Comparing Bayes estimation with Maximum Likelihood Estimation of Generalized Inverted Exponential Distribution in Case of Fuzzy Data

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Abstract:

In this paper, the generalized inverted exponential distribution is considered as one of the most important distributions in studying failure times. A shape and scale parameters of the distribution have been estimated after removing the fuzziness that characterizes its data because they are triangular fuzzy numbers. To convert the fuzzy data to crisp data the researcher has used the centroid method. Hence the studied distribution has two parameters which show a difficulty in separating and estimating them directly of the MLE method. The Newton-Raphson method has been used.

For the Bayesian method, the gamma distribution has been proposed as a prior distribution for the two parameters with a quadratic loss function and by using Metropolis-Hasting algorithm to find the Bayesian parameters estimators. Different samples have been generated to represent the population under study by using simulation approach. After estimating the parameters, the results of the two methods have been compared according to the Mean Squared Error measurement. And the researcher concluded that the best estimation method is the MLE followed by the Bayesian.

Keywords/ **Newton-Raphson method; Metropolis-Hasting algorithm; triangular fuzzy numbers; centroid method.**

*البحث مستل من رسالة ماجستير

1- Introduction

Inference of the properties of a population by estimating the unknown parameters of a sample drawn from it is the most important branch of statistical inference. The problem of estimating parameters for a distribution which represents a real life phenomenon with uncertain data quality is one of the major problems that a fuzzy theory (fuzzy theory is introduced by Zadeh in 1965) describes. Yager in (1981) proposed a centroid method to transfer fuzzy numbers to crisp numbers.

Abouammoh and Alshingiti in (2009) have proposed the generalized inverted exponential (GIE) distribution by introducing a shape parameter of the inverted exponential (IE) distribution. They studied parameters estimation by using maximum likelihood estimation and least square estimation methods. Also, they discussed statistical and reliability properties of the distribution. This lifetime distribution is capable of modeling various shapes of failure rates and hence various shapes of ageing criteria [1].

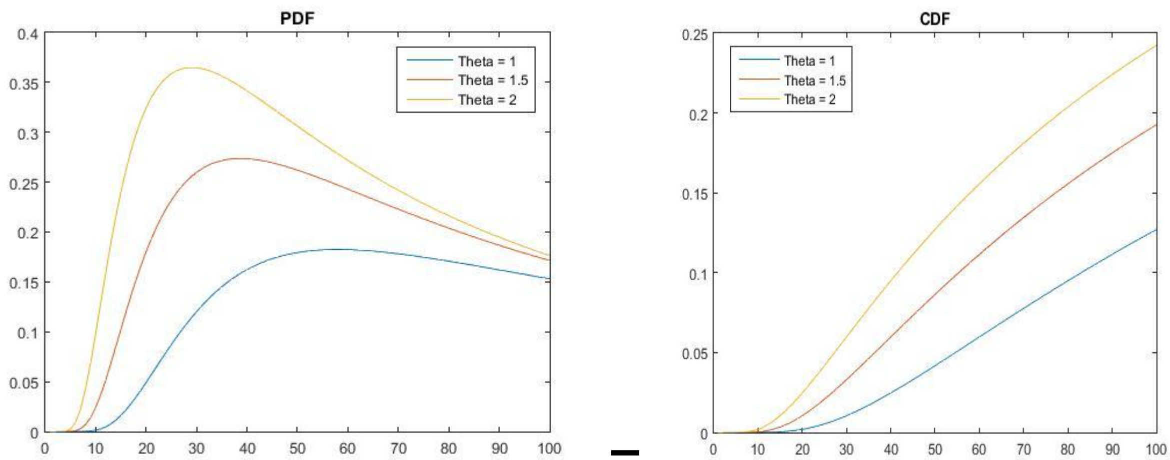
This paper is organized as follows: - In section (2), the model of the (GIE) distribution and some of its statistical properties defined and explained. Sections (3 and 4) contain the estimation of parameters of this distribution using the methods of maximum likelihood estimation and Bayes estimation depending on Quadratic Loss function. Section (5) introduces some concepts of the fuzzy set theory. Section (6) the simulation experiments. Finally, the conclusions of this paper are in section (7).

2- The Model

The probability density function (PDF) and the cumulative distribution function (CDF) of the (GIE) distribution are given by the forms respectively [2,4]:

$$f(t;\lambda,\theta) = \frac{\lambda}{\theta t^2}\, e^{-1/\theta t}\left(1-e^{-1/\theta t}\right)^{\lambda-1} \quad ; t \geq 0, \lambda, \theta > 0 \qquad \dots(1)$$

$$F(t;\lambda,\theta) = 1-\left(1-e^{-1/\theta t}\right)^{\lambda} \quad ; t \geq 0, \lambda, \theta > 0 \qquad \dots(2)$$



Where λ is a shape parameter and θ is a scale parameter.
Figure (1) represents the PDF and the CDF of the (GIE) distribution respectively when $\lambda = 0.3$ and $\theta = 1,1.5$ and 2 [2].

