ON DISCRETE WEIBULL DISTRIBUTION

Ali A. S. Al-Wakeel[a]

ABSTRACT

Most of the Weibull models studied in the literature were appropriate for modelling a continuous random variable which assume the variable takes on real values over the interval [0,∞]. One of the new studies in statistics is when the variables takes on discrete values. The idea was first introduced by Nakagawa and Osaki, as they introduced discrete Weibull distribution with two shape parameters q and β where $0 < q < 1$ and $\beta > 0$. Weibull models for modelling discrete random variables assume only non-negative integer values. Such models are useful for modelling for example; the number of cycles to failure when components are subjected to cyclical loading. Discrete Weibull models can be obtained as the discrete counter parts of either the distribution function or the failure rate function of the standard Weibull model. Which lead to different models. This paper discusses the discrete model which is the counter part of the standard two-parameter Weibull distribution. It covers the determination of the probability mass function, cumulative distribution function, survivor function, hazard function, and the pseudo-hazard function.

Keywords; Functions of discrete Weibull distribution, pseudo-hazard function, failure studies.

مجلة العلوم
الاقتصادية والإدارية
المجلد 20
العدد 79
لسنة 2014
الصفحات ١ – ٩

INTRODUCTION

This paper considers failure studies in which the time to failure is often measures the number of cycles to failure and therefore, becomes a discrete random variable. In some situations, system lifetime is expressed by discrete random variable. The failure data in failure studies are mainly measured in discrete time such as runs, cycles, blows, shocks, or revolutions (Nakagawa and Osaki 1975).

The discrete Weibull arises in reliability problems when the observed variable is discrete. Geometric and negative binomial distributions are known to be discrete alternatives, for the exponential and gamma distributions respectively. Discrete Weibull is an alternative to the Weibull distribution. There are different forms of discrete Weibull distributions, one of these forms from Nakagawa and Osaki (1975) and others from Stein and Dettero (1984), Padgett and Spurrier(1985).

In situations where the observed data values are very large (in thousands of cycles, etc.) a continuous distribution is considered an adequate model for the discrete random variable, for example when an equipment operates in cycles or demand, and the number of cycles or demands prior to failure is observed, then the usual .reliability concepts for continuous lifetimes have to be defined again to be adapted to discrete time (Xie et al. 2002).

Nakagawa and Osaki (1975) who pioneered the work on the discrete Weibull distribution and submitted a form of discrete Weibull distribution with two parameters.

They investigated, from the view points of both theory and practice, what discrete distribution corresponds to the Weibull and then explained that failure data in failure studies are mainly measured in discrete time such as cycles, blows and shocks.

Stein and Dattero (1984) submitted a paper that defined another discrete Weibull distribution and compared it to the distribution of Nakagawa and Osaki (1975). They said that discrete Weibull distribution should possess three properties; The hazard rate should be similar to that of continuous Weibull. It should provide the exact lifetime distribution of a specific system, and, the lifetime should, in some sense, converge to that given by the continuous Weibull, thus showing the connection between the two distributions.

Padgett and Spurrier (1985) introduced a paper that provided three families of discrete parametric distribution which are versatile in fitting increasing, decreasing and constant failure rate models to either uncensored or right–censored discrete life test data.

Ali Khan et al. (1989) discussed two discrete distributions and a simple method is presented to estimate the parameters for one of them. They compared this method with the method of moments, and they concluded that the estimates appear to have almost similar properties.

Salvia (1996) referred to a new hazard function of discrete Weibull distribution with the parameter $\beta > -1$. The hazard function is increasing when $\beta > 0$ and decreasing when $\beta < 0$.

DISCRETE WEIBULL DISTRIBUTION

Discrete Weibull models can be obtained as the discrete counter parts of either the distribution function or the failure rate function of the standard Weibull model.

Let $\beta > 0$ be the shape parameter and 0<q<1 also the shape parameter, then the probability mass function of discrete Weibull distribution is given by :

$$p(x) = q^{(x-1)^{\beta}} - q^{x^{\beta}} \quad , \quad x=0,1,2,\dots \tag{1}$$

where x is a nonnegative integer value.

