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Comparison of Bayes' Estimators for the Pareto Type-I Reliability Function Under Different Double Informative Priors Functions

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Abstract

The comparison of double informative priors which are assumed for the reliability function of Pareto type I distribution. To estimate the reliability function of Pareto type I distribution by using Bayes estimation, will be used two different kind of information in the Bayes estimation; two different priors have been selected for the parameter of Pareto type I distribution . Assuming distribution of three double prior's chi- gamma squared distribution, gamma - erlang distribution, and erlang- exponential distribution as double priors. The results of the derivaties of these estimators under the squared error loss function with two different double priors. Using the simulation technique, to compare the performance for each estimator, several cases from pareto type I distribution for data generating, and for different samples sizes (small, medium, and large).

It has been obtained from the simulation results the double prior distribution of gamma-erlang distribution with (a,b,λ) give a good estimation for reliability function $R(t)$ when the true value for $(\alpha = 1 \& 1.5, \theta = 0.5)$ for all t .Also the double prior distribution chi- gamma square distribution with (v, a, b) give good estimation for reliability function $(R(t))$ when the true value $(\alpha = 1, \theta = 1.5) \& (\alpha = 1.5, \theta = 2.5)$ for all t. And the same thing for $(\alpha = 1.5, \theta = 1.5)$ with the values of the parameters (v, a, b) and for all t except t=1.3. It has obtained a good estimation for reliability function ($R(t)$), when the double prior distribution is chi-gamma square distribution with (v, a, b) at the true value for $(\alpha = 1, \theta = 2.5)$ for all t.

Key words: **Pareto type I distribution, reliability function, bayes method, double prior distributions (chi-gamma square distribution. gamma- erlang distribution, erlang exponential distribution, the squared error loss function.**

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1. Introduction

The Pareto type I distribution is a member of continuous probability distributions. It has been introduced by Pareto in (1897). In addition to economics, its models is now being used in a wide range of fields in several different forms such as insurance, business, engineering, survival analysis, reliability and life testing. The Pareto type I distribution is studied as a lifetime testing experiments, the failure times of units placed on the test are not always observed by the experimenter. Some of studies mentioned in a brief manner: In (2003) Nadarajah and Kotzy [5] estimated the reliability function of several Pareto distributions and derived the corresponding forms. Regard the estimation of the reliability $R = Pr(X_2 < X_1)$ when X_1 and X_2 are independent random variables belonging to the same univariate family of distributions .In (2006) Saleh[9] estimated the reliability function of Pareto type I by using bayes estimation, comparing it with different estimation methods and obtaining the results using simulation .In (2007) Howlader and other [4] used natural conjugate and minimal information priors for Bayesian estimation and prediction from Pareto type I distribution to obtain, compare the highest posterior density intervals for the parameter β and also for the associated reliability function. Also, they derived the highest posterior density intervals for a future observation. In (2010) Odat [6] estimated the reliability function on estimation of $p(x > y)$ when x and y are two independent Pareto type I distribution, deriving the maximum likelihood and its asymptotical distribution for the reliability function. In (2014) Basim [2] estimated the reliability function of the Pareto type I distribution by using three methods includes maximum likelihood method, the first modification maximum likelihood method and method of moments. Using simulation to find the best method of estimation this function according to the smallest value of MSE. In (2014) Gazi and Rasheed [8] estimated the reliability function of the Pareto type I distribution under Generalized square error loss function in addition to Quadratic loss function, with informative and non-informative prior, assuming that the scale parameter (α) is known. They used Monte Carlo simulation to obtain the estimators and compared empirically according to the smallest value of the Integral mean squares error (IMSE). In (2015) Usta and Gezer [10] estimated the parameters and reliability function, hazard rate function and mean time system to failure, of Pareto-I distribution based on progressively type-II censored sample with random removals. They assumed that the number of units removed at each failure time follows a binomial distribution. They used the maximum likelihood method to obtain the estimators of parameters and reliability characteristics functions of Pareto-I distribution. They obtained the results by using simulation and compared the performance of maximum likelihood estimates under progressively type-II censoring with the different random schemes.

The Aim of this paper is to obtain bayes Estimators for the reliability function of Pareto type I distribution under different double informative priors. A few studies present in double informative priors, mentioned above: Abdul Haq and Muhammad Aslam [1], they used double prior selection for discrete case in the case of Poisson distribution. Radha and Vekatesan (2015), they studied double prior selection for continuous case in the case of Maxwell distribution [7]. They assumed that generalized uniform-inverted Gamma distribution as double priors.

Studying double prior selection for continuous case in the case of the Pareto type I distribution. Assuming that Chi-squared - Gamma distribution and- Gamma- Erlang distribution as double priors, to find best method to estimate the reliability function of Pareto type I distribution .According to the smallest value of Mean Square Errors (MSE) were calculated to compare the methods of estimation. Several cases from Pareto type I distribution for data generating, of different samples sizes (small, medium, and large). The results were obtained by using simulation technique.

2. The Pareto Type I Distribution

Let us consider $t_1, t_2, \ldots, t_n$ is a random sample of n independent observations from a Pareto type I distribution having the probability density function (pdf) define as :

$$f(t;\theta,\alpha) = \frac{\theta\,\alpha^{\theta}}{t^{(\theta+1)}} \quad , \quad t \ge \alpha,\ \alpha > 0\ ,\ \theta > 0 \qquad \ldots (1)$$

Where α is scale parameter and θ is the shape parameter. So, the cumulative distribution function (cdf) is given as;

$$F(t;\theta,\alpha) = 1 - \left(\frac{\alpha}{t}\right)^{\theta} \quad , \quad t \ge \alpha\ , \quad \alpha, \theta > 0 \qquad \ldots (2)$$

Also, the Reliability function is

$$R(t) = \left(\frac{\alpha}{t}\right)^{\theta} \quad , \quad t \ge \alpha\ , \quad \alpha, \theta > 0 \qquad \ldots (3)$$

3. Bayes Estimation Method

In this section, we used several methods to estimate Reliability function ($R(t)$). Let $(t_1, t_2, \ldots, t_n)$ be a random sample of size n with probability density function given in equation (1) and likelihood function from the Pareto type I pdf given in (1) will be as follows [3]:

$$L(t|\ \theta) = \prod_{i=1}^{n} f(t;\theta) = \prod_{i=1}^{n} \frac{\theta\,\alpha^{\theta}}{t^{(\theta+1)}} = \theta^{n} \alpha^{n\theta} \prod_{i=1}^{n} t_i^{-(\theta+1)}$$

We can rewrite it as follow:

$$L(t|\theta)=\theta^{n}\exp(n\theta\ln(\alpha))\exp(-(\theta+1)\ln\prod_{i=1}^{n}t_i)$$

$$L(t|\theta)=\theta^{n}\exp(n\theta\ln(\alpha))\exp(-(\theta+1)\sum_{i=1}^{n}\ln t_i)$$

$$L(t|\theta)=\theta^{n}\exp(n\theta\ln(\alpha))\exp(-\theta\sum_{i=1}^{n}\ln t_i-\sum_{i=1}^{n}\ln t_i) \qquad ...(4)$$

In this paper the posterior distributions for the unknown parameter θ are derived using the following three types of double informative priors, here we have assumed

1. **Chi-squared – Gamma distribution.**
2. **Gamma- Erlang distribution.**
3. **Exponential - Erlang distribution.**

As double priors, and then get bayes estimation.

3.1 <u>The posterior distribution using different double priors</u>

It is assumed that θ follows four types of prior distributions with pdf as given in table -1:

Table -1: The four types of prior distributions ($f_i(\theta)$) with pdf for θ.

Prior distribution	$f_i(\theta)$, $i=1,2,3,4$
$\theta\sim$ **Chi-square** (v)	$f_1(\theta)=\frac{1}{\Gamma(\frac{v}{2})}(\frac{1}{2})^{-\frac{v}{2}}\theta^{\frac{v}{2}-1}\exp(-\frac{1}{2}\theta)$ for $\theta\ge 0$, $v=1,2,....v$
$\theta\sim$**Gamma** (a,b)	$f_2(\theta)=\frac{b^a}{\Gamma a}\theta^{a-1}\exp(-b\theta)$ for $\theta\ge 0$, $a,b>0$
$\theta\sim$**Erlang** (λ)	$f_3(\theta)=\lambda^2\theta\exp(-\lambda\theta)$ for $\theta\ge 0$, $\lambda>0$
$\theta\sim$ **Exponential** (λ_1)	$f_4(\theta)=\lambda_1\exp(-\lambda_1\theta)$ for $\theta\ge 0$, $\lambda_1>0$

And their double prior's distributions with pdf as given in table -2:

Table -2: The three types of double prior distributions ($P_i(\theta)$) with pdf for θ .

Prior distribution	$P_i(\theta) \quad , i = 1, 2, 3$
$\theta \sim$ **Chi-square** (v) **-** $\theta \sim$**Gamma** (a, b)	$P_1(\theta) \; \alpha \; f_1(\theta) \; f_2(\theta)$ $P_1(\theta) \, \alpha [\frac{1}{\Gamma(\frac{v}{2})} (\frac{1}{2})^{-\frac{v}{2}} \frac{b^a}{\Gamma a}] \, \theta^{\frac{v}{2}+a-2} \exp(-\theta(\frac{1}{2}+b))$
$\theta \sim$**Gamma** (a, b) **-** $\theta \sim$**Erlang** (λ)	$P_2(\theta) \; \alpha \; f_2(\theta) \; f_3(\theta)$ $P_2(\theta) \; \alpha [\frac{b^a}{\Gamma a} \lambda^2] \, \theta^a \exp(-\theta(b+\lambda))$
$\theta \sim$**Erlang** (λ) **-** $\theta \sim$**Exponential** (λ_1)	$P_3(\theta) \; \alpha \; f_3(\theta) \; f_4(\theta)$ $P_3(\theta) \; \alpha [\lambda^2 \; \lambda_1] \, \theta \exp(-\theta(\lambda+\lambda_1))$

Then the posterior distribution of θ for the given the data $\underset{\sim}{t} = (t_1, t_2, \ldots, t_n)$ **is given by:**

$$P(\theta | t) = \frac{L(t|\theta) \; P(\theta)}{\int_{\theta} L(t|\theta) \; P(\theta) d\theta} \qquad \ldots(5)$$

Substituting the equation (4) and for each $P(\theta)$ **as shown in table -2 in equation (5), we get the posterior distributions for the unknown parameter θ are derived using the following three types of double priors (for more details see Appendix-A).**

Table -3: The posterior distributions ($P(\theta \backslash t)$) for the unknown parameter (θ) are derived using the following three types of double priors.

Double prior dist^n.	**The posterior distribution (** $P(\theta | t)$ **)**
$\theta \sim$ **Chi-square** (v)**-** $\theta \sim$**Gamma** (a, b)	$P_1(\theta | t) \sim \text{gamma distribution} \; (a_{(new)} = (n + a + \frac{v}{2} - 1), b_{(new)} = (\sum_{i=1}^{n} \ln t_i - n \ln(\alpha) + \frac{1}{2} + b))$ $P_1(\theta | t) = \frac{(\sum_{i=1}^{n} \ln t_i - n \ln(\alpha) + \frac{1}{2} + b)^{(n+a+\frac{v}{2}-1)}}{\Gamma(n + a + \frac{v}{2} - 1)} \theta^{(n+\frac{v}{2}+a-1)-1} \exp(-\theta(\sum_{i=1}^{n} \ln t_i - n \ln(\alpha) + \frac{1}{2} + b))$ $\alpha, a, n, b, v > 0 , \quad \theta \ge 0$

$\theta \sim$**Gamma** (a, b)- $\theta \sim$**Erlang** (λ)	$P_2(\theta \mid t) \sim \text{gamma distribution}\left(a_{(new)} = (n+a+1),\ b_{(new)} = \left(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + b + \lambda\right)\right)$ $P_2(\theta \mid t) = \frac{\left(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + b + \lambda\right)^{(n+a+1)}}{\Gamma(n+a+1)} \theta^{(n+a+1)-1} \exp\left(-\theta\left(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + b + \lambda\right)\right)$, $\alpha, a, b, n, \lambda > 0 \quad , \quad \theta \geq 0$
$\theta \sim$**Erlang** (λ) - $\theta \sim$**Exponential** (λ_1)	$P_3(\theta \mid t) \sim \text{gamma distribution}\left(a_{(new)} = (n+2),\ b_{(new)} = \left(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1\right)\right)$ $P_3(\theta \mid t) = \frac{\left(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1\right)^{(n+2)}}{\Gamma(n+2)} \theta^{(n+2)-1} \exp\left(-\theta\left(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1\right)\right)$ $\alpha, , n, \lambda, \lambda_1 > 0 \quad , \quad \theta \geq 0$

3.2 Bayes' Estimators

Bayes' estimators for Reliability function (R= $R(t)$**), was considered with two different double priors and under the squared error loss function** $L(\hat{R}, R) = (\hat{R} - R)^2$. **Where** $\hat{R}$ **an estimator for** R**, was considered with two different double priors, and under the squared error loss function. The following is the derivation of these estimators:**

3.2.1 The squared error loss function

To obtain the Bayes' estimator, we minimize the posterior expected loss given by:

$$L(\hat{R}, R) = (\hat{R} - R)^2 \qquad \dots (6)$$

After simplified steps, we get Bayes estimator of $R(t)$ **denoted by** $\hat{R}_i(t)$ **for the above prior as follows**

$$\hat{R}_i(t) = \int_0^{\infty} R(t) P_i(\theta \backslash t) d\theta \quad , \quad i = 1,2,3 \qquad \dots (7)$$

So, the following results are the derivations of these estimators under the squared error loss function with three different double priors (for more details see Appendix-B).

Table -4: The estimators ($\hat{R}_i(t)$) under the squared error loss function With three different double priors.