The reliability (survival) function and the hazard (failure) rate function of the (GIE) distribution are given by respectively [2]:

$$R(t) = 1 - F(t)$$

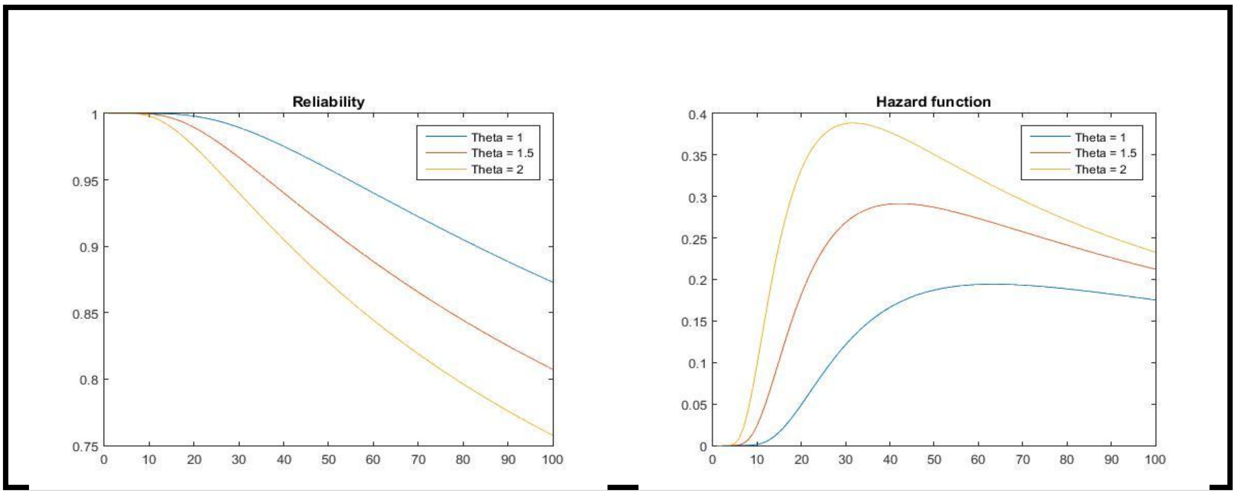
$$= \left(1 - e^{-1/\theta t}\right)^{\lambda} \qquad ; t \geq 0, \lambda, \theta > 0 \qquad \ldots(3)$$

$$h(t) = \frac{f(t)}{R(t)}$$

$$= \frac{\lambda}{\theta t^2 \ (e^{1/\theta t} - 1)} \qquad ; t \geq 0, \lambda, \theta > 0 \qquad \ldots(4)$$

Figure (2-2) represents the reliability function and the hazard function of the (GIE) distribution respectively when $\lambda = 0.3$ and $\theta = 1,1.5$ and 2 [2].

The median is the value of t which satisfies the inverse image of the following



equation:

$$F(t) = \frac{1}{2}$$

$$t = F^{-1}\left(\frac{1}{2}\right)$$

$$t = \frac{-1}{\theta \log(1-(2)^{-1/\lambda})} \quad \dots(5)$$

The mode [5] is the value of t which satisfies the following equation

$$\frac{\partial \log f(t)}{\partial t} = 0$$

Where

$$\log f(t) = \log\left(\frac{\lambda}{\theta}\right) - 2\log(t) - \frac{1}{\theta t} + (\lambda-1)\log\left(1-e^{-1/\theta t}\right) \quad \dots(6)$$

$$1 - 2\theta t - (\lambda-1)e^{-1/\theta t}\left(1-e^{-1/\theta t}\right)^{-1} = 0 \quad \dots(7)$$

Eq. (7) can be solved numerically to get the mode.

The formula of a random number generation of the (GIE) distribution as follows [2]:

$$t_i = \frac{-1}{\theta \log(1-(1-u_i)^{1/\lambda})}; i = 1,2,\dots,n \quad \dots(8)$$

Where u_i is a random variable uniformly distributed on (0, 1).

3- Maximum Likelihood Estimation

Let $t_1, t_2, \dots, t_n$ be a random sample of size n drawn from a population with probability density function $f(t_i;\beta)$ where β is a vector of unknown parameters. The likelihood function is defined by

$$L(t_i;\beta) = \prod_{i=1}^{n} f(t_i;\beta)$$

The maximum likelihood estimator of β is the solution of the following equation:

$$\frac{\partial \log L(t_i;\beta)}{\partial \beta} = 0$$

Therefore; the likelihood function of the (GIE) distribution is given as follows [2]

$$L(t_i;\lambda,\theta) = \prod_{i=1}^{n} f(t_i;\lambda,\theta)$$

Let $L = L(t_i;\lambda,\theta)$

$$L=\left(\frac{\lambda}{\theta}\right)^n \prod_{i=1}^{n}\left(\frac{1}{t_i{}^2}\right) e^{-\sum_{i=1}^{n}(1/\theta t_i)} \prod_{i=1}^{n}\left(1-e^{-1/\theta t_i}\right)^{\lambda-1} \quad \ldots(9)$$

By taking the log function to equation (9) we get

$$\log L = n\log\lambda - n\log\theta + \sum_{i=1}^{n}\log\left(\frac{1}{t_i^2}\right) - \sum_{i=1}^{n}\left(\frac{1}{\theta t_i}\right) + (\lambda - 1)\sum_{i=1}^{n}\log\left(1-e^{-1/\theta t_i}\right) \quad \ldots(10)$$

