,2} + x_{7,3} = 10.0$$

$$x_{8,1} + x_{9,1} = 126.6$$

$$x_{8,2} + x_{9,1} + x_{9,2} = 126.6$$

$$x_{8,3} + x_{9,1} + x_{9,2} + x_{9,3} = 126.6$$

$$x_{10,1} + x_{11,1} = 5.6$$

$$x_{10,2} + x_{11,1} + x_{11,2} = 5.6$$

$$x_{10,3} + x_{11,1} + x_{11,2} + x_{11,3} = 5.6$$





These constraints are simply definitional, requiring that the total amount of treated and untreated land of each type be equal to the amount of land of that type in the watershed.

The capital constraints are as follows:

$$f_{2j}(115 - 39 \ln x_{2j})x_{2j} + f_{4j}x_{4j} + f_{5j}(115 - 39 \ln x_{5j})x_{5j} + f_{7j}x_{7j} + f_{9j}x_{9j} + f_{11j}x_{11j} \leq D_j$$

for  $j = 1, 2, \& 3$ . The function  $h_j(x)$  in the above inequality,  $h_j(x) \leq D_j$ , is a convex function which can be verified by noting that the Hessian matrix of  $h_j(x)$  is positive semidefinite for achievable values of  $x_{ij}$ . The parameter  $f_{ij}$  represents the cost of land treatment (in \$1000 per square mile) associated with pair  $i$  of vegetation type and land treatment at the beginning of period  $j$  and  $D_j$  is the or capital available for period  $j$ . This constraints corresponds to an approximate curve-fitting of experimental field data, and reflects the fact that the unit cost of treatment is high for the first few acres. To search for nondominated solutions the algorithm described in this paper was applied to our problem, now formulated as a multiobjective problem with five objective functions and 18 constraints. Associated with each land treatment and vegetation type here is a collection of (given known) data parameters representing water runoff, sediment, wildlife, recreation, and commercial levels over three 10-years periods. A computer program was then using the CUTTING PLANE technique to solve the nonlinear model in the various steps of the algorithm.

Maximization of the individual objective functions yields vector  $Z$  ( see [2] )

$$Z^* = \begin{bmatrix} Z_1^* \\ Z_2^* \\ Z_3^* \\ Z_4^* \\ Z_5^* \end{bmatrix} = \begin{bmatrix} 586.4 * 10^3 \\ 0.0 \\ 0.0 \\ 1200.0 * 10^3 \\ 6627.0 * 10^3 \end{bmatrix}$$

for acre-feet of runoff, acre-feet of sediment, pound-years wildlife, dollars for recreation , and dollars for commercial, respectively. While, our approach yields the following results:

$$Z^e = \begin{bmatrix} Z_1^e \\ Z_2^e \\ Z_3^e \\ Z_4^e \\ Z_5^e \end{bmatrix} = \begin{bmatrix} 605.3 * 10^3 \\ 0.888 \\ 0.0 \\ 1011.2 * 10^3 \\ 6122.7 * 10^3 \end{bmatrix}$$

with convergence  $\bar{d} \%$  is 0.35.

### Discussion and Conclusions:



Although there have been several decades of research in MODM, the reported successful applications are far less than what had been promised. Recent developments in computer technology bring new promises to the applications of MODM models. Specifically, the use of computer graphics may greatly facilitate the process of interactive decision making. As we mentioned before, there is a lack of guidelines in helping users to select the appropriate MODM algorithm. From the results of the problem application and table (1) we found that the use of our model and the procedure is preferred when the accuracy of the solution is the critical factor in selecting an interactive multi – objectives decision making. In any case, the performance of our procedure is quite good.

We are suggested that, further development area is in creating software, which permit the application of such technique in stand – alone decision support system.

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