Double Prior distribution	$\hat{R}_i(t) = \int_0^\infty R(t) P_i(\theta \mid t) d\theta \quad , \quad i = 1,2,3$
$\theta \sim$ **Chi-square** (v)- $\theta \sim$**Gamma** (a, b)	$\hat{R}_{SE1}(t) = \left(\dfrac{\sum_{i=1}^{n} \ln t_i - n \ln(\alpha) + \frac{1}{2} + b}{\sum_{i=1}^{n} \ln t_i - (n+1)\ln(\alpha) + \ln(t) + \frac{1}{2} + b} \right)^{(n+a+\frac{v}{2}-1)}$, $t, \alpha, n, b, a, v > 0$
$\theta \sim$**Gamma** (a, b)- $\theta \sim$**Erlang** (λ)	$\hat{R}_{SE2}(t) = \left(\dfrac{\sum_{i=1}^{n} \ln t_i - n \ln(\alpha) + b + \lambda}{\sum_{i=1}^{n} \ln t_i - (n+1)\ln(\alpha) + \ln(t) + b + \lambda} \right)^{(n+a+1)}$, $t, \alpha, n, b, a, \lambda > 0$
$\theta \sim$**Erlang** (λ) - $\theta \sim$**Exponential** (λ_1)	$\hat{R}_{SE3}(t) = \dfrac{\left(\sum_{i=1}^{n} \ln t_i - n \ln(\alpha) + \lambda + \lambda_1\right)^{(n+2)}}{\left(\sum_{i=1}^{n} \ln t_i - (n+1)\ln(\alpha) + \ln(t) + \lambda + \lambda_1\right)^{(n+2)}}$, $t, \alpha, n, \lambda, \lambda_1 > 0$

4. Simulation Study

In this study, we have generated random samples from a Pareto type I distribution and compared the performance of bayes estimator based on them. So, we have considered several steps to perform simulation study as follow:

1. **We have chosen sample size n = 30, 60, 90 and 120 to represent small, moderate and large sample size.**
2. **We generated data from a Pareto type I distribution for the scale parameter, according to the following cdf** $F(t;\theta,\alpha) = 1-(\frac{\alpha}{t})^{\theta}$ **, by setting** $F_i = u_i$ **where** $u_i \sim$ uniform dist.$^n(0,1)$**, we have** $t_i = \alpha(1-u_i)^{-(1/\theta)}$**.We have considered randomly several values for the shape parameter of Pareto type I distribution** $\theta = 0.5,\ 1.5,\ 2.5$ **and the scale parameter** $\alpha = 1, 1.5$ **.**
3. **We used randomly the values for the parameters of the Chi-square** (v)**-Gamma** (a, b) **distribution (v= 1,3) and (a, b) = (3,2), (3,4) as double prior distribution for** θ**.**

4. **We used randomly the values for the parameters of the Gamma(a , b)-Erlang (λ) distribution (a , b) = (3,2) ,(3,4) and (λ =4,6) as double prior distribution for θ.**
5. **We used randomly the values for the parameters of the Erlang (λ) - Exponential (λ_1) distribution (λ =3, 4) and (λ_1 =0.5, 1) as double prior distribution for θ.**
6. **The true $R(t)$ is computed according to the formula (3) with $\theta = 0.5,\ 1.5,\ 2.5$ and the true t is $t = 1.3, 1.6, 1.9, 2.1, 2.4$.**
7. **The number of replication used was ($r = 1000$) for each sample size (n).**

We obtained estimators for reliability function ($R(t)$), the estimators in the table-4 in section (3.2.1), it means the estimators $\hat{R}_i(t)$ under the squared error loss function with three different double priors .The simulation program was written by using MATLAB-R2008a program .After the reliability function ($R(t)$), was estimated, Mean Square Errors (MSE) was calculated to compare between the bayes estimators, So we have the following criterion.As the following stepes:

1. **We generated data t_i for each sample size, let n=30.**
2. **Then computed $\sum_{i=1}^{n} \ln t_i$.**
3. **computed $R(t)$ according to the formula (3) at $t = 1.3$ for L=1.**
4. **computed $\hat{R}_1(t)$ according to the formula in table (4) at $t = 1.3$ for L=1.**
5. **Computed $(\hat{R}_L(t) - R(t))^2$ for L=1.**
6. **The number of replication used for ($L = 1000$) for each sample size (n=30).**
7. **Then computed the $MSE(\hat{R}(t))$ according to the formula (8) as shown bellows.**

$$MSE(\hat{R}(t)) = \frac{1}{L}\sum_{L=1}^{1000} (\hat{R}_L(t) - R(t))^2 \qquad ...(8)$$

See appendix-C for the programs algorithm .The results of the simulation study are summarized and tabulated in tables (4-1) - (4-9).In each row of tables (4-1) - (4-9) ,we have four estimated values for $R(t)$ $\hat{R}_i(t)$ with MSE for all samples sizes (n) and values (v , a , b, λ & λ_1) respectively. By using bayes estimation method under the squared error loss function with three different double priors. So, our criterion is the best method that gives the smallest value of (MSE). We list the results in the following tables (4-1) - (4-9).

4-1: MSE of estimated Pareto type I reliability function using Bayes with respect to MSE($\hat{R}(t)$)

when double prior distribution is (Chi-square (v) -Gamma (a, b)) distribution.

parameters						R(t)	$\hat{R}(t)$				MSE($\hat{R}(t)$)			
							Sample Size(n)				Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	v	a	b		($P_1(\theta \mid x)$) when double prior distribution is (Chi-square(v) -Gamma (a ,b)) dist^n.							
1	0.5	1.3	1	3	2	0.87706	0.86971	0.87336	0.87423	0.87532	0.00049	0.00024	0.00015	0.00011
		1.6				0.79057	0.7794	0.78495	0.78623	0.78793	0.00122	0.00063	0.00040	0.00028
		1.9				0.72548	0.71218	0.71879	0.72027	0.72233	0.00185	0.00097	0.00062	0.00044
		2.1				0.69007	0.67589	0.68294	0.68448	0.68671	0.00220	0.00117	0.00075	0.00053
		2.4				0.6455	0.63049	0.63796	0.63955	0.64195	0.00261	0.00140	0.00090	0.00064
1	0.5	1.3	1	3	4	0.87706	0.87366	0.87532	0.87553	0.87629	0.00039	0.00022	0.00014	0.00010
		1.6				0.79057	0.78569	0.78808	0.78832	0.78948	0.00099	0.00057	0.00037	0.00027
		1.9				0.72548	0.71997	0.72269	0.72287	0.72427	0.00153	0.00089	0.00058	0.00042
		2.1				0.69007	0.68441	0.68721	0.68734	0.68884	0.00183	0.00106	0.00070	0.00051
		2.4				0.6455	0.63982	0.64265	0.6427	0.6443	0.00219	0.00129	0.00084	0.00061
1	0.5	1.3	3	3	2	0.87706	0.86599	0.87148	0.87296	0.87437	0.00057	0.00027	0.00016	0.00011
		1.6				0.79057	0.77347	0.78192	0.78419	0.7864	0.00143	0.00069	0.00043	0.00029
		1.9				0.72548	0.70482	0.71501	0.71772	0.72042	0.00217	0.00106	0.00066	0.00046
		2.1				0.69007	0.62167	0.6788	0.68169	0.68461	0.00303	0.00126	0.00079	0.00055
		2.4				0.6455	0.62167	0.63341	0.63648	0.63964	0.00303	0.00151	0.00096	0.00067
1	0.5	1.3	3	3	4	0.87706	0.87004	0.87345	0.87427	0.87534	0.00045	0.00023	0.00015	0.00010
		1.6				0.79057	0.7799	0.78509	0.78629	0.78796	0.00113	0.00061	0.00039	0.00027
		1.9				0.72548	0.71277	0.71895	0.72035	0.72237	0.00172	0.00094	0.00061	0.00043
		2.1				0.69007	0.67651	0.68311	0.68457	0.68675	0.00204	0.00112	0.00073	0.00052
		2.4				0.6455	0.63115	0.63814	0.63964	0.642	0.00243	0.00135	0.00088	0.00063

Continue for Table 4-1

4-2: MSE of estimated Pareto type I reliability function using Bayes with respect to MSE($\hat{R}(t)$)
When double prior distribution is (Gamma (a, b) - Erlang (λ)) distribution.

parameters						R(t)	$\hat{R}(t)$				MSE($\hat{R}(t)$)			
							Sample Size(n)				Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	a	b	λ		($P_2(\theta \mid x)$) when double prior distribution is (Gamma(a ,b) - Erlang (λ)) dist^n.							
1	0.5	1.3	3	2	4	0.87706	0.87116	0.87398	0.87462	0.87559	0.00041	0.00022	0.00014	0.00010
		1.6				0.79057	0.78166	0.78592	0.78684	0.78836	0.00103	0.00058	0.00038	0.00027
		1.9				0.72548	0.71493	0.71999	0.72103	0.72287	0.00158	0.00090	0.00059	0.00042
		2.1				0.69007	0.67887	0.68424	0.68532	0.6873	0.00188	0.00108	0.00071	0.00051
		2.4				0.6455	0.63371	0.63938	0.64046	0.6426	0.00224	0.00130	0.00086	0.00062
1	0.5	1.3	3	2	6	0.87706	0.87485	0.87586	0.87589	0.87654	0.00033	0.00020	0.00013	0.00010
		1.6				0.79057	0.78754	0.78895	0.78889	0.78989	0.00086	0.00053	0.00035	0.00026
		1.9				0.72548	0.72224	0.72376	0.72358	0.72478	0.00133	0.00083	0.00055	0.00040
		2.1				0.69007	0.68686	0.68838	0.68812	0.6894	0.00159	0.00099	0.00066	0.00049
		2.4				0.6455	0.64248	0.64393	0.64354	0.64491	0.00192	0.00120	0.00081	0.00059
1	0.5	1.3	4	3	4	0.87706	0.86956	0.8731	0.87402	0.87513	0.00042	0.00023	0.00015	0.00010
		1.6				0.79057	0.77908	0.78452	0.78588	0.78762	0.00107	0.00059	0.00038	0.00027
		1.9				0.72548	0.71171	0.71823	0.71982	0.72194	0.00163	0.00092	0.00060	0.00043
		2.1				0.69007	0.67533	0.68231	0.68399	0.68629	0.00194	0.00110	0.00072	0.00051
		2.4				0.6455	0.62982	0.63725	0.639	0.64148	0.00231	0.00132	0.00087	0.00062
1	0.5	1.3	4	3	6	0.87706	0.87323	0.87499	0.87529	0.87608	0.00034	0.00020	0.00013	0.00010
		1.6				0.79057	0.78493	0.78753	0.78792	0.78915	0.00087	0.00053	0.00035	0.00026
		1.9				0.72548	0.71897	0.72199	0.72237	0.72385	0.00135	0.00083	0.00055	0.00041
		2.1				0.69007	0.68327	0.68643	0.68678	0.68838	0.00161	0.00100	0.00067	0.00049
		2.4				0.6455	0.63851	0.64178	0.64207	0.64378	0.00193	0.00120	0.00081	0.00060

parameters						$R(t)$	$(\hat{R}(t))$ Sample Size(n)				$MSE(\hat{R}(t))$ Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	v	a	b		$(P_1(\theta \mid x))$ when double prior distribution is (Chi-square(v) -Gamma(a ,b)) dist^n.							
1.5	0.5	1.3	1	3	2	-	-	-	-	-	-	-	-	-
		1.6				0.96825	0.96614	0.96702	0.96758	0.96779	$3.7554e^{-5}$	$1.8939e^{-5}$	$1.0926e^{-5}$	$8.7861e^{-6}$
		1.9				0.88852	0.88177	0.88456	0.88638	0.88708	0.00040	0.00020	0.00012	$9.8354e^{-5}$
		2.1				0.84515	0.83627	0.83992	0.84233	0.84326	0.00073	0.00037	0.00022	0.00017
		2.4				0.79057	0.7794	0.78395	0.78702	0.7882	0.00122	0.00063	0.00037	0.00030
1.5	0.5	1.3	1	3	4	-	-	-	-	-	-	-	-	-
		1.6				0.96825	0.96722	0.96756	0.96793	0.96805	$2.9969e^{-5}$	$1.6777e^{-5}$	$1.0128e^{-5}$	$8.3383e^{-6}$
		1.9				0.88852	0.88538	0.88636	0.88756	0.88796	0.00032	0.00018	0.00011	$9.3602e^{-5}$
		2.1				0.84515	0.84112	0.84234	0.84393	0.84445	0.00059	0.00033	0.00020	0.00017
		2.4				0.79057	0.78569	0.78711	0.7891	0.78975	0.00099	0.00057	0.00035	0.00029
1.5	0.5	1.3	3	3	2	-	-	-	-	-	-	-	-	-
		1.6				0.96825	0.96511	0.9665	0.96723	0.96753	$4.4905e^{-5}$	$2.1024e^{-5}$	$1.1725e^{-5}$	$9.2233e^{-6}$
		1.9				0.88852	0.87837	0.88283	0.88523	0.88621	0.00048	0.00023	0.00013	0.00010
		2.1				0.84515	0.83169	0.83758	0.84077	0.84209	0.00086	0.00041	0.00023	0.00018
		2.4				0.79057	0.77347	0.78091	0.78499	0.78667	0.00143	0.00070	0.00039	0.00031
1.5	0.5	1.3	3	3	4	-	-	-	-	-	-	-	-	-
		1.6				0.96825	0.96623	0.96705	0.96759	0.9678	$3.4715e^{-5}$	$1.8249e^{-5}$	$1.067e^{-5}$	$8.6335e^{-6}$
		1.9				0.88852	0.88207	0.88465	0.88642	0.8871	0.00037	0.00020	0.00011	$9.6659e^{-5}$
		2.1				0.84515	0.83667	0.84004	0.84238	0.84328	0.00068	0.00036	0.00021	0.00017
		2.4				0.79057	0.7799	0.7841	0.78708	0.78823	0.00113	0.00061	0.00036	0.00029

Note: When double prior distribution is (Chi- square (v) –Gamma (a, b)) dist^n., at t=1.3, we obtained

un suitable value for $R(t)$ & $\hat{R}(t)$, which is more than one. So, we put dash (-).

Continue for Table 4-2

parameters						$R(t)$	$(\hat{R}(t))$ Sample Size(n)				$MSE(\hat{R}(t))$ Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	a	b	λ		$(P_2(\theta \mid x))$ when double prior distribution is (Gamma(a ,b) - Erlang (λ)) dist^n.							
1.5	0.5	1.3	3	2	4	-	-	-	-	-	-	-	-	-
		1.6				0.96825	0.96654	0.96719	0.96768	0.96787	$3.1511e^{-5}$	$1.7353e^{-5}$	$1.033e^{-5}$	$8.437e^{-6}$
		1.9				0.88852	0.88309	0.88514	0.88672	0.88732	0.00034	0.00019	0.00011	$9.4528e^{-5}$
		2.1				0.84515	0.83803	0.84069	0.8428	0.84359	0.00062	0.00034	0.00021	0.00017
		2.4				0.79057	0.78166	0.78495	0.78762	0.78863	0.00103	0.00058	0.00035	0.00029
1.5	0.5	1.3	3	2	6	-	-	-	-	-	-	-	-	-
		1.6				0.96825	0.96756	0.96771	0.96803	0.96812	$2.5685e^{-5}$	$1.5507e^{-5}$	$9.6367e^{-6}$	$8.0423e^{-6}$
		1.9				0.88852	0.88647	0.88688	0.88788	0.88819	0.00028	0.00017	0.00010	$9.0359e^{-5}$
		2.1				0.84515	0.84257	0.84304	0.84435	0.84476	0.00051	0.00031	0.00019	0.00016
		2.4				0.79057	0.78754	0.788	0.78965	0.79016	0.00086	0.00053	0.00033	0.00028
1.5	0.5	1.3	4	3	4	-	-	-	-	-	-	-	-	-
		1.6				0.96825	0.96611	0.96696	0.96752	0.96774	$3.2965e^{-5}$	$1.7867e^{-5}$	$1.0529e^{-5}$	$8.5404e^{-6}$
		1.9				0.88852	0.88163	0.88434	0.88618	0.88691	0.00035	0.00019	0.00011	$9.557e^{-5}$
		2.1				0.84515	0.83605	0.83962	0.84205	0.84303	0.00064	0.00035	0.00021	0.00017
		2.4				0.79057	0.77908	0.78355	0.78665	0.78789	0.00107	0.00060	0.00036	0.00029
1.5	0.5	1.3	4	3	6	-	-	-	-	-	-	-	-	-
		1.6				0.96825	0.96712	0.96747	0.96786	0.968	$2.6343e^{-5}$	$1.5786e^{-5}$	$9.7263e^{-6}$	$8.0829e^{-6}$
		1.9				0.88852	0.88499	0.88608	0.88733	0.88777	0.00029	0.00017	0.00010	$9.0696e^{-5}$
		2.1				0.84515	0.84057	0.84196	0.84361	0.84419	0.00052	0.00031	0.00019	0.00016
		2.4				0.79057	0.78493	0.78659	0.78868	0.78942	0.00087	0.00053	0.00033	0.00028

Note: When double prior distribution is ((Gamma (a, b) - Erlang (λ)) distn., at t=1.3, we obtained

un suitable value for $R(t)$ & $\hat{R}(t)$, which is more than one. So, we put dash (-).

4-3: MSE of estimated Pareto type I reliability function using Bayes with respect to $MSE(\hat{R}(t))$ when double prior distribution is (Erlang (λ)-Exponential (λ_1)) distribution.