Then the distribution function is given by:

$$F(x) = 1 - q^{x^{\beta}} \quad , \quad x=0,1,2,\dots \tag{2}$$
$$= 0 \quad , \quad x < 0$$

The survivor function is given by :

$$\bar{F}(x) = 1 - F(x) \tag{3}$$

$$\bar{F}(x) = q^{x^{\beta}} \tag{4}$$

and the hazard function (failure rate) is given by:

$$h(x) = 1 - q^{x^{\beta} - (x-1)^{\beta}} \tag{5}$$

The hazard function can be increasing, decreasing or constant depending on the value of *β*, similar to the continuous Weibull distribution, and the distribution function is similar to the two-parameter Weibull distribution. It has application in reliability when the response of interest is a discrete variable.

Roy and Gupta (1992) proposed an alternate discrete failure r(x). It was called second rate of failure and was defined as follows:

$$r(x) = \ln\left[\frac{\bar{F}(x-1)}{\bar{F}(x)}\right] \tag{6}$$

From (6) we have .

$$r(x) = \ln\left[\frac{q^{(x-1)^{\beta}}}{q^{x^{\beta}}}\right]$$

$$r(x) = \ln\left[q^{(x-1)^{\beta} - x^{\beta}}\right] \tag{7}$$

Xie et al.(2002) advocate that r(x) can be called the discrete failure rate. However it is important to note that r(x) is not a conditional probability whereas h(x) is a conditional probability. He used the term pseudo-hazard function for r(x) so as to differentiate it from the hazard function h(x). The pseudo-hazard function can be increasing, decreasing and constant depending on the value of *β* being greater, less than or equal to one. When *β*=1 the discrete Weibull distribution reduces to the geometric distribution. When *β*=2, it increases linearly which is similar to the continuous case. This is the counter part of the power law relationship linking the exponential and the Weibull distribution in the continuous case.

Weibull distribution has been used to analyze failure in electronic components, ball- bearings, etc. Failure of some devices often depends more on the total number of cycles than on the total time that they have been seen. Such examples are switching devices, railroad tracks, and tires of automobiles. In this case, discrete Weibull distribution will be a good approximation for such devices, materials or structures.

RESULTS AND DISCUSSION

By taking different values for the shape parameters β and q, we can see the variation of the mass function and other functions. Assuming a sample size of 15 with the shape parameters β = 0.5, 1, 1.5 and q = 0.3, 0.7

The following graphs show probability mass function p(x), cumulative distribution function F(x), survivor function $\bar{F}(x)=1-F(x)$, hazard function h(x) and pseudo- hazard function r(x);