Differentiating eq. (10) with respect to the parameters λ and θ gives [2]

$$\frac{\partial \log L}{\partial\lambda} = \frac{n}{\lambda} + \sum_{i=1}^{n}\log\left(1-e^{-1/\theta t_i}\right) \quad \ldots(11)$$

$$\frac{\partial \log L}{\partial\theta} = \frac{-n}{\theta} + \sum_{i=1}^{n}\frac{1}{\theta^2 t_i} - (\lambda-1)\sum_{i=1}^{n}\frac{e^{-1/\theta t_i}}{\theta^2 t_i\left(1-e^{-1/\theta t_i}\right)} \quad \ldots(12)$$

Equating eq. (11) and eq. (12) to zero. They are nonlinear equations, therefore, any iterative procedure such as Newton-Raphson method can be used to get a solution, and the formula of this method is given by

$$\begin{bmatrix}\lambda_{k+1}\\ \theta_{k+1}\end{bmatrix} = \begin{bmatrix}\lambda_k\\ \theta_k\end{bmatrix} - J^{-1}\begin{bmatrix}g_1(\lambda)\\ g_2(\theta)\end{bmatrix} \quad \ldots(13)$$

Where

$$g_1(\lambda) = \frac{\partial \log L}{\partial\lambda} \text{ and } g_2(\theta) = \frac{\partial \log L}{\partial\theta}$$

$$J = \begin{bmatrix}\frac{\partial g_1(\lambda)}{\partial\lambda} & \frac{\partial g_1(\lambda)}{\partial\theta}\\ \frac{\partial g_2(\theta)}{\partial\lambda} & \frac{\partial g_2(\theta)}{\partial\theta}\end{bmatrix} \quad \ldots(14)$$

$$\frac{\partial g_1(\lambda)}{\partial\lambda} = \frac{-n}{\lambda^2} \quad \ldots(15)$$

$$\frac{\partial g_1(\lambda)}{\partial\theta} = \frac{\partial g_2(\theta)}{\partial\lambda} = \sum_{i=1}^{n} \frac{-e^{-1/\theta t_i}}{\theta^2 t_i\,(1-e^{-1/\theta t_i})} \qquad \dots(16)$$

$$\frac{\partial g_2(\theta)}{\partial\theta} = \frac{n}{\theta^2} - \sum_{i=1}^{n}\frac{2}{\theta^3 t_i} - (\lambda-1)\sum_{i=1}^{n}\frac{e^{-1/\theta t_i}\left[\left(1-e^{-1/\theta t_i}\right)(1-2\theta t_i)-1\right]}{[\theta^2 t_i\,(1-e^{-1/\theta t_i})]^2}$$

$\dots(17)$

Where λ_k and θ_k are the initial values, which are imposed. This iteration process continues until convergence (i.e.)

$$\left\|\begin{bmatrix}\lambda_{k+1}\\ \theta_{k+1}\end{bmatrix} - \begin{bmatrix}\lambda_k\\ \theta_k\end{bmatrix}\right\| < \begin{bmatrix}\varepsilon\\ \varepsilon\end{bmatrix} \qquad \dots(18)$$

Where ε is pre-fixed greater than zero. Let $\varepsilon = 10^{-4}$. After satisfying the condition (18), the result $\hat{\lambda}$ and $\hat{\theta}$ are the maximum likelihood estimators of λ and θ respectively.

4- Bayes Estimation

In Bayesian method must assume that the parameter (parameters) is a random variable and has a prior distribution to be determined from previous information and experience. This approach is a combination between the prior distribution and the likelihood function.

To find the Bayesian estimator of the (GIE) distribution we assume the parameters of this distribution are random variables, It may be noted that the gamma distribution is flexible enough to cover wide variety of the prior experiments believes [7]. Therefore, λ and θ have prior gamma distributions as follows:

$$h_1(\lambda) = \frac{u_1^{v_1} e^{-u_1\lambda}\lambda^{v_1-1}}{\Gamma(v_1)} \qquad ;\ \lambda, u_1, v_1 > 0 \qquad \dots(19)$$

$$h_2(\theta) = \frac{u_2^{v_2}\, e^{-u_2\theta}\theta^{v_2-1}}{\Gamma(v_2)} \qquad ;\ \theta, u_2, v_2 > 0 \qquad \dots(20)$$

The hyper- parameters u_1, v_1, u_2 and v_2 are known.

The joint prior PDF of λ and θ is

$$h(\lambda,\theta) = h_1(\lambda)h_2(\theta) \qquad \dots(21)$$

$$= \frac{u_1^{v_1}u_2^{v_2}}{\Gamma(v_1)\Gamma(v_2)}\,\lambda^{v_1-1}\,\theta^{v_2-1}e^{-(u_1\lambda+u_2\theta)} \qquad \dots(22)$$

A joint posterior distribution of λ and θ is:

$$\pi(\lambda,\theta|t_i)=\frac{h(\lambda,\theta)L(t_i;\lambda,\theta)}{\iint_0^\infty h(\lambda,\theta)L(t_i;\lambda,\theta)\,d\lambda\,d\theta} \quad ...(23)$$

$$\pi(\lambda,\theta|t_i)=\frac{\lambda^{v_1-1}\,\theta^{v_2-1}e^{-(u_1\lambda+u_2\theta)}\left(\frac{\lambda}{\theta}\right)^n\prod_{i=1}^n\left(\frac{1}{t_i^2}\right)e^{-\sum_{i=1}^n(1/\theta t_i)}\prod_{i=1}^n\left(1-e^{-1/\theta t_i}\right)^{\lambda-1}}{\iint_0^\infty \lambda^{v_1-1}\,\theta^{v_2-1}e^{-(u_1\lambda+u_2\theta)}\left(\frac{\lambda}{\theta}\right)^n\prod_{i=1}^n\left(\frac{1}{t_i^2}\right)e^{-\sum_{i=1}^n(1/\theta t_i)}\prod_{i=1}^n\left(1-e^{-1/\theta t_i}\right)^{\lambda-1}d\lambda\,d\theta} \quad ...(24)$$