Parameters					$R(t)$	$(\hat{R}(t))$ Sample Size(n)				$MSE(\hat{R}(t))$ Sample Size(n)			
						30	60	90	120	30	60	90	120
α	θ	t	λ	λ_1		$(P_3(\theta \mid x))$ when double prior distribution is (Erlang (λ)-Exponential (λ_1)) dist^n.							
1	0.5	1.3	3	0.5	0.87706	0.87356	0.87529	0.87552	0.87628	0.00040	0.00022	0.00014	0.00010
		1.6			0.79057	0.78554	0.78804	0.78829	0.78947	0.00103	0.00058	0.00037	0.00027
		1.9			0.72548	0.7198	0.72264	0.72285	0.72426	0.00158	0.00090	0.00058	0.00042
		2.1			0.69007	0.68423	0.68716	0.68732	0.68883	0.00189	0.00108	0.00070	0.00051
		2.4			0.6455	0.63964	0.6426	0.64267	0.64429	0.00227	0.00131	0.00085	0.00062
1	0.5	1.3	3	1	0.87706	0.87452	0.87577	0.87584	0.87652	0.00039	0.00022	0.00014	0.00010
		1.6			0.79057	0.78708	0.78881	0.78881	0.78985	0.00099	0.00057	0.00037	0.00027
		1.9			0.72548	0.72172	0.7236	0.7235	0.72474	0.00152	0.00088	0.00058	0.00042
		2.1			0.69007	0.68632	0.68822	0.68803	0.68936	0.00182	0.00106	0.00069	0.00051
		2.4			0.6455	0.64194	0.64377	0.64345	0.64487	0.00219	0.00129	0.00084	0.00061
1	0.5	1.3	4	0.5	0.87706	0.87547	0.87625	0.87616	0.87676	0.00037	0.00021	0.00014	0.00010
		1.6			0.79057	0.7886	0.78958	0.78933	0.79024	0.00095	0.00056	0.00036	0.00026
		1.9			0.72548	0.72361	0.72456	0.72414	0.72522	0.00147	0.00087	0.00057	0.00041
		2.1			0.69007	0.68839	0.68927	0.68873	0.68989	0.00176	0.00105	0.00069	0.00050
		2.4			0.6455	0.64421	0.64492	0.64423	0.64545	0.00212	0.00127	0.00083	0.00061
1	0.5	1.3	4	1	0.87706	0.87641	0.87672	0.87648	0.877	0.00036	0.00021	0.00014	0.00010
		1.6			0.79057	0.7901	0.79034	0.78984	0.79062	0.00092	0.00055	0.00036	0.00026
		1.9			0.72548	0.72547	0.72551	0.72478	0.7257	0.00143	0.00086	0.00056	0.00041
		2.1			0.69007	0.69043	0.69031	0.68944	0.69042	0.00171	0.00103	0.00068	0.00050
		2.4			0.6455	0.64645	0.64607	0.64501	0.64604	0.00207	0.00125	0.00082	0.00061

Continue for Table 4-3

4-4: MSE of estimated Pareto type I reliability function using Bayes with respect to $MSE(\hat{R}(t))$ when double prior distribution is (Chi-square(v) -Gamma (a, b)) distribution.

parameters						$R(t)$	$(\hat{R}(t))$ Sample Size(n)				$MSE(\hat{R}(t))$ Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	v	a	b		$(P_1(\theta \mid x))$ when double prior distribution is (Chi-square(v) -Gamma(a ,b)) dist^n.							
1	1.5	1.3	1	3	2	0.67466	0.67949	0.67688	0.67732	0.67709	0.00182	0.00104	0.00067	0.00057
		1.6				0.49411	0.50346	0.49868	0.49869	0.49817	0.00314	0.00178	0.00116	0.00098
		1.9				0.38183	0.39432	0.38809	0.38762	0.38689	0.00355	0.00199	0.00130	0.00110
		2.1				0.3286	0.34254	0.33567	0.33492	0.33407	0.00356	0.00197	0.00129	0.00109
		2.4				0.26896	0.28438	0.27685	0.27577	0.27481	0.00341	0.00186	0.001224	0.00102
1	1.5	1.3	1	3	4	0.67466	0.70197	0.68909	0.68562	0.68341	0.00209	0.00110	0.00072	0.00059
		1.6				0.49411	0.53298	0.51469	0.5096	0.50646	0.00393	0.00198	0.00129	0.00106
		1.9				0.38183	0.42562	0.40502	0.39916	0.39565	0.00473	0.00230	0.00148	0.00121
		2.1				0.3286	0.37381	0.35254	0.34642	0.34281	0.00491	0.00234	0.00150	0.00122
		2.4				0.26896	0.3148	0.2932	0.28692	0.28327	0.00488	0.00227	0.00144	0.00116
1	1.5	1.3	3	3	2	0.67466	0.6715	0.67268	0.67448	0.67494	0.00187	0.00106	0.00067	0.00057
		1.6				0.49411	0.49304	0.49319	0.49496	0.49535	0.00311	0.00178	0.00115	0.00097
		1.9				0.38183	0.38332	0.3823	0.38369	0.38391	0.00341	0.00195	0.00127	0.00107
		2.1				0.3286	0.33159	0.3299	0.331	0.33111	0.00336	0.00192	0.00126	0.00106
		2.4				0.26896	0.27376	0.27127	0.27198	0.27195	0.00314	0.00178	0.00117	0.00099
1	1.5	1.3	3	3	4	0.67466	0.6944	0.68501	0.68284	0.68129	0.00179	0.00101	0.00067	0.00057
		1.6				0.49411	0.52283	0.50927	0.50591	0.50367	0.00330	0.00180	0.00119	0.00100
		1.9				0.38183	0.41468	0.39924	0.39524	0.39268	0.00392	0.00207	0.00136	0.00114
		2.1				0.3286	0.36279	0.34674	0.34249	0.33984	0.00403	0.00209	0.00137	0.00114
		2.4				0.26896	0.30394	0.28755	0.28309	0.28038	0.00398	0.00201	0.00131	0.00108

Parameters					$R(t)$	$(\hat{R}(t))$ Sample Size(n)				$MSE(\hat{R}(t))$ Sample Size(n)			
						30	60	90	120	30	60	90	120
α	θ	t	λ	λ_1		$(P_3(\theta \mid x))$ when double prior distribution is (Erlang (λ)-Exponential (λ_1)) dist^n.							
1.5	0.5	1.3	3	0.5	-	-	-	-	-	-	-	-	-
		1.6			0.96825	0.96719	0.96755	0.96793	0.96805	$3.1122e^{-5}$	$1.7084e^{-5}$	$1.0246e^{-5}$	$8.4105e^{-6}$
		1.9			0.88852	0.88528	0.88633	0.88755	0.88795	0.00034	0.00019	0.00011	$9.4407e^{-5}$
		2.1			0.84515	0.841	0.8423	0.84391	0.84444	0.00061	0.00034	0.00020	0.00017
		2.4			0.79057	0.78554	0.78706	0.78908	0.78974	0.00103	0.00058	0.00035	0.00029
1.5	0.5	1.3	3	1	-	-	-	-	-	-	-	-	-
		1.6			0.96825	0.96746	0.96768	0.96801	0.96812	$2.9642e^{-5}$	$1.6641e^{-5}$	$1.0087e^{-5}$	$8.3208e^{-6}$
		1.9			0.88852	0.88617	0.88677	0.88784	0.88817	0.00032	0.00018	0.00011	$9.3468e^{-5}$
		2.1			0.84515	0.84219	0.8429	0.84431	0.84474	0.00059	0.00033	0.00020	0.00017
		2.4			0.79057	0.78708	0.78784	0.78959	0.79013	0.00099	0.00056	0.00035	0.00029
1.5	0.5	1.3	4	0.5	-	-	-	-	-	-	-	-	-
		1.6			0.96825	0.96772	0.96781	0.9681	0.96818	$2.8349e^{-5}$	$1.6239e^{-5}$	$9.9449e^{-6}$	$8.2403e^{-6}$
		1.9			0.88852	0.88704	0.88721	0.88813	0.88839	0.00031	0.00018	0.00011	$9.2632e^{-5}$
		2.1			0.84515	0.84336	0.8435	0.8447	0.84503	0.00056	0.00032	0.00020	0.00016
		2.4			0.79057	0.7886	0.78861	0.79011	0.79051	0.00095	0.00055	0.00034	0.00028
1.5	0.5	1.3	4	1	-	-	-	-	-	-	-	-	-
		1.6			0.96825	0.96798	0.96794	0.96819	0.96825	$2.7234e^{-5}$	$1.588e^{-5}$	$9.8192e^{-6}$	$8.1691e^{-6}$
		1.9			0.88852	0.88789	0.88765	0.88842	0.88861	0.00030	0.00017	0.00011	$9.1898e^{-5}$
		2.1			0.84515	0.84451	0.84409	0.84509	0.84532	0.00054	0.00032	0.00020	0.00016
		2.4			0.79057	0.7901	0.78938	0.79061	0.79089	0.00092	0.00054	0.00034	0.00028

Note: When double prior distribution is (Erlang (λ)-Exponential (λ_1)) dist^n., at t=1.3, we obtained

un suitable value for $R(t)$ & $\hat{R}(t)$, which is more than one. So, we put dash (-).

Continue for Table 4-4

parameters						$R(t)$	$(\hat{R}(t))$				$MSE(\hat{R}(t))$			
							Sample Size(n)				Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	v	a	b		$(P_1(\theta \mid x))$ when double prior distribution is (Chi-square(v) -Gamma(a ,b)) dist^n.							
1.5	1.5	1.3	1	3	2	-	-	-	-	-	-	-	-	-
		1.6				0.90773	0.90859	0.90833	0.90833	0.90812	0.00020	0.00011	7.8052e-5	6.0992e-5
		1.9				0.70147	0.70571	0.70413	0.70382	0.70308	0.00160	0.00090	0.00062	0.00048
		2.1				0.60368	0.61017	0.60764	0.60705	0.60602	0.00239	0.00135	0.00093	0.00073
		2.4				0.49411	0.50346	0.49969	0.49868	0.49734	0.00314	0.00176	0.00123	0.00095
1.5	1.5	1.3	1	3	4	-	-	-	-	-	-	-	-	-
		1.6				0.90773	0.91606	0.91236	0.91108	0.91022	0.00021	0.00011	8.0004e-5	6.1648e-5
		1.9				0.70147	0.72678	0.71552	0.71161	0.70902	0.00182	0.00096	0.00066	0.00050
		2.1				0.60368	0.63596	0.62159	0.61658	0.6133	0.00284	0.00148	0.00100	0.00076
		2.4				0.49411	0.53298	0.51564	0.50959	0.50566	0.00393	0.00199	0.00135	0.00102
1.5	1.5	1.3	3	3	2	-	-	-	-	-	-	-	-	-
		1.6				0.90773	0.90592	0.90693	0.90738	0.90741	0.00021	0.00011	7.9336e-5	6.1845e-5
		1.9				0.70147	0.69822	0.7002	0.70116	0.70106	0.00165	0.00091	0.00062	0.00048
		2.1				0.60368	0.60102	0.60284	0.60379	0.60356	0.00243	0.00135	0.00093	0.00073
		2.4				0.49411	0.49304	0.49421	0.49495	0.49452	0.00311	0.00174	0.00122	0.00095
1.5	1.5	1.3	3	3	4	-	-	-	-	-	-	-	-	-
		1.6				0.90773	0.9136	0.91102	0.91016	0.90952	0.00018	0.00010	7.6055e-5	5.9481e-5
		1.9				0.70147	0.71971	0.71171	0.709	0.70703	0.00156	0.00088	0.00062	0.00048
		2.1				0.60368	0.62721	0.61689	0.61337	0.61086	0.00241	0.00135	0.00094	0.00073
		2.4				0.49411	0.52283	0.51023	0.5059	0.50286	0.00330	0.00181	0.00126	0.00096

Note: When double prior distribution is (Chi- square (v) –Gamma (a, b)) dist^n., at t=1.3, we obtained

un suitable value for $R(t)$ **&** $\hat{R}(t)$**, which is more than one. So, we put dash (-).**

4-5: MSE of estimated Pareto type I reliability function using Bayes with respect to $MSE(\hat{R}(t))$ **when double prior distribution is (Gamma (a, b) - Erlang (λ)) distribution.**

parameters						R(t)	$(\hat{R}(t))$				$MSE(\hat{R}(t))$			
							Sample Size(n)				Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	a	b	λ		$(P_2(\theta \mid x))$ when double prior distribution is (Gamma(a ,b) - Erlang (λ)) dist^n.							
1	1.5	1.3	3	2	4	0.67466	0.70597	0.69168	0.68746	0.68484	0.00215	0.00111	0.00073	0.00060
		1.6				0.49411	0.53819	0.51806	0.51201	0.50835	0.00407	0.00202	0.00132	0.00107
		1.9				0.38183	0.43108	0.40857	0.4017	0.39764	0.00492	0.00236	0.00152	0.00123
		2.1				0.3286	0.37924	0.35606	0.34895	0.34479	0.00511	0.00241	0.00154	0.00124
		2.4				0.26896	0.32003	0.2966	0.28936	0.28518	0.00510	0.00234	0.00149	0.00119
1	1.5	1.3	3	2	6	0.67466	0.7242	0.70256	0.69514	0.69079	0.00336	0.00149	0.00093	0.00072
		1.6				0.49411	0.56289	0.53259	0.52221	0.51623	0.00645	0.00277	0.00171	0.00131
		1.9				0.38183	0.45793	0.42415	0.41261	0.40604	0.00788	0.00329	0.00200	0.00152
		2.1				0.3286	0.40644	0.37172	0.35988	0.35319	0.00824	0.00338	0.00205	0.00154
		2.4				0.26896	0.34699	0.31194	0.30003	0.29337	0.00827	0.00332	0.00199	0.00149
1	1.5	1.3	4	3	4	0.67466	0.70839	0.69331	0.68864	0.68578	0.00220	0.00112	0.00074	0.00061
		1.6				0.49411	0.54134	0.5202	0.51356	0.50957	0.00419	0.00206	0.00134	0.00109
		1.9				0.38183	0.43441	0.41082	0.40334	0.39893	0.00507	0.00242	0.00156	0.00125
		2.1				0.3286	0.38254	0.3583	0.35058	0.34607	0.00527	0.00247	0.00158	0.00126
		2.4				0.26896	0.32323	0.29877	0.29094	0.28642	0.00526	0.00241	0.00153	0.00121
1	1.5	1.3	4	3	6	0.67466	0.72583	0.70392	0.69619	0.69164	0.00345	0.00153	0.00096	0.00074
		1.6				0.49411	0.56505	0.53439	0.5236	0.51735	0.00663	0.00285	0.00176	0.00134
		1.9				0.38183	0.46025	0.42606	0.41408	0.40723	0.00809	0.00339	0.00207	0.00156
		2.1				0.3286	0.40876	0.37363	0.36135	0.35438	0.00845	0.00349	0.00211	0.00158
		2.4				0.26896	0.34926	0.31381	0.30147	0.29452	0.00849	0.00343	0.00205	0.00153

Continue for Table 4-5

parameters						R(t)	$(\hat{R}(t))$				$MSE(\hat{R}(t))$			
							Sample Size(n)				Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	a	b	λ		$(P_2(\theta \mid x))$ when double prior distribution is (Gamma(a ,b) - Erlang (λ)) dist^n.							
1.5	1.5	1.3	3	2	4	-	-	-	-	-	-	-	-	-
		1.6				0.90773	0.91741	0.91321	0.91169	0.9107	0.00021	0.00011	$8.0492e^{-5}$	$6.1783e^{-5}$
		1.9				0.70147	0.73054	0.71792	0.71334	0.71038	0.00186	0.00097	0.00066	0.00050
		2.1				0.60368	0.64054	0.62451	0.61869	0.61496	0.00292	0.00150	0.00102	0.00077
		2.4				0.49411	0.53819	0.51898	0.512	0.50756	0.00407	0.00204	0.00138	0.00103
1.5	1.5	1.3	3	2	6	-	-	-	-	-	-	-	-	-
		1.6				0.90773	0.92327	0.91675	0.91421	0.91266	0.00033	0.00015	$9.9699e^{-5}$	$7.2766e^{-5}$
		1.9				0.70147	0.74755	0.72805	0.72052	0.71596	0.00291	0.00131	0.00084	0.00060
		2.1				0.60368	0.66169	0.63703	0.62755	0.62183	0.00460	0.00205	0.00129	0.00093
		2.4				0.49411	0.56289	0.53346	0.52221	0.51546	0.00645	0.00281	0.00176	0.00125
1.5	1.5	1.3	4	3	4	-	-	-	-	-	-	-	-	-
		1.6				0.90773	0.91821	0.91375	0.91209	0.91101	0.00022	0.00011	$8.1284e^{-5}$	$6.2159e^{-5}$
		1.9				0.70147	0.7328	0.71943	0.71445	0.71126	0.00191	0.00099	0.00067	0.00051
		2.1				0.60368	0.6433	0.62636	0.62005	0.61604	0.00300	0.00153	0.00103	0.00078
		2.4				0.49411	0.54134	0.52109	0.51355	0.50879	0.00419	0.00208	0.00140	0.00104
1.5	1.5	1.3	4	3	6	-	-	-	-	-	-	-	-	-
		1.6				0.90773	0.92381	0.91719	0.91455	0.91294	0.00034	0.00015	0.00010	$7.4244e^{-5}$
		1.9				0.70147	0.74908	0.72931	0.7215	0.71677	0.00299	0.00135	0.00086	0.00062
		2.1				0.60368	0.66357	0.63857	0.62876	0.62282	0.00473	0.00211	0.00133	0.00095
		2.4				0.49411	0.56505	0.53524	0.52359	0.51659	0.00663	0.00290	0.00181	0.00128

Note: When double prior distribution is ((Gamma (a, b) - Erlang (λ))dist^n., at t=1.3, we obtained

un suitable value for $R(t)$ & $\hat{R}(t)$, which is more than one. So, we put dash (-).