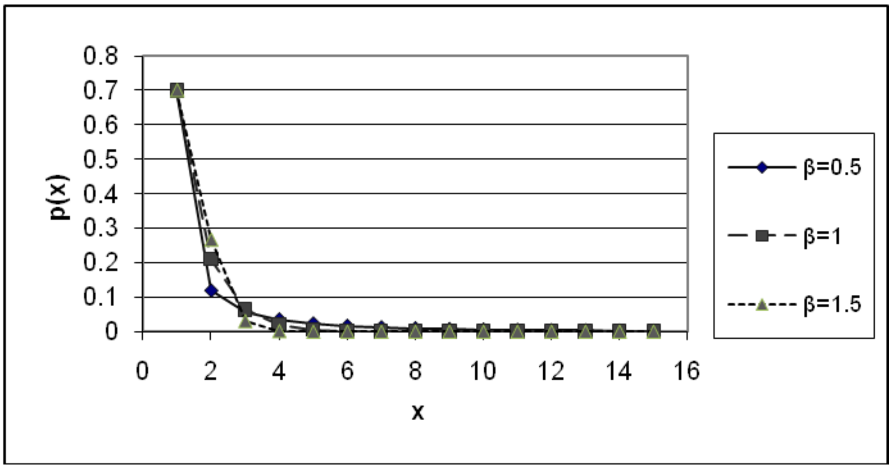

FIGURE 1a. Probability mass function with q=0.3, β=0.5,1,1.5

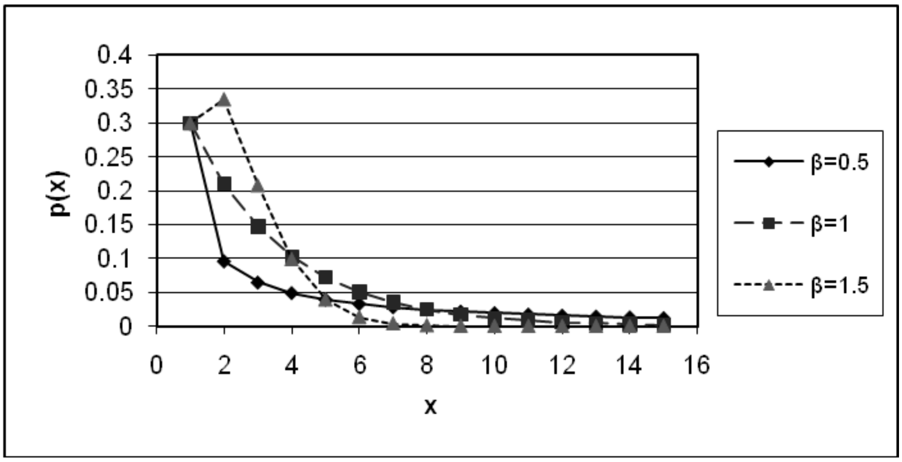

FIGURE 1b. Probability mass function with q=0.7, β=0.5,1,1.5

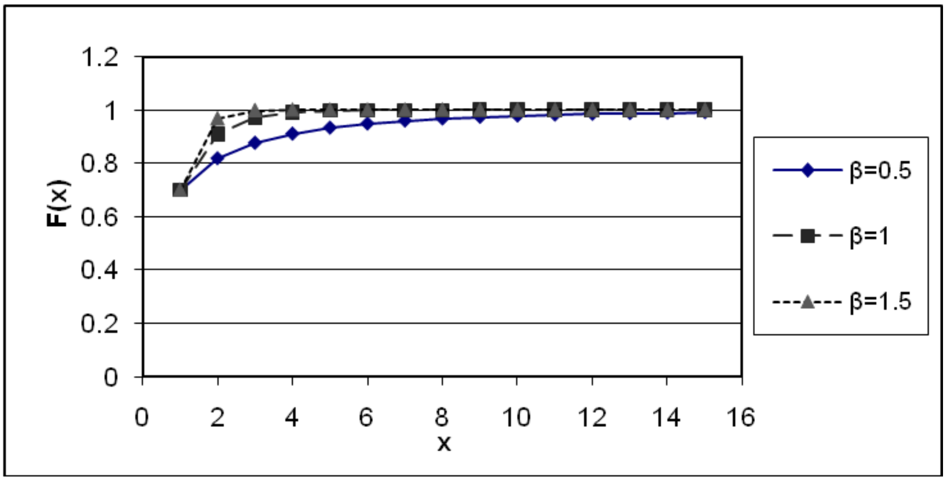

FIGURE 2a. Cumulative distribution function with q=0.3, β=0.5,1,1.5

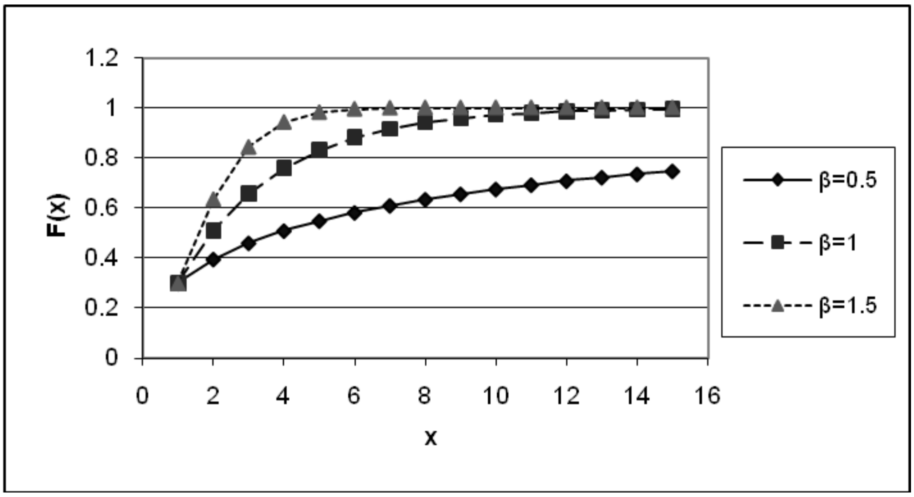

FIGURE 2b. Cumulative distribution function with q=0.7, β=0.5,1,1.5

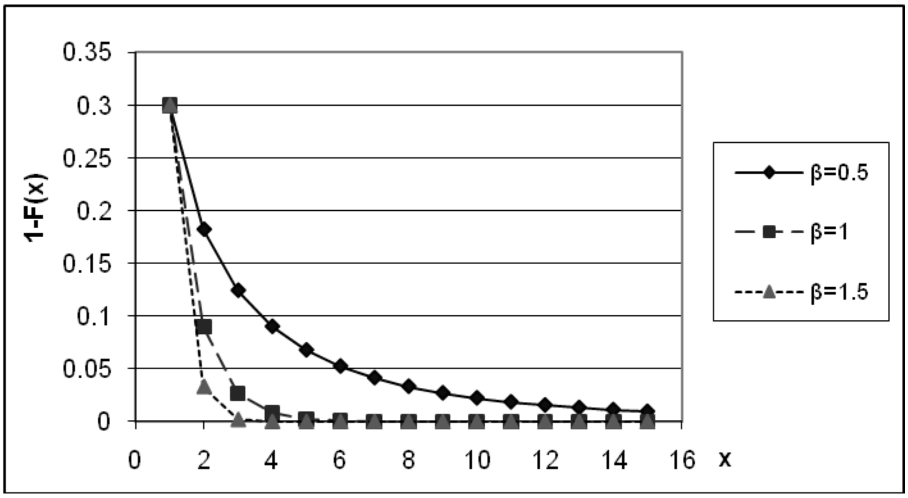

FIGURE 3a. Reliability function with q=0.3, β=0.5,1,1.5

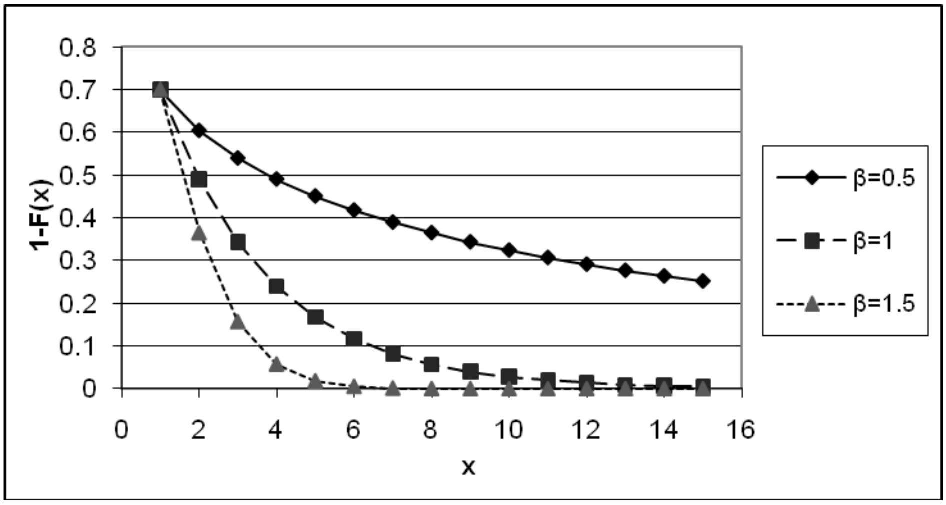

FIGURE 3b. Reliability function with q=0.7, β=0.5,1,1.5

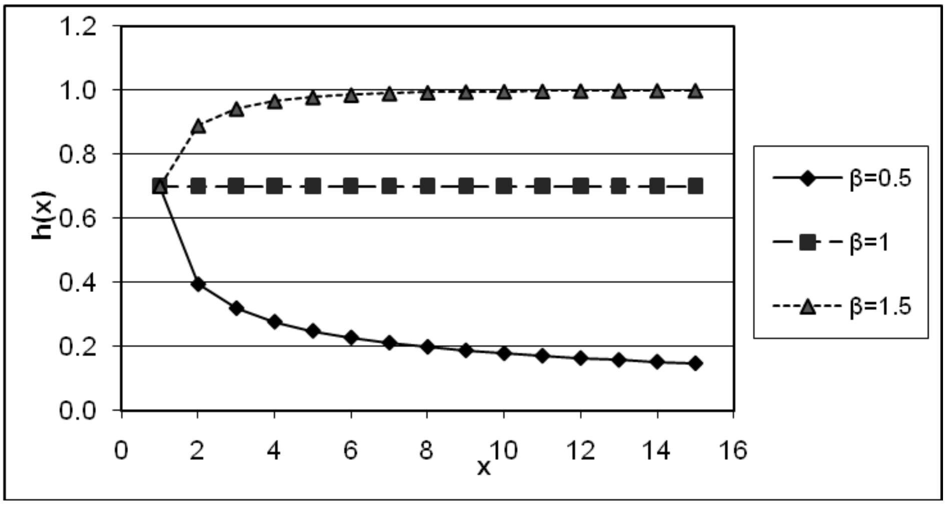

FIGURE 4a. Hazard function with q=0.3, β=0.5,1,1.5

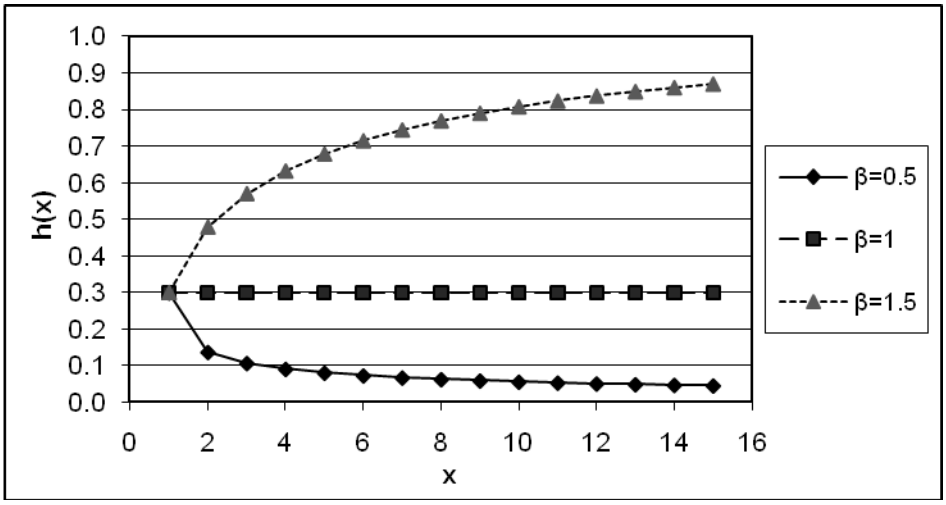

FIGURE 4b. Hazard function with q=0.7, β=0.5,1,1.5

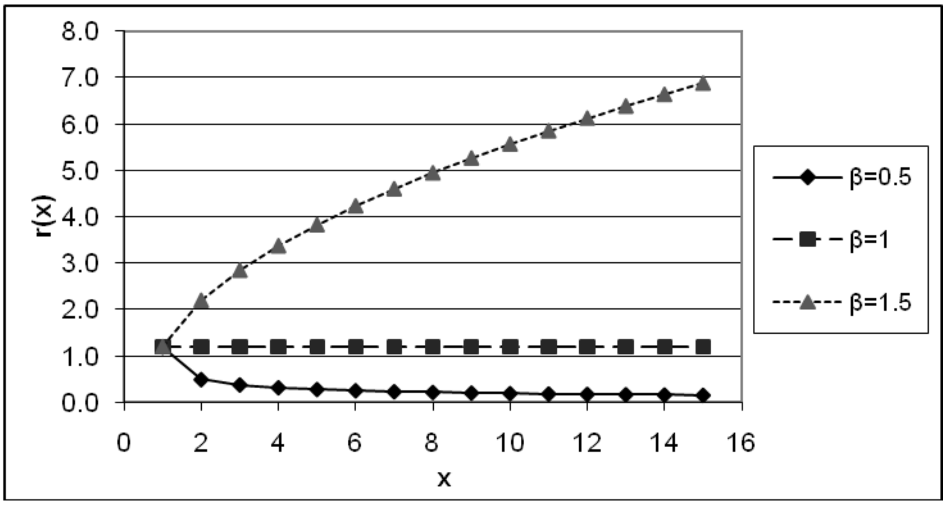