The marginal posterior PDFs of λ and θ are given respectively by [5,7]

$$\pi_1(\lambda|t_i)=\int_0^\infty \pi(\lambda,\theta|t_i)d\theta \quad ...(25)$$

$$\pi_2(\theta|t_i)=\int_0^\infty \pi(\lambda,\theta|t_i)d\lambda \quad ...(26)$$

Note that the above integrals cannot be obtained directly, therefore; we must resort to numerical methods at their evaluation. By using Markov Chain Monte Carlo (MCMC) methods and specially Metropolis-Hasting algorithm to generate random samples from posterior distributions for two the parameters under study in order to be used as Bayesian estimators [6,8].

Posterior distributions can be written as follows:

$$\pi_1^*(\lambda|\theta,t_i)\ \propto\ \lambda^{v_1+n-1}e^{-\left[u_1-\sum_{i=1}^n\log\left(1-e^{-1/\theta t_i}\right)\right]\lambda} \quad ...(27)$$

$$\pi_2^*(\theta|\lambda,t_i)\ \propto\ \theta^{v_2-n-1}e^{-u_2\theta}\prod_{i=1}^n e^{-(1/\theta t_i)}\left(1-e^{-1/\theta t_i}\right)^{\lambda-1} \quad ...(28)$$

The following steps show how to get the above estimators

1- Set λ_0 and θ_0 as an initial values for λ and θ respectively.

2- Set $i=1$.

3- Generate a random sample for λ and θ from equations (27) and (28) respectively.

4- Repeat steps 2 and 3, m times.

5- To compute the Bayesian estimators of λ and θ by using the following equations:

$$\hat{\lambda} = \frac{1}{m-m_0}\sum_{i=1}^{m-m_0}\lambda_i \qquad \ldots(29)$$

$$\hat{\theta} = \frac{1}{m-m_0}\sum_{i=1}^{m-m_0}\theta_i \qquad \ldots(30)$$

This means that each of $\hat{\lambda}$ and $\hat{\theta}$ represent the average number of samples generated randomly from the posterior distributions and m_0 represent the number of random samples that have been generated at the beginning of the experiment, which are neglected in order not to cause abnormalities in the estimated values (burn-in- period).

It is worth mentioning that the Metropolis-Hasting algorithm depends (in addition to the posterior distribution) on additional distributions, the first is called the random generating distribution, which gives an initial value to base the generating process on, and the second is called the proposal distribution, which is based on the selection of a posterior value of the random sample depending on the current value $\rho(t_{i+1}|t_i)$.

Both distributions (random generation distribution and the proposal distribution) Requires that their corresponding ranges are equivalent to the parameters ranges under study ($\lambda, \theta > 0$). In this paper, both of the proposal and the random generation distributions were distributed according the gamma distribution with 5000 iterations for each random sample.

4- Fuzzy Set Theory

The following introduces some concepts of fuzzy set theory.

Definition: Let T be a nonempty set. A fuzzy set $\tilde{A}$ in T is defined as a set of ordered pairs; $\tilde{A} = \{(t, \mu_{\tilde{A}}(t)) \mid t \in T\}$, where $\mu_{\tilde{A}}(t)$ is called the membership function for the fuzzy set $\tilde{A}$, it's defined as [2] :-

$$\mu_{\tilde{A}}(t): T \to [0,1]$$

Definition: a fuzzy subset $\tilde{A}$ of a real number R with membership function $\mu_{\tilde{A}}(t): R \to [0,1]$ is said to be a fuzzy number if

i. **$\tilde{A}$ is normal, i.e. $\exists$ an element t_0 s.t. $\mu_{\tilde{A}}(t_0) = 1$.**
ii. **$\mu_{\tilde{A}}(t)$ is upper semi-continuous membership function.**
iii. **$\tilde{A}$ is fuzzy convex, i.e.**

iv. $\mu_{\tilde{A}}(\delta t_1 + (\delta - 1)t_2) \geq \min\left(\mu_{\tilde{A}}(t_1), \mu_{\tilde{A}}(t_2)\right) \ \forall\, t_1, t_2 \in T, \forall\, \delta \in [0,1]$.

v. **Support of** $\tilde{A} = \{t \in T: \mu_{\tilde{A}}(t) > 0\}$ **is bounded** [3].