4-6: MSE of estimated Pareto type I reliability function using Bayes with respect to $MSE(\hat{R}(t))$ when double prior distribution is (Erlang (λ)-Exponential (λ_1)) distribution.

Parameters					$R(t)$	$(\hat{R}(t))$ Sample Size(n)				$MSE(\hat{R}(t))$ Sample Size(n)			
						30	60	90	120	30	60	90	120
α	θ	t	λ	λ_1		$(P_3(\theta \mid x))$ when double prior distribution is (Erlang (λ)-Exponential (λ_1)) dist^n.							
1	1.5	1.3	3	0.5	0.67466	0.69506	0.68518	0.68293	0.68135	0.00194	0.00106	0.00069	0.00058
		1.6			0.49411	0.52387	0.50956	0.50606	0.50376	0.00357	0.00188	0.00123	0.00102
		1.9			0.38183	0.41594	0.39959	0.39542	0.39279	0.00424	0.00217	0.00141	0.00116
		2.1			0.3286	0.36413	0.34712	0.34269	0.33996	0.00437	0.00219	0.00142	0.00117
		2.4			0.26896	0.30536	0.28794	0.2833	0.28051	0.00432	0.00211	0.00135	0.00111
1	1.5	1.3	3	1	0.67466	0.70052	0.68819	0.68499	0.68292	0.00209	0.00110	0.00072	0.00059
		1.6			0.49411	0.53111	0.51352	0.50877	0.50582	0.00391	0.00197	0.00128	0.00105
		1.9			0.38183	0.42366	0.40379	0.39829	0.39498	0.00469	0.00229	0.00147	0.00120
		2.1			0.3286	0.37187	0.35131	0.34556	0.34214	0.00486	0.0023	0.00149	0.00121
		2.4			0.26896	0.31294	0.29202	0.28608	0.28262	0.00483	0.00225	0.00143	0.00115
1	1.5	1.3	4	0.5	0.67466	0.70579	0.69114	0.68702	0.68447	0.00229	0.00115	0.00075	0.00061
		1.6			0.49411	0.53813	0.51742	0.51145	0.50787	0.00433	0.00209	0.00134	0.00109
		1.9			0.38183	0.4312	0.40794	0.40114	0.39715	0.00524	0.00244	0.00156	0.00125
		2.1			0.3286	0.37946	0.35547	0.3484	0.3443	0.00545	0.00249	0.00158	0.00126
		2.4			0.26896	0.32038	0.29607	0.28885	0.28473	0.00544	0.00242	0.00152	0.00121
1	1.5	1.3	4	1	0.67466	0.71087	0.69404	0.68902	0.686	0.00254	0.00122	0.00079	0.00063
		1.6			0.49411	0.54496	0.52126	0.51411	0.5099	0.00485	0.00224	0.00142	0.00114
		1.9			0.38183	0.43856	0.41204	0.40397	0.3993	0.00589	0.00263	0.00165	0.00131
		2.1			0.3286	0.38689	0.35958	0.35123	0.34646	0.00615	0.00269	0.00168	0.00132
		2.4			0.26896	0.32771	0.30008	0.2916	0.28682	0.00615	0.00263	0.00162	0.00127

Continue for Table 4-6

Parameters					$R(t)$	$(\hat{R}(t))$				$MSE(\hat{R}(t))$			
						Sample Size(n)				Sample Size(n)			
						30	60	90	120	30	60	90	120
α	θ	t	λ	λ_1		$(P_3(\theta \mid x))$ when double prior distribution is (Erlang (λ)-Exponential (λ_1)) $dist^n$.							
1.5	1.5	1.3	3	0.5	-	-	-	-	-	-	-	-	-
		1.6			0.90773	0.91377	0.91107	0.91019	0.90953	0.00020	0.00011	$7.8405e^{-5}$	$6.0897e^{-5}$
		1.9			0.70147	0.72031	0.71188	0.70909	0.70708	0.00169	0.00092	0.00064	0.00049
		2.1			0.60368	0.62802	0.61713	0.61349	0.61092	0.00261	0.00141	0.00097	0.00074
		2.4			0.49411	0.52387	0.51054	0.50605	0.50294	0.00357	0.00189	0.00130	0.00099
1.5	1.5	1.3	3	1	-	-	-	-	-	-	-	-	-
		1.6			0.90773	0.91557	0.91206	0.91087	0.91005	0.00021	0.00011	$8.0047e^{-5}$	$6.1727e^{-5}$
		1.9			0.70147	0.72542	0.71468	0.71102	0.70855	0.00181	0.00096	0.00065	0.00050
		2.1			0.60368	0.6343	0.62057	0.61586	0.61273	0.00283	0.00147	0.00100	0.00076
		2.4			0.49411	0.53111	0.51448	0.50876	0.50501	0.00391	0.00198	0.00134	0.00101
1.5	1.5	1.3	4	0.5	-	-	-	-	-	-	-	-	-
		1.6			0.90773	0.9173	0.91303	0.91154	0.91057	0.00023	0.00012	$8.2614e^{-5}$	$6.3101e^{-5}$
		1.9			0.70147	0.73035	0.71743	0.71292	0.71001	0.00199	0.00101	0.00068	0.00051
		2.1			0.60368	0.64038	0.62395	0.61819	0.61452	0.00312	0.00156	0.00104	0.00078
		2.4			0.49411	0.53813	0.51837	0.51144	0.50706	0.00433	0.00211	0.00141	0.00105
1.5	1.5	1.3	4	1	-	-	-	-	-	-	-	-	-
		1.6			0.90773	0.91895	0.91397	0.9122	0.91108	0.00025	0.00012	$8.6065e^{-5}$	$6.5e^{-5}$
		1.9			0.70147	0.7351	0.72013	0.71479	0.71146	0.00220	0.00107	0.00071	0.00053
		2.1			0.60368	0.64626	0.62727	0.6205	0.61629	0.00347	0.00166	0.00110	0.00081
		2.4			0.49411	0.54496	0.5222	0.51409	0.5091	0.00485	0.00227	0.00148	0.00109

Note: When double prior distribution is (Erlang (λ)-Exponential (λ_1))$dist^n$., at t=1.3, we obtained

un suitable value for $R(t)$ & $\hat{R}(t)$, which is more than one. So, we put dash (-).

4-7: MSE of estimated Pareto type I reliability function using Bayes with respect to $MSE(\hat{R}(t))$ **when double prior distribution is (Chi-square(v) -Gamma (a, b)) distribution.**

parameters						R(t)	$\hat{R}(t)$				$MSE(\hat{R}(t))$			
							Sample Size(n)				Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	v	a	b		$(P_1(\theta \mid x))$ when double prior distribution is (Chi-square(v) -Gamma(a ,b)) dist^n.							
1	2.5	1.3	1	3	2	0.51897	0.55191	0.53723	0.53218	0.52879	0.00347	0.00182	0.00125	0.00095
		1.6				0.30882	0.34934	0.33118	0.32488	0.3208	0.00457	0.00225	0.00152	0.00113
		1.9				0.20096	0.24122	0.22305	0.21675	0.21275	0.00413	0.00192	0.00128	0.00094
		2.1				0.15648	0.19506	0.17755	0.1715	0.16769	0.00364	0.00164	0.00109	0.00078
		2.4				0.11207	0.14744	0.13126	0.12569	0.12224	0.00292	0.00126	0.00082	0.00059
1	2.5	1.3	1	3	4	0.51897	0.59387	0.56138	0.54908	0.54183	0.00725	0.00301	0.00184	0.00129
		1.6				0.30882	0.39677	0.35778	0.34334	0.33495	0.01000	0.00393	0.00234	0.00161
		1.9				0.20096	0.28582	0.24746	0.23353	0.22555	0.00933	0.00349	0.00204	0.00138
		2.1				0.15648	0.23667	0.19998	0.18683	0.17934	0.00836	0.00304	0.00176	0.00117
		2.4				0.11207	0.18447	0.1508	0.13893	0.13225	0.00684	0.00239	0.00136	0.00089
1	2.5	1.3	3	3	2	0.51897	0.54198	0.53194	0.52857	0.52605	0.00297	0.00167	0.00117	0.00091
		1.6				0.30882	0.33835	0.32542	0.32098	0.31785	0.00379	0.00202	0.00141	0.00107
		1.9				0.20096	0.23105	0.21781	0.21322	0.2101	0.00335	0.00170	0.00117	0.00088
		2.1				0.15648	0.18566	0.17276	0.16828	0.16528	0.00292	0.00143	0.00099	0.00073
		2.4				0.11207	0.13918	0.12711	0.12293	0.12018	0.00232	0.00109	0.00074	0.00054
1	2.5	1.3	3	3	4	0.51897	0.58447	0.55623	0.54554	0.53913	0.00597	0.00261	0.00165	0.00118
		1.6				0.30882	0.38573	0.35198	0.33941	0.33198	0.00819	0.00339	0.00209	0.00146
		1.9				0.20096	0.27513	0.24204	0.22991	0.22284	0.00760	0.00300	0.00181	0.00124
		2.1				0.15648	0.22653	0.19495	0.1835	0.17686	0.00678	0.00260	0.00155	0.00105
		2.4				0.11207	0.17525	0.14635	0.13602	0.1301	0.00552	0.00204	0.00120	0.00080

Continue for Table 4-7

parameters						$R(t)$	$\hat{R}(t)$				$MSE(\hat{R}(t))$			
							Sample Size(n)				Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	v	a	b		$(P_1(\theta \mid x))$ when double prior distribution is (Chi-square(v) -Gamma(a ,b)) dist^n.							
1.5	2.5	1.3	1	3	2	-	-	-	-	-	-	-	-	-
		1.6				0.851	0.86244	0.85697	0.85543	0.85446	0.00050	0.00027	0.00019	0.00014
		1.9				0.55379	0.58487	0.57012	0.56579	0.56313	0.00316	0.00166	0.00112	0.00082
		2.1				0.4312	0.4682	0.45068	0.44544	0.44225	0.00415	0.00211	0.00141	0.00103
		2.4				0.30882	0.34934	0.33017	0.32433	0.32082	0.00457	0.00223	0.00147	0.00106
1.5	2.5	1.3	1	3	4	-	-	-	-	-	-	-	-	-
		1.6				0.851	0.87861	0.86652	0.86215	0.85965	0.00098	0.00042	0.00026	0.00018
		1.9				0.55379	0.62495	0.59334	0.58203	0.57563	0.00654	0.00269	0.00164	0.00113
		2.1				0.4312	0.51379	0.47678	0.46363	0.45623	0.00881	0.00353	0.00212	0.00145
		2.4				0.30882	0.39677	0.35684	0.34281	0.33497	0.01000	0.00386	0.00227	0.00154
1.5	2.5	1.3	3	3	2	-	-	-	-	-	-	-	-	-
		1.6				0.851	0.85853	0.85485	0.85399	0.85337	0.00044	0.00026	0.00018	0.00013
		1.9				0.55379	0.57536	0.56503	0.56233	0.5605	0.00272	0.00154	0.00105	0.00078
		2.1				0.4312	0.45748	0.445	0.44158	0.43933	0.00351	0.00193	0.00132	0.00097
		2.4				0.30882	0.33835	0.32441	0.32043	0.31787	0.00379	0.00201	0.00136	0.00100
1.5	2.5	1.3	3	3	4	-	-	-	-	-	-	-	-	-
		1.6				0.851	0.87512	0.86454	0.86076	0.85859	0.00081	0.00037	0.00023	0.00017
		1.9				0.55379	0.61601	0.58843	0.57865	0.57305	0.00540	0.00234	0.00146	0.00103
		2.1				0.4312	0.50343	0.47119	0.45981	0.45332	0.00725	0.00306	0.00189	0.00132
		2.4				0.30882	0.38573	0.35104	0.33888	0.332	0.00819	0.00333	0.00202	0.00140

Note: When double prior distribution is (Chi- square (v) –Gamma (a, b)) dist^n., at t=1.3, we obtained un suitable value for $R(t)$ **&** $\hat{R}(t)$**, which is more than one. So, we put dash (-).**

4-8: MSE of estimated Pareto type I reliability function using Bayes with respect to $MSE(\hat{R}(t))$ **when double prior distribution is (Gamma (a, b) - Erlang (λ)) distribution.**

parameters						$R(t)$	$(\hat{R}(t))$				$MSE(\hat{R}(t))$			
							Sample Size(n)				Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	a	b	λ		$(P_2(\theta \mid x))$ when double prior distribution is (Gamma(a ,b) - Erlang (λ)) dist[n].							
1	2.5	1.3	3	2	4	0.51897	0.60716	0.57042	0.55579	0.54717	0.00909	0.00371	0.00221	0.00151
		1.6				0.30882	0.41222	0.36792	0.35075	0.3408	0.01258	0.00487	0.00283	0.00190
		1.9				0.20096	0.30065	0.25689	0.24034	0.23088	0.01177	0.00436	0.00248	0.00164
		2.1				0.15648	0.25066	0.20872	0.19309	0.18422	0.01055	0.00380	0.00214	0.00140
		2.4				0.11207	0.1971	0.15848	0.14438	0.13646	0.00865	0.00300	0.00166	0.00107
1	2.5	1.3	3	2	6	0.51897	0.63862	0.59093	0.57088	0.55912	0.01527	0.00606	0.00344	0.00225
		1.6				0.30882	0.45042	0.39158	0.36779	0.35411	0.02154	0.00806	0.00446	0.00287
		1.9				0.20096	0.33866	0.27943	0.25627	0.24319	0.0205	0.00728	0.00394	0.00249
		2.1				0.15648	0.28727	0.22987	0.20787	0.19556	0.01857	0.00639	0.00341	0.00214
		2.4				0.11207	0.23103	0.17741	0.1574	0.14637	0.01545	0.00509	0.00267	0.00165
1	2.5	1.3	4	3	4	0.51897	0.61496	0.57602	0.56004	0.55059	0.01036	0.00423	0.00249	0.00168
		1.6				0.30882	0.42142	0.37427	0.35549	0.34457	0.01437	0.00558	0.00321	0.00212
		1.9				0.20096	0.3096	0.26286	0.24473	0.23434	0.01347	0.00500	0.00281	0.00183
		2.1				0.15648	0.25917	0.21427	0.19713	0.18739	0.01209	0.00436	0.00243	0.00157
		2.4				0.11207	0.20485	0.16339	0.14791	0.13922	0.00993	0.00345	0.00189	0.00120
1	2.5	1.3	4	3	6	0.51897	0.64443	0.59573	0.5747	0.56228	0.01658	0.00671	0.00381	0.00249
		1.6				0.30882	0.45758	0.39719	0.37214	0.35766	0.02347	0.00894	0.00495	0.00318
		1.9				0.20096	0.34584	0.28481	0.26037	0.24648	0.02239	0.00810	0.00438	0.00276
		2.1				0.15648	0.29422	0.23495	0.21169	0.19861	0.02031	0.00712	0.00380	0.00237
		2.4				0.11207	0.23751	0.18198	0.16079	0.14905	0.01694	0.00568	0.00297	0.00183