FIGURE 5a. Pseudo hazard function with q=0.3, β=0.5,1,1.5

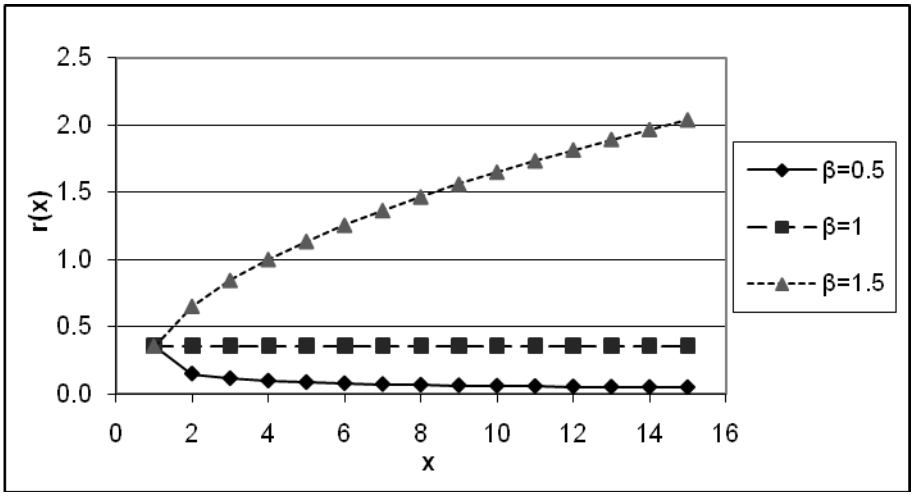

FIGURE 5b. Pseudo hazard function with q=0.7, β=0.5,1,1.5

CONCLUSION

Discrete Weibull distribution is the counter part of the standard two-parameter Weibull distribution. From Figures 1a and 1b we can conclude that increasing the shape parameter β causes the probability mass function to approach zero faster. It also shows that the shape parameter, q, has no effect on the probability mass function.

Figures 2a and 2b show that increasing the shape parameter β causes the distribution function reach one faster. Also the shape parameter, q, has no effect on the distribution function.

Figures 3a and 3b indicate that the survivor function reaches zero faster when the shape parameter β increases.