Definition: A fuzzy number $\tilde{A}$**is a triangular fuzzy number denoted by (a, b, c) where a, b and c are real numbers and their membership function** $\mu_{\tilde{A}}(t)$ **is given by:-**

$$\mu_{\tilde{A}}(t) = \begin{cases} \dfrac{t-a}{b-a} & a \leq t \leq b \\ 1 & t = b \\ \dfrac{c-t}{c-b} & b \leq t \leq c \end{cases} \qquad (31)$$

Remark: The triangular fuzzy number $\tilde{A} = (a, b, c)$**, in this paper The researcher assume that**

$a = t - 0.05t, b = t$ and $c = t + 0.07t$ **, then**

$$\tilde{A} = (t - 0.05t, t, t + 0.07t) \qquad \dots(32)$$

Eq. (32) called is a triangular fuzzy data this is because t **represented a data satisfying the (GIE) distribution. The defuzzification methods must be to convert a fuzzy number approximately to a crisp number so that this can be used efficiently in practical applications** [3]**.**

One important defuzzification methods is centroid method where the centroid $C(\tilde{A})$ **of fuzzy number** $\tilde{A}$ **is defined for continuous case by** [3,9,]**.**

$$C(\tilde{A}) = \frac{\int t\, \mu_{\tilde{A}}(t)dt}{\int \mu_{\tilde{A}}(t)dt} \qquad \dots(33)$$

Then the Centroid Method of triangular fuzzy number is defined as

$$C(\tilde{A}) = \frac{\int_a^c t\, \mu_{\tilde{A}}(t)dt}{\int_a^c \mu_{\tilde{A}}(t)dt} \qquad \dots(34)$$

$$C(\tilde{A}) = \frac{\int_a^b t\left[\frac{(t-a)}{(b-a)}\right]dt + \int_b^c t\left[\frac{(c-t)}{(c-b)}\right]dt}{\int_a^b \left[\frac{(t-a)}{(b-a)}\right]dt + \int_b^c \left[\frac{(c-t)}{(c-b)}\right]dt} \qquad \dots(35)$$

$$C(\tilde{A}) = \frac{1}{3}[a + b + c] \qquad \dots(36)$$

6- Simulation Experiments

To apply what has been mentioned, the necessary data has been simulated using MATLAB 2015a.Choosing the sample sizes n= 14, 25, 50 and100 and the default parameters values $\lambda = 0.3, 0.5$ and 0.8 and $\theta = 1, 1.5$and 2 and by using eq. (8) to generate a random number follows the (GIE) distribution. To obtain a fuzzy data set for all sizes by using eq. (32). Fuzzy data are converted to crisp data by using eq. (36). Estimating the model parameters by using the MLE and Bayesian methods. Finally, the estimators have been compared by using the mean squared error of the model, which it is given by

$$MSE\left(\hat{f}(t)\right) = \frac{1}{r}\sum_{i=1}^{r}\left[\hat{f}(t_i) - f(t_i)\right]^2$$

Where r represents the number of experiment replicates (In this paper r =500) while $\hat{f}(t_i)$ is obtained by substituting the estimates of λ and θ (for each method) in eq.(2).

The mean squared error for λ and θ given respectively by [2]

$$MSE(\hat{\lambda}) = \frac{1}{r}\sum_{i=1}^{r}\left[\hat{\lambda}_i - \lambda_i\right]^2$$

$$MSE(\hat{\theta}) = \frac{1}{r}\sum_{i=1}^{r}\left[\hat{\theta}_i - \theta_i\right]^2$$

The following contains the tables and the figures for the simulation results.

Table (1) Simulation results for sample size 14 and different values of λ and θ

N	estimators method	$\lambda=0.3$			$\lambda=0.5$			$\lambda=0.8$		
		$\theta=1$	$\theta=1.5$	$\theta=2$	$\theta=1$	$\theta=1.5$	$\theta=2$	$\theta=1$	$\theta=1.5$	$\theta=2$
	estimators method	MLE								
1	$\hat{\lambda}$	0.340947977	0.324949453	0.322151993	0.600603195	0.579313044	0.571078007	0.775363713	0.864873595	1.018299516
	$MSE(\hat{\lambda})$	3.86E-07	8.08E-07	1.42E-06	1.36E-06	2.97E-06	5.13E-06	2.72E-06	7.28E-06	1.67E-05
	$\hat{\theta}$	0.985260954	1.48817165	1.992363675	0.964245669	1.471669143	1.972375532	0.998088292	1.464814626	1.9244915
	$MSE(\hat{\theta})$	2.22E-13	2.92E-11	1.53E-12	6.47E-07	6.46E-10	1.63E-16	3.85E-15	6.59E-10	7.67E-12
	$MSE(\hat{f}(t))$	4.36E-05	9.89E-05	0.000178532	4.73E-05	9.71E-05	0.000174071	4.44E-05	9.56E-05	0.000166476
	estimators method	Bayes								
	$\hat{\lambda}$	0.340294957	0.326951783	0.324781246	0.587434661	0.566977317	0.5657196	0.76953052	0.844501905	0.934649824
	$MSE(\hat{\lambda})$	0.000562793	0.000300951	0.000655221	0.001401671	0.003958425	0.005378434	0.000308833	0.002088346	0.015546328
	$\hat{\theta}$	1.214284791	1.597232148	1.886092607	1.158476486	1.682099177	2.070136458	1.040620144	1.576485885	2.098517105
	$MSE(\hat{\theta})$	0.000479402	7.61E-05	0.000280085	0.000936744	0.002782309	0.001072981	0.000448777	0.001472485	0.047818488
	$MSE(\hat{f}(t))$	0.080772728	0.071548805	0.100815225	0.054415459	0.157732572	0.128928842	0.006576752	0.031795216	0.200415761