Continue for Table 4-8

parameters						$R(t)$	$(\hat{R}(t))$				$MSE(\hat{R}(t))$			
							Sample Size(n)				Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	a	b	λ		$(P_2(\theta \mid x))$ when double prior distribution is (Gamma(a ,b) - Erlang (λ)) dist^n.							
1.5	2.5	1.3	3	2	4	-	-	-	-	-	-	-	-	-
		1.6				0.851	0.88359	0.87004	0.86478	0.86175	0.00123	0.00052	0.00031	0.00021
		1.9				0.55379	0.6376	0.60204	0.58847	0.58074	0.00821	0.00331	0.00197	0.00133
		2.1				0.4312	0.52839	0.48663	0.47088	0.46197	0.01107	0.00436	0.00256	0.00172
		2.4				0.30882	0.41222	0.36702	0.35024	0.34082	0.01258	0.00479	0.00276	0.00184
1.5	2.5	1.3	3	2	6	-	-	-	-	-	-	-	-	-
		1.6				0.851	0.89486	0.87778	0.87058	0.86638	0.00204	0.00084	0.00048	0.00032
		1.9				0.55379	0.66736	0.62165	0.60291	0.59217	0.01374	0.00541	0.00308	0.00202
		2.1				0.4312	0.56348	0.50918	0.48732	0.4749	0.01871	0.00717	0.00402	0.00262
		2.4				0.30882	0.45042	0.39075	0.3673	0.35414	0.02154	0.00794	0.00437	0.00281
1.5	2.5	1.3	4	3	4	-	-	-	-	-	-	-	-	-
		1.6				0.851	0.88646	0.8722	0.86643	0.86309	0.00140	0.00059	0.00035	0.00024
		1.9				0.55379	0.645	0.60742	0.59255	0.58402	0.00935	0.00378	0.00222	0.00149
		2.1				0.4312	0.53699	0.49277	0.4755	0.46565	0.01262	0.00498	0.00289	0.00193
		2.4				0.30882	0.42142	0.3734	0.35499	0.34459	0.01437	0.00549	0.00313	0.00207
1.5	2.5	1.3	4	3	6	-	-	-	-	-	-	-	-	-
		1.6				0.851	0.89692	0.87958	0.87203	0.8676	0.00221	0.00092	0.00054	0.00035
		1.9				0.55379	0.67286	0.62624	0.60657	0.59519	0.01493	0.00599	0.00341	0.00223
		2.1				0.4312	0.57	0.51449	0.4915	0.47832	0.02034	0.00795	0.00446	0.00290
		2.4				0.30882	0.45758	0.39637	0.37167	0.35768	0.02347	0.00882	0.00486	0.00312

Note: When double prior distribution is ((Gamma (a, b) - Erlang (λ))dist^n., at t=1.3, we obtained

un suitable value for $R(t)$ & $\hat{R}(t)$, which is more than one. So, we put dash (-).

4-9: MSE of estimated Pareto type I reliability function using Bayes with respect to $MSE(\hat{R}(t))$ when double prior distribution is (Erlang (λ)-Exponential (λ_1)) distribution.

Parameters					$R(t)$	$(\hat{R}(t))$				$MSE(\hat{R}(t))$			
						Sample Size(n)				Sample Size(n)			
						30	60	90	120	30	60	90	120
α	θ	t	λ	λ_1		$(P_3(\theta \mid x))$ when double prior distribution is (Erlang (λ)-Exponential (λ_1)) dist[n].							
1	2.5	1.3	3	0.5	0.51897	0.57879	0.55224	0.54257	0.53676	0.00552	0.00244	0.00155	0.00112
		1.6			0.30882	0.37947	0.34763	0.33619	0.32942	0.00757	0.00314	0.00196	0.00138
		1.9			0.20096	0.26936	0.23808	0.227	0.22053	0.00702	0.00277	0.00169	0.00117
		2.1			0.15648	0.22122	0.19133	0.18085	0.17477	0.00627	0.00240	0.00145	0.00099
		2.4			0.11207	0.17061	0.14323	0.13375	0.12831	0.00511	0.00188	0.00112	0.00075
1	2.5	1.3	3	1	0.51897	0.58894	0.55818	0.54675	0.53999	0.00666	0.00280	0.00173	0.00122
		1.6			0.30882	0.39113	0.35423	0.34078	0.33295	0.00919	0.00364	0.00220	0.00152
		1.9			0.20096	0.28046	0.24418	0.2312	0.22373	0.00857	0.00323	0.00192	0.00130
		2.1			0.15648	0.23164	0.19696	0.18469	0.17769	0.00766	0.00281	0.00165	0.00110
		2.4			0.11207	0.17997	0.14815	0.13708	0.13082	0.00627	0.00221	0.00127	0.00084
1	2.5	1.3	4	0.5	0.51897	0.59863	0.56397	0.55086	0.54318	0.00796	0.00323	0.00194	0.00135
		1.6			0.30882	0.40241	0.36072	0.34532	0.33644	0.01102	0.00422	0.00249	0.00169
		1.9			0.20096	0.29132	0.25022	0.23536	0.22691	0.01031	0.00377	0.00217	0.00145
		2.1			0.15648	0.24191	0.20255	0.18852	0.1806	0.00925	0.00328	0.00187	0.00123
		2.4			0.11207	0.18926	0.15307	0.14041	0.13334	0.00759	0.00258	0.00145	0.00094
1	2.5	1.3	4	1	0.51897	0.60788	0.56961	0.5549	0.54633	0.00938	0.00371	0.00219	0.00149
		1.6			0.30882	0.41333	0.3671	0.3498	0.3399	0.01304	0.00487	0.00281	0.00187
		1.9			0.20096	0.30195	0.2562	0.23951	0.23008	0.01225	0.00436	0.00246	0.00161
		2.1			0.15648	0.25202	0.20811	0.19233	0.1835	0.01101	0.00381	0.00212	0.00138
		2.4			0.11207	0.19848	0.15799	0.14374	0.13586	0.00906	0.00301	0.00165	0.00105

Continue for Table 4-9

Parameters					$R(t)$	$(\hat{R}(t))$				$MSE(\hat{R}(t))$			
						Sample Size(n)				Sample Size(n)			
						30	60	90	120	30	60	90	120
α	θ	t	λ	λ_1		$(P_3(\theta \mid x))$ when double prior distribution is (Erlang (λ)-Exponential (λ_1)) dist^n.							
1.5	2.5	1.3	3	0.5	-	-	-	-	-	-	-	-	-
		1.6			0.851	0.87289	0.86293	0.85957	0.85764	0.00075	0.00035	0.00022	0.00016
		1.9			0.55379	0.61057	0.58456	0.57578	0.57078	0.00499	0.00219	0.00138	0.00098
		2.1			0.4312	0.49731	0.46687	0.45661	0.45079	0.00670	0.00285	0.00178	0.00125
		2.4			0.30882	0.37947	0.34666	0.33565	0.32945	0.00757	0.00309	0.00189	0.00132
1.5	2.5	1.3	3	1	-	-	-	-	-	-	-	-	-
		1.6			0.851	0.87674	0.86526	0.86122	0.85892	0.00091	0.00039	0.00025	0.00017
		1.9			0.55379	0.62025	0.59027	0.5798	0.57387	0.00602	0.00251	0.00154	0.00107
		2.1			0.4312	0.50841	0.47331	0.46112	0.45426	0.00810	0.00328	0.00199	0.00137
		2.4			0.30882	0.39113	0.35327	0.34025	0.33297	0.00919	0.00358	0.00213	0.00146
1.5	2.5	1.3	4	0.5	-	-	-	-	-	-	-	-	-
		1.6			0.851	0.88036	0.86751	0.86284	0.86018	0.00108	0.00045	0.00027	0.00019
		1.9			0.55379	0.62947	0.59582	0.58374	0.57693	0.00718	0.00288	0.00173	0.00119
		2.1			0.4312	0.51905	0.4796	0.46555	0.45769	0.00969	0.00379	0.00224	0.00153
		2.4			0.30882	0.40241	0.35978	0.34479	0.33646	0.01102	0.00415	0.00241	0.00162
1.5	2.5	1.3	4	1	-	-	-	-	-	-	-	-	-
		1.6			0.851	0.88377	0.86969	0.86441	0.86141	0.00126	0.00051	0.00031	0.00021
		1.9			0.55379	0.63825	0.60123	0.58761	0.57994	0.00846	0.00331	0.00195	0.00132
		2.1			0.4312	0.52927	0.48576	0.46992	0.46108	0.01143	0.00436	0.00253	0.00170
		2.4			0.30882	0.41333	0.36617	0.34928	0.33993	0.01304	0.00479	0.00273	0.00181

Note: When double prior distribution is (Erlang (λ)-Exponential (λ_1)) dist^n., at t=1.3, we obtained

un suitable value for $R(t)$ & $\hat{R}(t)$, which is more than one. So, we put dash (-).

5. Discussion

In general, as we see in the tables (4-1to 4-9) by using different estimation methods, we find the Mean Square Errors (MSE) is decreased when sample size increased in all cases. That means the estimation of $(\hat{R}(t))$ get better for the large sample sizes. We obtained a good estimation according to the smallest values of MSE for all samples sizes (n) comparative to the other estimated values for MSE. As we see <u>in table (4-1)</u>, when the true value for $(\alpha = 1, \theta = 0.5)$ and the double prior distribution for θ is Chi-square(v)-Gamma(a, b) distribution with (v=1, a=3,b=4) for all t=1.3,1.6,1.9,2.1,2.4 . And the same thing for $(\alpha = 1.5, \theta = 0.5)$ and for all t except t=1.3.

And in table (4-2), when the true value for $(\alpha = 1, \theta = 0.5)$ and the double prior distribution for θ is Gamma(a, b)-Erlang (λ) distribution with (a=3, b=2, $\lambda = 6$) for all t. The same thing for $(\alpha = 1.5, \theta = 0.5)$ and for all t except t=1.3.

In table (4-3), when the true value for $(\alpha = 1, \theta = 0.5)$ and the double prior distribution for θ is Erlang (λ) - Exponential (λ_1) distribution with ($\lambda = 4$, $\lambda_1 = 1$) for all t. The same thing for $(\alpha = 1.5, \theta = 0.5)$ and for all t except t=1.3.

In table (4-4), when the true value for $(\alpha = 1, \theta = 1.5)$ and the double prior distribution for θ is Chi-square(v)-Gamma(a, b) distribution with (v=1, a=3, b=2) for all t. The same thing for $(\alpha = 1.5, \theta = 1.5)$ with (v=3, a=3, b=4) and for all t except t=1.3.

In table (4-5), when the true value for $(\alpha = 1, \theta = 1.5)$ and the double prior distribution for θ is Gamma(a, b)-Erlang (λ) distribution with (a=3, b=2, $\lambda = 4$) for all t. The same thing for $(\alpha = 1.5, \theta = 1.5)$ and for all t except t=1.3.

And in table (4-6), when the true value for $(\alpha = 1, \theta = 1.5)$ and the double prior distribution for θ is Erlang(λ) - Exponential (λ_1) distribution with ($\lambda = 3$, $\lambda_1 = 0.5$) for all t. The same thing for $(\alpha = 1.5, \theta = 1.5)$ and for all t except t=1.3.

In table (4-7), when the true value for $(\alpha = 1, \theta = 2.5)$ and the double prior distribution for θ is Chi-square(v)-Gamma(a, b) distribution with (v=3, a=3, b=2) for all t. The same thing for $(\alpha = 1.5, \theta = 2.5)$ with (v=1, a=3, b=2) and for all t except t=1.3.

In table (4-8), when the true value for $(\alpha = 1, \theta = 2.5)$ and the double prior distribution for θ is Gamma(a, b)-Erlang (λ) distribution with (a=3, b=2, $\lambda = 4$) for all t. The same thing for $(\alpha = 1.5, \theta = 2.5)$ and for all t except t=1.3.

And in table (4-9), when the true value for $(\alpha = 1, \theta = 2.5)$ and the double prior distribution for θ is Erlang (λ) - Exponential (λ_1) distribution with ($\lambda = 3$, $\lambda_1 = 0.5$) for all t. The same thing for $(\alpha = 1.5, \theta = 2.5)$ and for all t except t=1.3.

See the summarized and tabulated the above discussion in table (5-1)- (5-3) in Appendix-D.

6. Conclusion

When we compared the estimated values for Reliability function $R(t)$ $\hat{R}_i(t)$ for the parameter of the Pareto type I distribution by using the methods in this study .We find that Mean Square Errors (MSE) is decreased when sample size increased in all cases, and the MSE is increased in all samples sizes (n) when the true value of θ increased .The best method is the bayes estimation according to the smallest values of MSE for all sample sizes (n), comparative to the other estimated values for MSE.

As we see in table (6-1), when the true value for $(\alpha=1,\theta=0.5)$ and the double prior distribution for θ is Gamma(a, b)-Erlang (λ) distribution with (a=3, b=2, $\lambda=6$) for all t=1.3,1.6,1.9,2.1,2.4 . And the same thing for $(\alpha=1.5,\theta=0.5)$ and for all t except t=1.3.

In table (6-1), when the true value for $(\alpha=1,\theta=1.5)$ and the double prior distribution for θ is Chi-square (v)-Gamma (a, b) distribution with (v=1, a=3, b=2) for all t. And the same thing for $(\alpha=1.5,\theta=1.5)$ with (v =3, a=3, b=4) and for all t except t=1.3.

And in table (6-1), when the true value for $(\alpha=1,\theta=2.5)$ and the double prior distribution for θ is Chi-square(v)-Gamma(a, b) distribution with (v=3, a=3, b=2) for all t. And the same thing for $(\alpha=1.5,\theta=2.5)$ with (v=1, a=3, b=2) and for all t except t=1.3.

See the summary of conclusion for $MSE(\hat{R}(t))$ in table (6-1) in Appendix-D.

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Appendix-A: The posterior distribution by using different double Priors.