Figures 4a and 4b show that the hazard function increases when the shape parameter $\beta > 1$. However, when the shape parameter $\beta = 1$ the hazard function is constant and when the shape parameter $\beta < 1$ the hazard function decreases. The same applies for the pseudo hazard function which is clear from Figures 5a and 5b.

As an overall conclusion, the results show that the shape parameter q has no effect on all functions while the shape parameter β has the greatest effect on these functions. This agrees with the idea of standard Weibull distribution (Murthy et al. 2004).

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حول توزيع ويبل المتقطع

أ.علي عبد الحسين الوكيل
كلية الادارة والاقتصاد/ جامعة بغداد

المستخلص

معظم نماذج ويبل التي تناولتها البحوث تلائم المتغيرات العشوائيه المستمره والتي تفترض بان المتغير العشوائي ياخذ قيم حقيقية من صفر الى ما لا نهاية $(0,\infty)$

الا ان هناك بعض الدراسات الاحصائيه الحديثة ياخذ فيها المتغير العشوائي قيم متقطعه. هذه الفكرة قدمت من قبل ناكاكاوا واوساكي (Nakagawa and Osaki) .

حيث قدما بحث لتوزيع ويبل بمعلمتين للشكل q و β

حيث $0 < q < 1$ و $\beta > 0$

ان نماذج ويبل للمتغيرات العشوائية المتقطعة تفترض قيم صحيحة غير سالبه وتفيد في نمذجة العديد من الحالات مثلا عدد الدورات لحين الفشل عند تعرض المكونات لضغط دوري. ويمكن الحصول على نماذج ويبل المتقطع باعتبارها مناظره لدالة التوزيع او دالة معدل الفشل لنموذج ويبل القياسي والذي يؤدي بالتالي الى العديد من النماذج المختلفه. يتناول هذا البحث نموذج ويبل المتقطع والمناظر لتوزيع ويبل القياسي ذو المعلمتين. ويوضح البحث احتساب الدالة الاحتمالية الكمية، دالة التوزيع التراكمية، دالة البقاء دالة الخطر ودالة شبه الخطر.

المصطلحات الرئيسية للبحث/ دوال توزيع ويبل المتقطع- دالة شبه الخطر- دراسات فشل.