Table (2) Simulation results for sample size 25 and different values of λ and θ

		$\lambda = 0.3$			$\lambda = 0.5$			$\lambda = 0.8$		
		$\theta = 1$	$\theta = 1.5$	$\theta = 2$	$\theta = 1$	$\theta = 1.5$	$\theta = 2$	$\theta = 1$	$\theta = 1.5$	$\theta = 2$
N	estimators method	MLE								
25	$\hat{\lambda}$	0.324968427	0.343194732	0.31742911	0.544933827	0.533502579	0.526301512	0.79383925	0.866963023	0.871219891
	$MSE(\hat{\lambda})$	3.49E-07	8.23E-07	1.29E-06	1.09E-06	2.39E-06	4.07E-06	2.61E-06	7.29E-06	1.32E-05
	$\hat{\theta}$	0.989066978	1.455356406	2.009834315	0.978488082	1.48184365	1.976325146	0.996399367	1.477310954	1.957498098
	$MSE(\hat{\theta})$	4.43E-07	7.50E-12	5.65E-16	3.98E-10	9.66E-14	1.98E-12	5.37E-09	4.65E-12	3.26E-07
	$MSE\left(\hat{f}(t)\right)$	4.36E-05	9.55E-05	0.000181256	4.29E-05	9.84E-05	0.000174414	4.42E-05	9.72E-05	0.000170039
	estimators method	Bayes								
	$\hat{\lambda}$	0.327249099	0.34071283	0.321578258	0.540251768	0.531195413	0.528476978	0.796864246	0.861857127	0.851543164
	$MSE(\hat{\lambda})$	0.000327986	0.000880874	0.000881059	0.000900452	0.00128912	0.001212135	0.000440347	0.002792854	0.004496749
	$\hat{\theta}$	0.989066978	1.455356406	2.009834315	0.978488082	1.48184365	1.976325146	0.996399367	1.477310954	1.957498098
	$MSE(\hat{\theta})$	0.000109252	4.66E-04	0.000108564	0.000457642	0.000819739	0.000269749	0.000588098	0.00103158	0.004636221
	$MSE\left(\hat{f}(t)\right)$	0.05041138	0.116983147	0.105368343	0.039400798	0.05348049	0.054071228	0.007075251	0.037475454	0.070405139

Table (3) Simulation results for sample size 50 and different values of λ and θ

		$\lambda = 0.3$			$\lambda = 0.5$			$\lambda = 0.8$		
		$\theta = 1$	$\theta = 1.5$	$\theta = 2$	$\theta = 1$	$\theta = 1.5$	$\theta = 2$	$\theta = 1$	$\theta = 1.5$	$\theta = 2$
N	estimators method	MLE								
50	$\hat{\lambda}$	0.313631671	0.304950696	0.320164155	0.538032837	0.526690313	0.521120545	0.772480952	0.840746386	0.846760763
	$MSE(\hat{\lambda})$	3.04E-07	6.14E-07	1.20E-06	1.08E-06	2.31E-06	4.03E-06	2.54E-06	6.64E-06	1.22E-05
	$\hat{\theta}$	0.999814625	1.500105565	1.981973821	0.984404744	1.485790897	1.957514332	1.002240674	1.483329936	1.979459607
	$MSE(\hat{\theta})$	5.35E-15	4.09E-19	3.42E-13	4.98E-12	4.43E-10	2.36E-07	9.72E-16	1.20E-13	1.06E-11
	$MSE\left(\hat{f}(t)\right)$	4.46E-05	1.00E-04	0.000175673	4.32E-05	9.86E-05	0.000166098	4.47E-05	9.80E-05	0.000174666
	estimators method	Bayes								
	$\hat{\lambda}$	0.31505561	0.306285206	0.321158947	0.536461655	0.525558559	0.518119712	0.776836994	0.837180374	0.837276644
	$MSE(\hat{\lambda})$	0.000114888	0.000143504	0.000366997	0.000397686	0.000594161	0.001264195	0.000226109	0.001168895	0.00367116
	$\hat{\theta}$	1.111308706	1.598346967	2.142182567	1.077063362	1.591394794	2.053643753	1.00406525	1.560410104	2.063848747
	$MSE(\hat{\theta})$	5.35E-15	4.09E-19	3.42E-13	4.98E-12	4.43E-10	2.36E-07	9.72E-16	1.20E-13	1.06E-11
	$MSE\left(\hat{f}(t)\right)$	4.46E-05	1.00E-04	0.000175673	4.32E-05	9.86E-05	0.000166098	4.47E-05	9.80E-05	0.000174666