1- The posterior distribution using chi-square - gamma distribution as double prior:

To find the posterior distribution using Chi-squared distribution–Gamma distribution, we follow these steps:

When the prior distribution of θ is chi-squared distribution, then the pdf is given by:

$$f_1(\theta)=\frac{1}{\Gamma(\frac{v}{2})}\left(\frac{1}{2}\right)^{\frac{v}{2}}\theta^{\frac{v}{2}-1}\exp(-\frac{1}{2}\theta) \qquad \text{for} \quad \theta\ge 0,\ v=1,2,\ \dots v \qquad \dots(A.1)$$

Again, when prior distribution of θ is gamma distribution, then the p.d.f. is given by:

$$f_2(\theta)=\frac{b^a}{\Gamma a}\theta^{a-1}\exp(-b\theta) \qquad \text{for} \quad \theta\ge 0,\ a,b>0 \qquad \dots(A.2)$$

We define the double prior for θ by combining these two priors as follows:

$$P_1(\theta)\ \alpha\ f_1(\theta)\ f_2(\theta) \qquad \dots(A.3)$$

$$P_1(\theta)\ \alpha\left[\frac{1}{\Gamma(\frac{v}{2})}\left(\frac{1}{2}\right)^{\frac{v}{2}}\theta^{\frac{v}{2}-1}\exp(-\frac{1}{2}\theta)\right]\left[\frac{b^a}{\Gamma a}\theta^{a-1}\exp(-b\theta)\right]$$

$$P_1(\theta)\ \alpha\left[\frac{1}{\Gamma(\frac{v}{2})}\left(\frac{1}{2}\right)^{-\frac{v}{2}}\frac{b^a}{\Gamma a}\right]\theta^{\frac{v}{2}+a-2}\exp(-\theta(\frac{1}{2}+b)) \qquad \dots(A.4)$$

$$P_1(\theta)\ \alpha k\ \theta^{\frac{v}{2}+a-2}\exp(-\theta(\frac{1}{2}+b)) \qquad \dots(A.5)$$

Where $k=\left[\frac{1}{\Gamma(\frac{v}{2})}\left(\frac{1}{2}\right)^{-\frac{v}{2}}\frac{b^a}{\Gamma a}\right]$

Then the posterior distribution of θ for the given the data $\underline{t}=(t_1,t_2,\dots,t_n)$ is given by:

$$P(\theta|\ \underline{t})=\frac{L(\underline{t}|\ \theta)\ P(\theta)}{\int_\theta L(\underline{t}|\ \theta)\ P(\theta)d\theta} \qquad \dots(A.6)$$

Substituting the equation (4) and (A.5) in equation (A.6), we get:

$$P_1(\theta|\ \underline{t})=\frac{\theta^n\exp(n\theta\ln(\alpha))\exp(\theta\sum_{i=1}^n \ln t_i-\sum_{i=1}^n \ln t_i)[k\ \theta^{\frac{v}{2}+a-2}\exp(-\theta(\frac{1}{2}+b))]}{\int_0^\infty \theta^n\exp(n\theta\ln(\alpha))\exp(\theta\sum_{i=1}^n \ln t_i-\sum_{i=1}^n \ln t_i)[k\ \theta^{\frac{v}{2}+a-2}\exp(-\theta(\frac{1}{2}+b))]d\theta} \qquad \dots(A.7)$$

$$P_1(\theta \mid t) = \frac{\theta^{(n+a+\frac{v}{2}-1)-1} \exp(-\theta(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \frac{1}{2} + b))}{\int_0^\infty \theta^{(n+a+\frac{v}{2}-1)-1} \exp(-\theta(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \frac{1}{2} + b))\, d\theta} \quad \text{... (A.8)}$$

By multiplying the integral in equation (A.8) by the quantity which equals to

$$\left(\frac{(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \frac{1}{2} + b)^{(n+a+\frac{v}{2}-1)}}{\Gamma(n+a+\frac{v}{2}-1)}\right)\left(\frac{\Gamma(n+a+\frac{v}{2}-1)}{(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \frac{1}{2} + b)^{(n+a+\frac{v}{2}-1)}}\right)$$, where

$\Gamma(.)$ is a gamma function. Then we get,

$$P_1(\theta \mid t) = \frac{(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \frac{1}{2} + b)^{(n+a+\frac{v}{2}-1)}}{\Gamma(n+a+\frac{v}{2}-1) \quad A(t,\theta)} \theta^{(n+a+\frac{v}{2}-1)-1} \exp(-\theta(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \frac{1}{2} + b)) \quad \text{...(A.9)}$$

Where $A(t,\theta)$ equals to

$$A(t,\theta) = \int_0^\infty \frac{(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \frac{1}{2} + b)^{(n+a+\frac{v}{2}-1)}}{\Gamma(n+a+\frac{v}{2}-1)} \theta^{(n+a+\frac{v}{2}-1)-1} \exp(-\theta(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \frac{1}{2} + b))\, d\theta = 1$$

Be the integral of the pdf of gamma distribution. Then we get the posterior distribution of θ given the data $\underline{t} = (t_1, t_2, \ldots, t_n)$ is

$$P_1(\theta \mid t) = \frac{(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \frac{1}{2} + b)^{(n+a+\frac{v}{2}-1)}}{\Gamma(n+a+\frac{v}{2}-1)} \theta^{(n+\frac{v}{2}+a-1)-1} \exp(-\theta(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \frac{1}{2} + b))$$

$$\alpha, a, n, b, v > 0 \;, \quad \theta \geq 0 \quad \text{... (A.10)}$$

It means that $P_1(\theta \mid t)) \sim$ gamma distribution with new parameters $(a_{(new)} = (n + a + \frac{v}{2} - 1),\ b_{(new)} = (\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \frac{1}{2} + b))$.

2- <u>The posterior distribution using gamma - erlang distribution as double prior:</u>

To find the posterior distribution using gamma distribution– Erlang distribution, we follow these steps:

When the prior distribution of θ is gamma distribution, then the pdf is given by:

$$f_1(\theta) = \frac{b^a}{\Gamma a} \theta^{a-1} \exp(-b\theta) \qquad \text{for} \quad \theta \geq 0,\ a, b > 0 \qquad \text{... (A.2)}$$

Again, when prior distribution of θ is erlang distribution, then the pdf is given by:

$$f_2(\theta)=\lambda^2\ \theta\ \exp(-\lambda\ \theta) \qquad \text{for} \quad \theta\geq 0,\ \lambda>0 \qquad \text{...(A.11)}$$

We define the double prior for θ by combining these two priors as follows:

$$P_2(\theta)\ \alpha\ \ f_2(\theta)\ \ f_2(\theta) \qquad \text{...(A.12)}$$

$$P_2(\theta)\ \alpha[\ \frac{b^a}{\Gamma a}\ \theta^{a-1}\ \exp(-b\theta)\ \ ][\lambda^2\ \ \theta\ \ \exp(-\lambda\ \theta)]$$

$$P_2(\theta)\ \ \alpha[\frac{b^a}{\Gamma a}\ \lambda^2]\ \theta^a\exp(-\theta(b+\lambda)) \qquad \text{...(A.13)}$$

$$P_2(\theta)\ \ \alpha\ \ k_1\ \ \theta^a\exp(-\theta(b+\lambda)) \qquad \text{...(A.14)}$$

Where $k_1=[\frac{b^a}{\Gamma a}\ \lambda^2]$

Then the posterior distribution of θ for the given the data $\underline{t}=(t_1,t_2,\dots,t_n)$ is given by:

$$P(\theta|\ t)=\frac{L(\underline{t}|\ \theta)\ \ P(\theta)}{\int_\theta L(\underline{t}|\ \theta)\ \ P(\theta)d\theta} \qquad \text{...(A.6)}$$

Substituting the equation (4) and (A.14) in equation (A.6), we get:

$$P_2(\theta|\ t)=\frac{\theta^n\exp(n\theta\ln(\alpha))\exp(-\theta\sum_{i=1}^{n}\ln t_i-\sum_{i=1}^{n}\ln t_i)[\ k_1\ \theta^a\exp(-\theta(b+\lambda))\ \ ]}{\int_0^\infty \theta^n\exp(n\theta\ln(\alpha))\exp(-\theta\sum_{i=1}^{n}\ln t_i-\sum_{i=1}^{n}\ln t_i)[\ k_1\ \theta^a\exp(-\theta(b+\lambda))\ ]d\theta} \qquad \text{...(A.15)}$$

$$P_2(\theta|\ t)=\frac{\theta^{(n+a)+1-1}\exp(-\theta(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda))}{\int_0^\infty \theta^{(n+a)+1-1}\exp(-\theta(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda))\ d\theta} \qquad \text{...(A.16)}$$

By multiplying the integral in equation (A.16) by the quantity which equals to

$$\left(\frac{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda)^{(n+a+1)}}{\Gamma(n+a+1)}\right)\left(\frac{\Gamma(n+a+1)}{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda)^{(n+a+1)}}\right)$$

, where $\Gamma(.)$ is a gamma function. Then we get,

$$P_2(\theta|\ t)=\frac{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\ \ \lambda)^{(n+a+1)}}{\Gamma(n+a+1)\quad A1(t,\theta)}\theta^{(n+a+1)-1}\exp(-\theta\ (\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\ \ \lambda)) \qquad \text{...(A.17)}$$

Where $A1(t,\theta)$ equals to

$$A1(t,\theta)=\int_0^{\infty}\frac{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda)^{(n+a+1)}}{\Gamma(n+a+1)}\theta^{(n+a+1)-1}\exp(-\theta(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda))d\theta=1$$

Be the integral of the pdf of gamma distribution. Then we get the posterior distribution of θ given the data $\underline{t}=(t_1,t_2,\ldots,t_n)$ is

$$P_2(\theta\mid t)=\frac{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda)^{(n+a+1)}}{\Gamma(n+a+1)}\theta^{(n+a+1)-1}\exp(-\theta(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda)),$$

$$\alpha,a,b,n,\lambda>0\quad,\quad\theta\geq 0\qquad\ldots(A.18)$$

It means that $P_2(\theta\mid t))\sim$ gamma distribution with new parameters $(a_{(new)}=(n+a+1), b_{(new)}=(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda))$.

3- <u>The posterior distribution using erlang - exponential distribution as double prior:</u>

To find the posterior distribution using erlang distribution- exponential distribution, we follow these steps:

When the prior distribution of θ is exponential distribution, then the pdf is given by:

$$f_4(\theta)=\lambda_1\exp(-\lambda_1\theta)\qquad\text{for}\quad\theta\geq 0,\ \lambda_1>0\qquad\ldots(A.19)$$

Again, when prior distribution of θ is erlang distribution, then the pdf is given by:

$$f_2(\theta)=\lambda^2\ \theta\ \exp(-\lambda\theta)\qquad\text{for}\quad\theta\geq 0,\ \lambda>0\qquad\ldots(A.11)$$

We define the double prior for θ by combining these two priors as follows:

$$P_3(\theta)\ \alpha\ f_2(\theta)\ f_4(\theta)\qquad\ldots(A.20)$$

$$P_3(\theta)\ \alpha\ [\lambda^2\ \theta\ \exp(-\lambda\theta)][\lambda_1\exp(-\lambda_1\theta)]$$

$$P_3(\theta)\ \alpha[\lambda^2\ \lambda_1]\ \theta\ \exp(-\theta(\lambda+\lambda_1))\qquad\ldots(A.21)$$

$$P_3(\theta)\ \alpha\ k_2\ \theta\ \exp(-\theta(\lambda+\lambda_1))\qquad\ldots(A.22)$$

Where $k_2=[\lambda^2\ \lambda_1]$

Then the posterior distribution of θ for the given the data $\underline{t}=(t_1,t_2,\ldots,t_n)$ is given by:

$$P(\theta\mid t)=\frac{L(\underline{t}\mid\theta)\ P(\theta)}{\int_{\theta}L(\underline{t}\mid\theta)\ P(\theta)d\theta}\qquad\ldots(A.6)$$

Substituting the equation (4) and (A.22) in equation (A.6), we get:

$$P_3(\theta\mid t)=\frac{\theta^{n}\exp(n\theta\ln(\alpha))\exp(-\theta\sum_{i=1}^{n}\ln t_i-\sum_{i=1}^{n}\ln t_i)[k_2\theta\exp(-\theta(\lambda+\lambda_1))]}{\int_0^{\infty}\theta^{n}\exp(n\theta\ln(\alpha))\exp(-\theta\sum_{i=1}^{n}\ln t_i-\sum_{i=1}^{n}\ln t_i)[k_2\theta\exp(-\theta(\lambda+\lambda_1))]d\theta}$$

$$\ldots(A.23)$$

$$P_2(\theta \mid t) = \frac{\theta^{(n+1)+1-1} \exp(-\theta(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1))}{\int_0^\infty \theta^{(n+1)+1-1} \exp(-\theta(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1))\, d\theta} \quad \dots (A.24)$$

By multiplying the integral in equation (A.24) by the quantity which equals to

$$\left(\frac{(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1)^{(n+2)}}{\Gamma(n+2)}\right)\left(\frac{\Gamma(n+2)}{(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1)^{(n+2)}}\right)$$

, where $\Gamma(.)$ is a gamma function. Then we get,

$$P_2(\theta \mid t) = \frac{(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1)^{(n+2)}}{\Gamma(n+2) \quad A2(t,\theta)} \theta^{(n+2)-1} \exp(-\theta(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1)) \quad \dots (A.25)$$

Where $A2(t,\theta)$ equals to

$$A2(t,\theta) = \int_0^\infty \frac{(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1)^{(n+2)}}{\Gamma(n+2)} \theta^{(n+2)-1} \exp(-\theta(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1))\, d\theta = 1$$

Be the integral of the pdf of gamma distribution. Then we get the posterior distribution of θ given the data $\underline{t} = (t_1, t_2, \dots, t_n)$ is

$$P_3(\theta \mid t) = \frac{(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1)^{(n+2)}}{\Gamma(n+2)} \theta^{(n+2)-1} \exp(-\theta(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1)),$$

$$\alpha, n, \lambda, \lambda_1 > 0 \quad , \quad \theta \geq 0 \quad \dots (A.26)$$

It means that $P_2(\theta \mid t)) \sim$ gamma distribution with new parameters$(a_{(new)} = (n+2), b_{(new)} = (\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1))$.

Appendix-B

The following is the derivation of these estimators under the squared error loss function.

Appendix-B

The following is the derivation of these estimators under the squared error loss function.

The squared error loss function

To obtain the Bayes' estimator, we minimize the posterior expected loss given by:

$L(\hat{R}, R) = (\hat{R} - R)^2$, **where** $R = R(t)$ & $\hat{R} = \hat{R}(t)$, **the risk function is:**

$$Risk = E[L(\hat{R}, R)] \quad \ldots(B.1)$$

$$Risk = E[(\hat{R} - R)^2]$$

$$Risk = \int_{\theta} (\hat{R} - R)^2 \; P(\theta \backslash t)\, d\theta$$

$$Risk = \int_{\theta} (\hat{R} - R)^2 \; P(\theta \backslash t)\, d\theta \;\Rightarrow Risk = \int_{\theta} (\hat{R}^2 - 2R\hat{R} + R^2)\; P(\theta \backslash t)\, d\theta$$

$$Risk = \hat{R}^2 \int_0^{\infty} P(\theta|\, t) d\theta - 2\hat{R} \int_0^{\infty} R\, P(\theta|\, t)\, d\theta + \int_0^{\infty} R^2\, P(\theta|\, t)\, d\theta \Rightarrow$$

$$Risk = \hat{R}^2 - 2\hat{R}\, E(R|\, t) + E(R^2|\, t) \quad \ldots (B.2)$$

Let $\frac{\partial}{\partial \hat{R}} Risk = 0$, **we get Bayes estimator of** R **denoted by** $\hat{R}_{Bayes}$ **for the above prior as follows**

$$\hat{R}_{ss}(t) = E(R|\, t) = \int_0^{\infty} R(t)\, P(\theta|\, t)\, d\theta \quad \ldots (B.3)$$

$$\hat{R}_{ssi}(t) = \int_0^{\infty} R(t) P_i(\theta|\, t) d\theta \qquad ; i = 1,2,3 \quad \ldots (B.4)$$

1. **Bayes estimation using Chi-square - Gamma distribution as double prior:**

To obtain the Bayes' estimator under Chi-square - Gamma distribution as double prior. Substituting the equation (A.10) in equation (B.4), we get:

$$\hat{R}_{ss1}(t) = \int_0^{\infty} R(t) P_1(\theta|\, t)\, d\theta \qquad for \quad i = 1 \quad \ldots (B.4)$$

Where $R(t)$ **is the Reliability function as follow**

$R(t) = (\frac{\alpha}{t})^{\theta}$, **we can rewrite it as follow**

$$R(t) = \exp(\theta(\ln(\alpha) - \ln(t)) \quad \ldots (B.5)$$

$$\hat{R}_{ss1}(t) = \int_0^{\infty} \exp(\theta(\ln(\alpha) - \ln(t)) \frac{(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \frac{1}{2} + b)^{(n+a+\frac{v}{2}-1)}}{\Gamma(n + a + \frac{v}{2} - 1)} \theta^{(n+a+\frac{v}{2}-1)-1}$$

$$\exp(-\theta(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \frac{1}{2} + b))d\theta \quad \ldots (B.6)$$

$$\hat{R}_{ss1}(t)=\int_0^\infty \frac{\left(\sum_{i=1}^{n}\ln t_i - n\ln(\alpha)+\frac{1}{2}+b\right)^{(n+a+\frac{v}{2}-1)}}{\Gamma(n+a+\frac{v}{2}-1)}\theta^{(n+a+\frac{v}{2}-1)-1}$$

$$\exp\left(-\theta\left(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\frac{1}{2}+b\right)\right)d\theta \qquad \ldots (B.7)$$

By multiplying the integral in equation (B.7) by the quantity which equals to

$$B_1=\left(\frac{\left(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\frac{1}{2}+b\right)^{n+a+\frac{v}{2}-1}}{\left(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\frac{1}{2}+b\right)^{n+a+\frac{v}{2}-1}}\right)$$, Then, we have

$$\hat{R}_{ss1}(t)=B_1\int_0^\infty \frac{\left(\sum_{i=1}^{n}\ln t_i - n\ln(\alpha)+\frac{1}{2}+b\right)^{(n+a+\frac{v}{2}-1)}}{\Gamma(n+a+\frac{v}{2}-1)}\theta^{(n+a+\frac{v}{2}-1)-1}$$

$$\exp\left(-\theta\left(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\frac{1}{2}+b\right)\right)d\theta \qquad \ldots (B.8)$$

Then, we have

$$\hat{R}_{ss1}(t)=\frac{\left(\sum_{i=1}^{n}\ln t_i - n\ln(\alpha)+\frac{1}{2}+b\right)^{(n+a+\frac{v}{2}-1)}}{\left(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\frac{1}{2}+b\right)^{(n+a+\frac{v}{2}-1)}}B_2(t,\theta) \qquad \ldots (B.9)$$

Where $B_2(t,\theta)$ equals to

$$B_2(t;\theta)=\int_0^\infty \frac{\left(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\frac{1}{2}+b\right)^{(n+a+\frac{v}{2}-1)}}{\Gamma(n+a+\frac{v}{2}-1)}\theta^{(n+\frac{v}{2}+a)-1}$$

$$\exp\left(-\theta\left(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\frac{1}{2}+b\right)\right)d\theta=1$$

Be the integral of the pdf of gamma distribution. Then we get the Bayes estimator of θ as the following formula:

$$\hat{R}_{ss1}(t)=\frac{\left(\sum_{i=1}^{n}\ln t_i - n\ln(\alpha)+\frac{1}{2}+b\right)^{(n+a+\frac{v}{2}-1)}}{\left(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\frac{1}{2}+b\right)^{(n+a+\frac{v}{2}-1)}}\ , t,\alpha,n,b,a,v>0 \qquad \ldots (B.10)$$

2. <u>Bayes estimation using gamma - Erlang distribution as double prior:</u>

To obtain the Bayes' estimator under Chi-square - Erlang distribution as double prior. Substituting the equation (A.18) in equation (B.4), we get:

$$\hat{R}_{ss2}(t)=\int_0^\infty R(t)\,P_2(\theta\mid t)\,d\theta \qquad \text{for}\quad i=2 \qquad \ldots (B.4)$$

Where $R(t)$ is the Reliability function as in in equation (B.5), we get:

$$\hat{R}_{ss2}(t)=\int_0^\infty \exp(\theta(\ln(\alpha)-\ln(t)))\frac{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda)^{(n+a+1)}}{\Gamma(n+a+1)}\theta^{(n+a+1)-1}\exp(-\theta(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda))\,d\theta \quad \dots (B.11)$$

$$\hat{R}_{ss2}(t)=\int_0^\infty \frac{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda)^{(n+a+1)}}{\Gamma(n+a+1)}\theta^{(n+a+1)-1}\exp(-\theta(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+b+\lambda))\,d\theta \quad \dots (B.12)$$

By multiplying the integral in equation (B.12) by the quantity which equals to

$$B_3=\left(\frac{(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+b+\lambda)^{(n+a+1)}}{(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+b+\lambda)^{(n+a+1)}}\right)$$, where $\Gamma(.)$ is a gamma function.

Then, we have

$$\hat{R}_{ss2}(t)=B3\int_0^\infty \frac{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda)^{(n+a+1)}}{\Gamma(n+a+1)}\theta^{(n+a+1)-1}\exp(-\theta(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+b+\lambda))\,d\theta \quad \dots (B.13)$$

Then, we have

$$\hat{R}_{ss2}(t)=\frac{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda)^{(n+a+1)}}{(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+b+\lambda)^{(n+a+1)}}B_4(t,\theta) \quad \dots (B.14)$$

Where $B_4(t,\theta)$ equals to

$$B_4(t,\theta)=\int_0^\infty \frac{(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+b+\lambda)^{(n+a+1)}}{\Gamma(n+a+1)}\theta^{(n+a+1)-1}\exp(-\theta(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+b+\lambda))\,d\theta=1$$

Be the integral of the pdf of gamma distribution. Then we get the Bayes estimator of θ as the following formula:

$$\hat{R}_{ss2}(t)=\frac{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+b+\lambda)^{(n+a+1)}}{(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+b+\lambda)^{(n+a+1)}}\quad ,t,\alpha,n,b,a,\lambda>0 \quad \dots (B.15)$$

3. Bayes estimation using erlang - exponential distribution as double prior:

To obtain the Bayes' estimator under erlang - exponential distribution as double prior. Substituting the equation (A.26) in equation (B.4), we get:

$$\hat{R}_{ss3}(t)=\int_0^\infty R(t)\,P_3(\theta\,|\,t)\,d\theta \qquad \text{for} \quad i=3 \qquad \ldots(B.4)$$

Where $R(t)$ is the Reliability function as in in equation (B.5), we get:

$$\hat{R}_{ss3}(t)=\int_0^\infty \exp(\theta\,(\ln(\alpha)-\ln(t))\,\frac{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+\lambda+\lambda_1)^{(n+2)}}{\Gamma(n+2)}\,\theta^{(n+2)-1}$$
$$\exp(-\theta\,(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+\lambda+\lambda_1))\;d\theta \quad \ldots(B.16)$$

$$\hat{R}_{ss3}(t)=\int_0^\infty \frac{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+\lambda+\lambda_1)^{(n+2)}}{\Gamma(n+2)}\,\theta^{(n+2)-1}$$
$$\exp(-\theta\,(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\lambda+\lambda_1))\;d\theta \quad \ldots(B.17)$$

By multiplying the integral in equation (B.17) by the quantity which equals to

$$B_5=\left(\frac{(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\lambda+\lambda_1)^{(n+2)}}{(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\lambda+\lambda_1)^{(n+2)}}\right),$$ where $\Gamma(.)$ is a gamma function.

Then, we have

$$\hat{R}_{ss3}(t)=B_5\int_0^\infty \frac{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+\lambda+\lambda_1)^{(n+2)}}{\Gamma(n+2)}\,\theta^{(n+2)-1}$$
$$\exp(-\theta\,(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\lambda+\lambda_1))\;d\theta \quad \ldots(B.18)$$

Then, we have

$$\hat{R}_{ss3}(t)=\frac{(\sum_{i=1}^{n}\ln t_i-n\ln(\alpha)+\lambda+\lambda_1)^{(n+2)}}{(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\lambda+\lambda_1)^{(n+2)}}\,B_6(t,\theta) \qquad \ldots(B.19)$$

Where $B_6(t,\theta)$ equals to

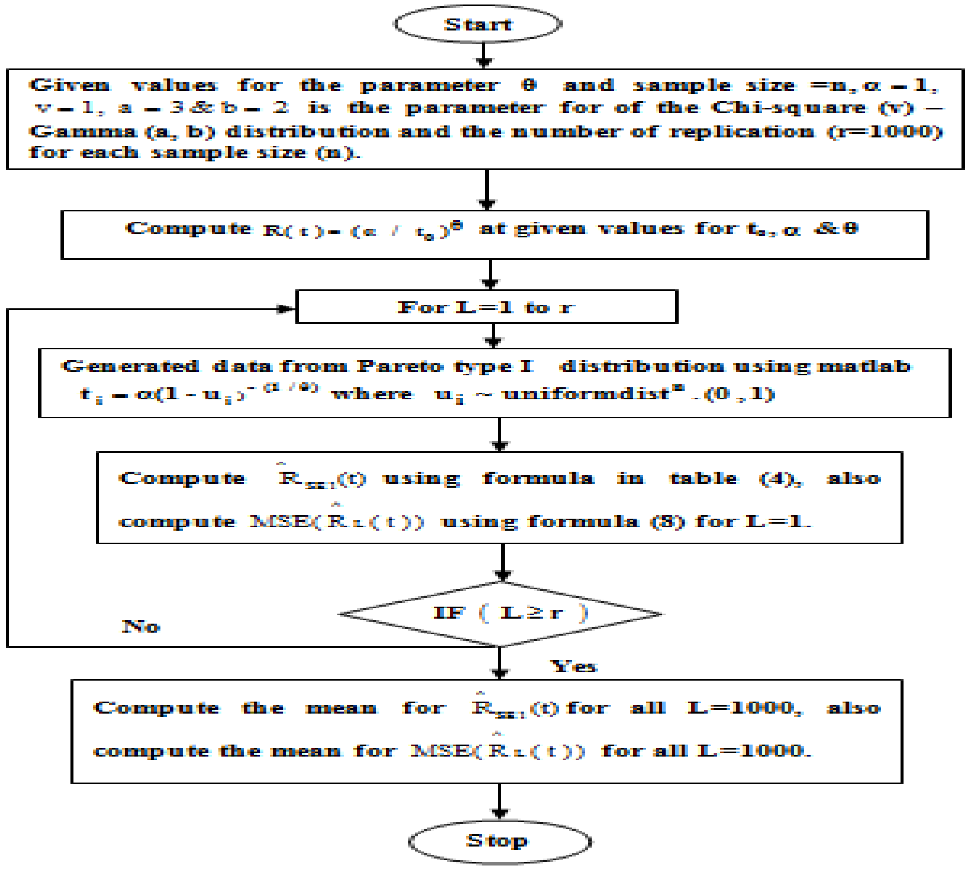
$$B_6(t,\theta)=\int_0^\infty \frac{(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\lambda+\lambda_1)^{(n+2)}}{\Gamma(n+2)}\,\theta^{(n+2)-1}$$
$$\exp(-\theta\,(\sum_{i=1}^{n}\ln t_i-(n+1)\ln(\alpha)+\ln(t)+\lambda+\lambda_1))\;d\theta=1$$

Be the integral of the pdf of gamma distribution. Then we get the Bayes estimator of θ as the following formula:

$$\hat{R}_{SE3}(t) = \frac{(\sum_{i=1}^{n} \ln t_i - n\ln(\alpha) + \lambda + \lambda_1)^{(n+2)}}{(\sum_{i=1}^{n} \ln t_i - (n+1)\ln(\alpha) + \ln(t) + \lambda + \lambda_1)^{(n+2)}} \quad ; t, \alpha, n, \lambda, \lambda_1 > 0 \quad \ldots (B.20)$$

Appendix-C: **The following is the program algorithm.**

Algorithm (1): To compute Bayes estimators ($\hat{R}_{SE1}(t)$) using the Chi-square (v)-Gamma (a, b)a double prior distribution for θ with MSE for $\hat{R}_{SE1}(t)$.



Note: we can reformulate the Algorithm (1) to compute Bayes estimators $\hat{R}_{SE1}(t), k = 2,3$ under using other distributions as double prior distribution for θ with MSE for $\hat{R}_{SEi}(t), i = 2,3$.

Appendix-D The summarized and tabulated discussions and conclusions.

5-1 Best Estimation according to the smallest value for $MSE(\hat{R}(t))$

parameters						$R(t)$	$(\hat{R}(t))$				$MSE(\hat{R}(t))$			
							Sample Size(n)				Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	v	a	b		$(P_1(\theta \mid x))$ when double prior distribution is (Chi-square(v) -Gamma(a ,b)) dist^n.							
1	0.5	1.3	1	3	4	0.87706	0.87366	0.87532	0.87553	0.87629	0.00039	0.00022	0.00014	0.00010
		1.6				0.79057	0.78569	0.78808	0.78832	0.78948	0.00099	0.00057	0.00037	0.00027
		1.9				0.72548	0.71997	0.72269	0.72287	0.72427	0.00153	0.00089	0.00058	0.00042
		2.1				0.69007	0.68441	0.68721	0.68734	0.68884	0.00183	0.00106	0.00070	0.00051
		2.4				0.6455	0.63982	0.64265	0.6427	0.6443	0.00219	0.00129	0.00084	0.00061
1.5	0.5	1.3	1	3	4	-	-	-	-	-	-	-	-	-
		1.6				0.96825	0.96722	0.96756	0.96793	0.96805	$2.9969e^{-5}$	$1.6777e^{-5}$	$1.0128e^{-5}$	$8.3383e^{-6}$
		1.9				0.88852	0.88538	0.88636	0.88756	0.88796	0.00032	0.00018	0.00011	$9.3602e^{-5}$
		2.1				0.84515	0.84112	0.84234	0.84393	0.84445	0.00059	0.00033	0.00020	0.00017
		2.4				0.79057	0.78569	0.78711	0.7891	0.78975	0.00099	0.00057	0.00035	0.00029
α	θ	t	a	b	λ	$(P_2(\theta \mid x))$ when double prior distribution is (Gamma(a ,b) - Erlang (λ)) dist^n.								
1	0.5	1.3	3	2	6	0.87706	0.87485	0.87586	0.87589	0.87654	0.00033	0.00020	0.00013	0.00010
		1.6				0.79057	0.78754	0.78895	0.78889	0.78989	0.00086	0.00053	0.00035	0.00026
		1.9				0.72548	0.72224	0.72376	0.72358	0.72478	0.00133	0.00083	0.00055	0.00040
		2.1				0.69007	0.68686	0.68838	0.68812	0.6894	0.00159	0.00099	0.00066	0.00049
		2.4				0.6455	0.64248	0.64393	0.64354	0.64491	0.00192	0.00120	0.00081	0.00059
1.5	0.5	1.3	3	2	6	-	-	-	-	-	-	-	-	-
		1.6				0.96825	0.96756	0.96771	0.96803	0.96812	$2.5685e^{-5}$	$1.5507e^{-5}$	$9.6367e^{-6}$	$8.0423e^{-6}$
		1.9				0.88852	0.88647	0.88688	0.88788	0.88819	0.00028	0.00017	0.00010	$9.0359e^{-5}$
		2.1				0.84515	0.84257	0.84304	0.84435	0.84476	0.00051	0.00031	0.00019	0.00016
		2.4				0.79057	0.78754	0.788	0.78965	0.79016	0.00086	0.00053	0.00033	0.00028
α	θ	t	λ	$λ_1$		$(P_3(\theta \mid x))$ when double prior distribution is (Erlang (λ)-Exponential ($λ_1$)) dist^n.								
1	0.5	1.3	4	1		0.87706	0.87641	0.87672	0.87648	0.877	0.00036	0.00021	0.00014	0.00010
		1.6				0.79057	0.7901	0.79034	0.78984	0.79062	0.00092	0.00055	0.00036	0.00026
		1.9				0.72548	0.72547	0.72551	0.72478	0.7257	0.00143	0.00086	0.00056	0.00041
		2.1				0.69007	0.69043	0.69031	0.68944	0.69042	0.00171	0.00103	0.00068	0.00050
		2.4				0.6455	0.64645	0.64607	0.64501	0.64604	0.00207	0.00125	0.00082	0.00061
1.5	0.5	1.3	4	1		-	-	-	-	-	-	-	-	-
		1.6				0.96825	0.96798	0.96794	0.96819	0.96825	$2.7234e^{-5}$	$1.588e^{-5}$	$9.8192e^{-6}$	$8.1691e^{-6}$
		1.9				0.88852	0.88789	0.88765	0.88842	0.88861	0.00030	0.00017	0.00011	$9.1898e^{-5}$
		2.1				0.84515	0.84451	0.84409	0.84509	0.84532	0.00054	0.00032	0.00020	0.00016
		2.4				0.79057	0.7901	0.78938	0.79061	0.79089	0.00092	0.00054	0.00034	0.00028