Table (4) Simulation results for sample size 100 and different values of λ and θ

		$\lambda = 0.3$			$\lambda = 0.5$			$\lambda = 0.8$		
		$\theta = 1$	$\theta = 1.5$	$\theta = 2$	$\theta = 1$	$\theta = 1.5$	$\theta = 2$	$\theta = 1$	$\theta = 1.5$	$\theta = 2$
N	**estimators method**	**MLE**								
100	$\hat{\lambda}$	**0.30378944**	**0.306250532**	**0.305403042**	**0.539112205**	**0.505604088**	**0.518816499**	**0.799174478**	**0.829342258**	**0.828470511**
	$MSE(\hat{\lambda})$	**2.59E-07**	**6.08E-07**	**1.04E-06**	**1.10E-06**	**2.12E-06**	**3.93E-06**	**2.67E-06**	**6.54E-06**	**1.18E-05**
	$\hat{\theta}$	**0.991041676**	**1.485146872**	**1.989360457**	**0.988936869**	**1.485427087**	**1.977753542**	**0.993771087**	**1.485510659**	**1.999784535**
	$MSE(\hat{\theta})$	**2.86E-11**	**2.11E-10**	**7.01E-12**	**1.52E-14**	**3.21E-07**	**7.22E-20**	**7.26E-17**	**3.91E-08**	**6.93E-12**
	$MSE(\hat{f}(t))$	**4.37E-05**	**9.85E-05**	**0.000176322**	**4.36E-05**	**9.72E-05**	**0.000174252**	**4.40E-05**	**9.84E-05**	**0.000178146**
	estimators method	**Bayes**								
	$\hat{\lambda}$	**0.304614608**	**0.306743968**	**0.306359495**	**0.53692058**	**0.504946449**	**0.517025444**	**0.797219883**	**0.825779284**	**0.829229224**
	$MSE(\hat{\lambda})$	**3.32945E-05**	**5.90775E-05**	**9.78869E-05**	**0.000309976**	**0.000162357**	**0.000382794**	**0.000126295**	**0.000766404**	**0.000978033**
	$\hat{\theta}$	**1.034129739**	**1.537667109**	**2.033862528**	**1.059569889**	**1.510797633**	**2.043632591**	**1.005185185**	**1.531751377**	**2.042713487**
	$MSE(\hat{\theta})$	**2.17967E-05**	**3.40E-05**	**2.37044E-05**	**0.000168734**	**0.000133799**	**0.000116051**	**0.000493027**	**0.000366953**	**0.000549643**
	$MSE(\hat{f}(t))$	**0.006050246**	**0.011531579**	**0.021644866**	**0.013024435**	**0.008877315**	**0.018707879**	**0.00229846**	**0.011860465**	**0.017076308**

Figure (1) an illustrative overview to curve of the (GIE) distribution for estimation methods when n = 100 and $\lambda = 0.3$ and $\theta = 1, 1.5$ and 2.

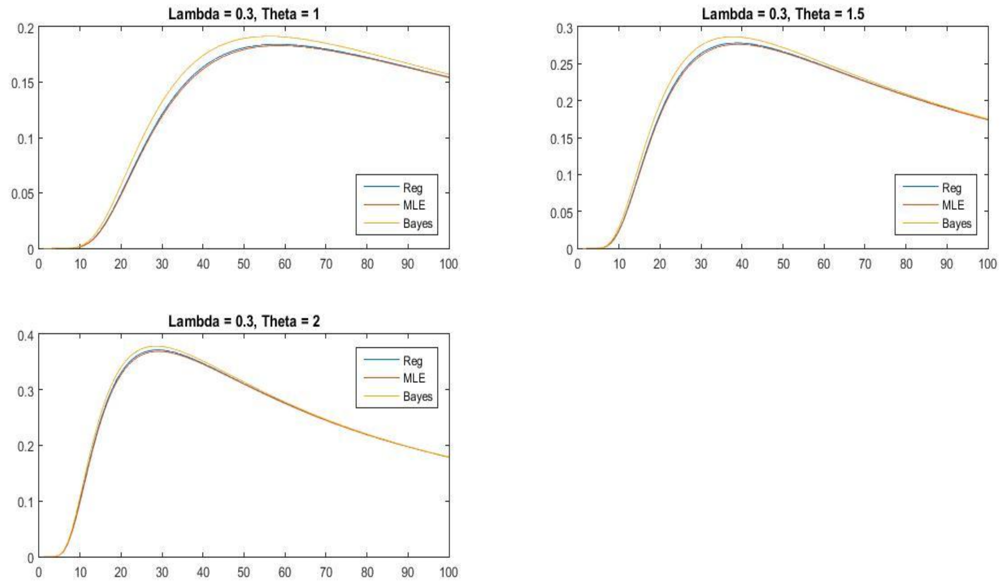


Figure (2) an illustrative overview to curve of the (GIE) distribution for estimation methods when n = 100 and $\lambda = 0.5$ and $\theta = 1, 1.5$ and 2.

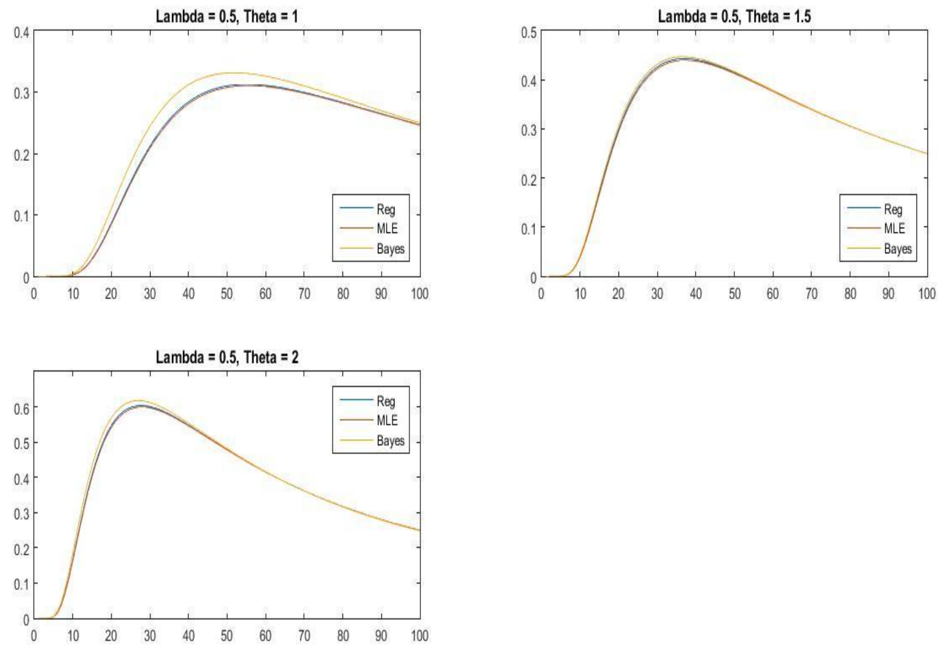
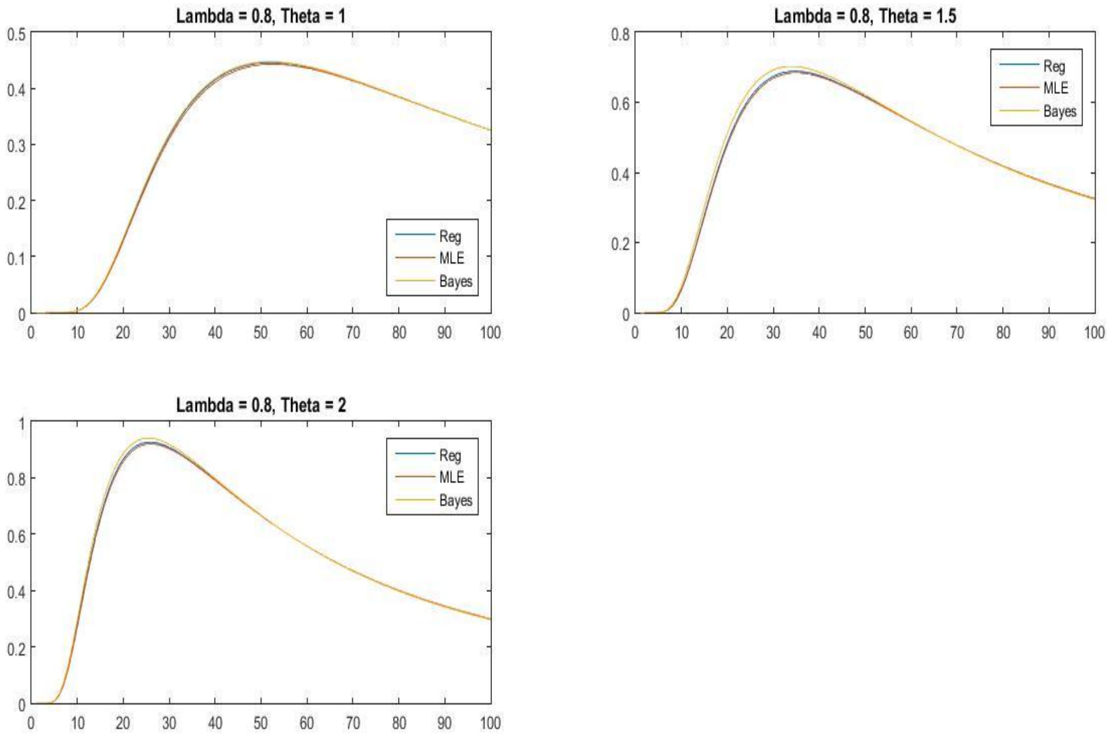


Figure (3) an illustrative overview to curve of the (GIE) distribution for estimation methods when n = 100 and $\lambda = 0.8$ and $\theta = 1, 1.5$ and 2.



7-Conclusions

It can be seen that after removing the fuzziness from the data by using the centroid method that the MLE method excels the Bayesian approximation method for all sample sizes and for all default values of shape and scale parameters. The effeciency of the MLE method decreases by the increase of the default value if the scale parmeter but they vary in the Bayesian method.

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مقارنة مقدر بيز مع طريقة الامكان الاعظم لتقدير معلمتي معكوس التوزيع الاسي المعمم في حالة ضبابية البيانات

المستخلص:

في هذا البحث تم تقدير معلمتي الشكل والقياس لمعكوس التوزيع الاسي المعمم والذي يعد من التوزيعات المهمة في دراسة اوقات الفشل ولكن بعد ازالة الضبابية التي تتصف بها بياناته إذ ان بياناته عبارة عن اعداد ضبابية ثلاثية ولتحويلها إلى اعداد اعتيادية تم استخدام (centroid method). وبما أن التوزيع المدروس ذو معلمتين فكان من الصعوبة الفصل بين المعلمتين وتقديرهما بشكل مباشر ففي طريقة الإمكان الاعظم تم الاستعانة بطريقة نيوتن رافسون التكرارية. اما المقدرات البيزية فقد تم الحصول عليها بفرض توزيع كاما كتوزيع اولي لمعلمتيه ومن ثم استعمال دالة الخسارة التربيعية وبالاعتماد على خوارزمية Metropolis-Hasting . وتم توليد عينات مختلفة تمثل المجتمع المدروس باستخدام اسلوب المحاكاة. وبعد تقدير معلمتي التوزيع ومقارنة نتائج طريقتي التقدير وفق مقياس متوسط مربعات الخطأ. تم التوصل الى أن افضل طريقة كانت طريقة الامكان الاعظم تليها الطريقة البيزية.

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