5-2 Best Estimation according to the smallest value for $MSE(\hat{R}(t))$

parameters						$R(t)$	$(\hat{R}(t))$				$MSE(\hat{R}(t))$			
							Sample Size(n)				Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	v	a	b		$(P_1(\theta \mid x))$ when double prior distribution is (Chi-square(v) -Gamma(a ,b)) dist^n.							
1	1.5	1.3	1	3	2	0.67466	0.67949	0.67688	0.67732	0.67709	0.00182	0.00104	0.00067	0.00057
		1.6				0.49411	0.50346	0.49868	0.49869	0.49817	0.00314	0.00178	0.00116	0.00098
		1.9				0.38183	0.39432	0.38809	0.38762	0.38689	0.00355	0.00199	0.00130	0.00110
		2.1				0.3286	0.34254	0.33567	0.33492	0.33407	0.00356	0.00197	0.00129	0.00109
		2.4				0.26896	0.28438	0.27685	0.27577	0.27481	0.00341	0.00186	0.00122	0.00102
1.5	1.5	1.3	3	3	4	-	-	-	-	-	-	-	-	-
		1.6				0.90773	0.9136	0.91102	0.91016	0.90952	0.00018	0.00010	$7.6055e^{-5}$	$5.9481e^{-5}$
		1.9				0.70147	0.71971	0.71171	0.709	0.70703	0.00156	0.00088	0.00062	0.00048
		2.1				0.60368	0.61017	0.60764	0.60705	0.60602	0.00239	0.00135	0.00093	0.00073
		2.4				0.49411	0.50346	0.49969	0.49868	0.49734	0.00314	0.00176	0.00123	0.00095
α	θ	t	a	b	λ	$(P_2(\theta \mid x))$ when double prior distribution is (Gamma(a ,b) - Erlang (λ)) dist^n.								
1	1.5	1.3	3	2	4	0.67466	0.70597	0.69168	0.68746	0.68484	0.00215	0.00111	0.00073	0.00060
		1.6				0.49411	0.53819	0.51806	0.51201	0.50835	0.00407	0.00202	0.00132	0.00107
		1.9				0.38183	0.43108	0.40857	0.4017	0.39764	0.00492	0.00236	0.00152	0.00123
		2.1				0.3286	0.37924	0.35606	0.34895	0.34479	0.00511	0.00241	0.00154	0.00124
		2.4				0.26896	0.32003	0.2966	0.28936	0.28518	0.00510	0.00234	0.00149	0.00119
1.5	1.5	1.3	3	2	4	-	-	-	-	-	-	-	-	-
		1.6				0.90773	0.91741	0.91321	0.91169	0.9107	0.00021	0.00011	$8.0492e^{-5}$	$6.1783e^{-5}$
		1.9				0.70147	0.73054	0.71792	0.71334	0.71038	0.00186	0.00097	0.00066	0.00050
		2.1				0.60368	0.64054	0.62451	0.61869	0.61496	0.00292	0.00150	0.00102	0.00077
		2.4				0.49411	0.53819	0.51898	0.512	0.50756	0.00407	0.00204	0.00138	0.00103
α	θ	t	λ	$λ_1$		$(P_3(\theta \mid x))$ when double prior distribution is (Erlang (λ)-Exponential ($λ_1$)) dist^n.								
1	1.5	1.3	3	0.5		0.67466	0.69506	0.68518	0.68293	0.68135	0.00194	0.00106	0.00069	0.00058
		1.6				0.49411	0.52387	0.50956	0.50606	0.50376	0.00357	0.00188	0.00123	0.00102
		1.9				0.38183	0.41594	0.39959	0.39542	0.39279	0.00424	0.00217	0.00141	0.00116
		2.1				0.3286	0.36413	0.34712	0.34269	0.33996	0.00437	0.00219	0.00142	0.00117
		2.4				0.26896	0.30536	0.28794	0.2833	0.28051	0.00432	0.00211	0.00135	0.00111
1.5	1.5	1.3	3	0.5		-	-	-	-	-	-	-	-	-
		1.6				0.90773	0.91377	0.91107	0.91019	0.90953	0.00020	0.00011	$7.8405e^{-5}$	$6.0897e^{-5}$
		1.9				0.70147	0.72031	0.71188	0.70909	0.70708	0.00169	0.00092	0.00064	0.00049
		2.1				0.60368	0.62802	0.61713	0.61349	0.61092	0.00261	0.00141	0.00097	0.00074
		2.4				0.49411	0.52387	0.51054	0.50605	0.50294	0.00357	0.00189	0.00130	0.00099

5-3 Best Estimation according to the smallest value for $MSE(\hat{R}(t))$

parameters						$R(t)$	$(\hat{R}(t))$				$MSE(\hat{R}(t))$			
							Sample Size(n)				Sample Size(n)			
							30	60	90	120	30	60	90	120
α	θ	t	v	a	b		$(P_1(\theta \mid x))$ when double prior distribution is (Chi-square(v) -Gamma(a ,b)) dist^n.							
1	2.5	1.3	3	3	2	0.51897	0.54198	0.53194	0.52857	0.52605	0.00297	0.00167	0.00117	0.00091
		1.6				0.30882	0.33835	0.32542	0.32098	0.31785	0.00379	0.00202	0.00141	0.00107
		1.9				0.20096	0.23105	0.21781	0.21322	0.2101	0.00335	0.00170	0.00117	0.00088
		2.1				0.15648	0.18566	0.17276	0.16828	0.16528	0.00292	0.00143	0.00099	0.00073
		2.4				0.11207	0.13918	0.12711	0.12293	0.12018	0.00232	0.00109	0.00074	0.00054
1.5	2.5	1.3	1	3	2	-	-	-	-	-	-	-	-	-
		1.6				0.851	0.86244	0.85697	0.85543	0.85446	0.00050	0.00027	0.00019	0.00014
		1.9				0.55379	0.58487	0.57012	0.56579	0.56313	0.00316	0.00166	0.00112	0.00082
		2.1				0.4312	0.4682	0.45068	0.44544	0.44225	0.00415	0.00211	0.00141	0.00103
		2.4				0.30882	0.34934	0.33017	0.32433	0.32082	0.00457	0.00223	0.00147	0.00106
α	θ	t	a	b	λ	$(P_2(\theta \mid x))$ when double prior distribution is (Gamma(a ,b) - Erlang (λ)) dist^n.								
1	2.5	1.3	3	2	4	0.51897	0.60716	0.57042	0.55579	0.54717	0.00909	0.00371	0.00221	0.00151
		1.6				0.30882	0.41222	0.36792	0.35075	0.3408	0.01258	0.00487	0.00283	0.00190
		1.9				0.20096	0.30065	0.25689	0.24034	0.23088	0.01177	0.00436	0.00248	0.00164
		2.1				0.15648	0.25066	0.20872	0.19309	0.18422	0.01055	0.00380	0.00214	0.00140
		2.4				0.11207	0.1971	0.15848	0.14438	0.13646	0.00865	0.00300	0.00166	0.00107
1.5	2.5	1.3	3	2	4	-	-	-	-	-	-	-	-	-
		1.6				0.851	0.88359	0.87004	0.86478	0.86175	0.00123	0.00052	0.00031	0.00021
		1.9				0.55379	0.6376	0.60204	0.58847	0.58074	0.00821	0.00331	0.00197	0.00133
		2.1				0.4312	0.52839	0.48663	0.47088	0.46197	0.01107	0.00436	0.00256	0.00172
		2.4				0.30882	0.41222	0.36702	0.35024	0.34082	0.01258	0.00479	0.00276	0.00184
α	θ	t	λ	λ_1		$(P_3(\theta \mid x))$ when double prior distribution is (Erlang (λ)-Exponential (λ_1)) dist^n.								
1	2.5	1.3	3	0.5		0.51897	0.57879	0.55224	0.54257	0.53676	0.00552	0.00244	0.00155	0.00112
		1.6				0.30882	0.37947	0.34763	0.33619	0.32942	0.00757	0.00314	0.00196	0.00138
		1.9				0.20096	0.26936	0.23808	0.227	0.22053	0.00702	0.00277	0.00169	0.00117
		2.1				0.15648	0.22122	0.19133	0.18085	0.17477	0.00627	0.00240	0.00145	0.00099
		2.4				0.11207	0.17061	0.14323	0.13375	0.12831	0.00511	0.00188	0.00112	0.00075
1.5	2.5	1.3	3	0.5		-	-	-	-	-	-	-	-	-
		1.6				0.851	0.87289	0.86293	0.85957	0.85764	0.00075	0.00035	0.00022	0.00016
		1.9				0.55379	0.61057	0.58456	0.57578	0.57078	0.00499	0.00219	0.00138	0.00098
		2.1				0.4312	0.49731	0.46687	0.45661	0.45079	0.00670	0.00285	0.00178	0.00125
		2.4				0.30882	0.37947	0.34666	0.33565	0.32945	0.00757	0.00309	0.00189	0.00132

6-1: the summary of conclusions for $MSE(\hat{R}(t))$.

parameters			The best estimation according to smallest value for $MSE(\hat{R}(t))$ when the double prior distribution is	$MSE(\hat{R}(t))$			
				Sample Size(n)			
α	θ	t		30	60	90	120
1	0.5	1.3	Gamma(a ,b) - Erlang (λ) distribution with (a=3,b=2, λ=6)	0.00033	0.00020	0.00013	0.00010
		1.6		0.00086	0.00053	0.00035	0.00026
		1.9		0.00133	0.00083	0.00055	0.00040
		2.1		0.00159	0.00099	0.00066	0.00049
		2.4		0.00192	0.00120	0.00081	0.00059
1.5	0.5	1.3	Gamma(a , b) - Erlang (λ) distribution with (a=3,b=2, λ=6)	-	-	-	-
		1.6		$2.5685e^{-5}$	$1.5507e^{-5}$	$9.6367e^{-6}$	$8.0423e^{-6}$
		1.9		0.00028	0.00017	0.00010	$9.0359e^{-5}$
		2.1		0.00051	0.00031	0.00019	0.00016
		2.4		0.00086	0.00053	0.00033	0.00028
1	1.5	1.3	Chi-square(v) - Gamma(a ,b) distribution with (v=1, a=3, b=2)	0.00182	0.00104	0.00067	0.00057
		1.6		0.00314	0.00178	0.00116	0.00098
		1.9		0.00355	0.00199	0.00130	0.00110
		2.1		0.00356	0.00197	0.00129	0.00109
		2.4		0.00341	0.00186	0.00122	0.00102
1.5	1.5	1.3	Chi-square(v) - Gamma(a ,b) distribution With (v=3, a=3 , b=4)	-	-	-	-
		1.6		0.00018	0.00010	$7.6055e^{-5}$	$5.9481e^{-5}$
		1.9		0.00156	0.00088	0.00062	0.00048
		2.1		0.00239	0.00135	0.00093	0.00073
		2.4		0.00314	0.00176	0.00123	0.00095
1	2.5	1.3	Chi-square(v) - Gamma(a , b) distribution with (v=3, a=3, b=2)	0.00297	0.00167	0.00117	0.00091
		1.6		0.00379	0.00202	0.00141	0.00107
		1.9		0.00335	0.00170	0.00117	0.00088
		2.1		0.00292	0.00143	0.00099	0.00073
		2.4		0.00232	0.00109	0.00074	0.00054
1.5	2.5	1.3	Chi-square(v) - Gamma(a ,b) distribution with (v=1, a=3 , b=2)	-	-	-	-
		1.6		0.00050	0.00027	0.00019	0.00014
		1.9		0.00316	0.00166	0.00112	0.00082
		2.1		0.00415	0.00211	0.00141	0.00103
		2.4		0.00457	0.00223	0.00147	0.00106

مقارنة مقدرات بيز لدالة المعولية لتوزيع باريتو من النوع الاول باستعمال دوال معلوماتية مضاعفة مختلفة

أ.م.د. جنان عباس ناصر العبيدي / الكلية التقنية الادارية / بغداد

الخلاصة

في هذا البحث ، نقدم مقارنة لتقديردالة المعولية لتوزيع باريتو من النوع الاول بالاعتماد على اسلوب بيز فقد استعملت الدوال معلوماتية مضاعفة التي تفترض لمعلمة الشكل (θ) لتوزيع باريتو من النوع الاول، لتقدير دالة المعولية لتوزيع باريتو من النوع الاول باستعمال تقدير بيز استخدمنا نوعين مختلفين من المعلومات في طريقة تقدير بيز، اختيرت نوعين مختلفين من الدوال الاولية لمعلمة الشكل (θ) لتوزيع باريتو من النوع الاول.هنا افترضنا توزيع مربع كاي – كاما ، وتوزيع كاما – ارلنك , وتوزيع ارلنك- الاسي كدوال معلوماتية مضاعفه، نتائج الاشتقاقات لتلك المقدرات باستعمال دالة الخسارة التربيعية مع ثلاثة دوال معلوماتية مضاعفه. استعمل اسلوب المحاكاة في مقارنة اداء كل مقدر، بافتراض عدة حالات لمعلمة الشكل (θ) لتوزيع باريتو من النوع الاول استعملت لتوليد البيانات ولاحجام مختلفة من العينات (صغيرة ، متوسطة ، كبيرة).

استحصلنا من نتائج المحاكاة بان التوزيع الاولي المضاعف توزيع كاما – ارلنك بالمعلمات (a,b,λ) يعطي تقدير جيد لدالة المعولية $(R(t))$ عندما تكون القيمة الحقيقية لـ $(\alpha=1\&1.5,\theta=0.5)$ لكل قيم t . وكذلك التوزيع الاولي المضاعف توزيع مربع كاي – كاما بالمعلمات (v,a,b) يعطي تقدير جيد لدالة المعولية $(R(t))$ عندما تكون القيمة الحقيقية لـ $(\alpha=1.5,\theta=2.5)\ \&\ (\alpha=1,\theta=1.5)$ لكل قيم t. والشئ نفسه لـ $(\alpha=1.5,\theta=1.5)$ بقيم المعلمات (v,a,b) ولكل قيم t عدا t=1.3. واستحصلنا على تقدير جيد لدالة المعولية $(R(t))$ عندما كان التوزيع الاولي المضاعف توزيع مربع كاي – كاما بالمعلمات (v,a,b) عند القيمة الحقيقة لـ $(\alpha=1,\theta=2.5)$ ولكل قيم t.

المصطلحات الرئيسة للبحث : توزيع باريتو من النوع الاول ، دالة المعولية ،طريقة بيز ،الدوال المعلوماتية المضاعفة : توزيع مربع كاي- كاما، ، توزيع كاما - ارلنك ، التوزيع ارلنك- الاسي ، دالة الخسارة التربيعية.