Journal of Economics and Administrative Sciences Vol.26 (NO. 123) 2020, pp. 158-175

Journal of Economics and Administrative Sciences (JEAS)

Available online at http://jeasiq.uobaghdad.edu.iq

Building a model to reduce the cost of hajj in the Iraqi Authority for Hajj and Umrah[1]

Haidar A. Abood(1)
University of Baghdad, College of Management and Economics, Department of Statistics, Baghdad, Iraq,
h.abdulretha76@gmail.com

Sabah M. Retha(2)
University of Baghdad, College of Management and Economics, Department of Statistics, Baghdad, Iraq,
drsabah@coadec.uobaghdad.edu.iq

Received: 25/4/2020 **Accepted :9/8/2020** **Published: November / 2020**

Abstract

The research is summarized in the construction of a mathematical model using the most common methods in the science of Operations Research, which are the models of transportation and linear programming to find the best solution to the problem of the high cost of hajj in Iraq, and this is done by reaching the optimum number of pilgrims traveling through both land ports and the number Ideal for passengers traveling through airports by Iraqi Airways, instead of relying on the personal experience of the decision-maker in Hajj and Umrah Authority by identifying the best port for pilgrim's travel, which can tolerate right or wrong, has been based on scientific methods of Operations Research, the researcher built two mathematical models, the first model was formulated in the form of a Transport problem and its goal is to determine the optimal number of traveling pilgrims through air or land ports and each province with the identification of the best port for travel (the least expensive), while the second model specifies the optimal duration of stay pilgrims in Saudi Arabia, taking into account the limitations of the problem, So it was the result reduce the total cost of hajj by (15,107,574.8) U.S. dollars or 11 percent. The results also included the preparation of a comprehensive plan for the development of transporting pilgrims, which the decision-makers of the Authority can use to transport pilgrims from Iraq to Saudi Arabia in the coming years.

Keywords: **Building Models, Linear Programming, Mathematical models, Operations Research, Transportation Problems.**

[1] The research is derived from a master's thesis

1. Introduction

Hajj, which is the fifth fundamentals in Islam, is considered one of the most important things in the Sunnah for Muslims in general and Iraqis in particular, and the cost of hajj is one of the most important obstacles that prevent many citizens, especially those with limited income from applying for hajj and completing this important fundamentals of Islam, and to avoid relying on personal experience in calculating the cost and not relying on modern scientific methods in calculating the real cost of hajj by the Authority, the researcher was born the desire to contribute to addressing this problem based on the successful experiences and many modern methods. For operations research that will help decision-makers to choose the best ways to solve the various administrative problems, including decision-making problems to choose the methods of the lowest cost, we can also determine the optimal number of passengers through airports or through land ports to take advantage of the cost difference between the two types of travel and reduce the number of losses resulting from the unnecessary preparations for some ports in the absence of the correct number of travel through them, and other things that can eventually reduce the total cost of hajj, which allows for a greater number of applying for hajj, especially those with limited income. During the research, some of the tools for operations research that wide-ranging processes, namely transport models and linear programming models, will be used extensively in addressing the problems facing management such as production, distribution, transportation, and other activities through which it seeks to maximize profits and returns or reduce losses and costs. One of the most important objectives achieved when using operations research is to assist the administration (be it senior management or otherwise) in making rational decisions (optimal), where the decision-making process is the basis or core of the administrative process, and the researcher's quest through the selection of the problem of research was to employ the available alternatives to help the decision-maker (management of the authority) in choosing the most suitable and the most correct ones for pilgrims of any province and according to his own vision. The researcher aims to build a mathematical model to find the optimal number of pilgrims traveling from each province in addition to determining the optimal number of travel through each of the Iraqi border ports, whether land ports or airport, which in turn is expected to reduce the total cost of hajj, which helps to reduce the amount of money that needs to be paid to the Authority and thus increase the number of applicant's hajj from the limited income citizens. The outcomes of the transport problem, which represents the preparation of an integrated transport plan for products and goods, taking into account the minimum cost of the problem and providing the requirements of the requests as much as possible, will also be used to prepare an integrated plan for the development transport of Iraqi pilgrims from all provinces and through all the available outlets (Bakhit & Hashim, 2016), (Noor & Sahad, 2012), (Mohammed, 2015).

2 .Methods and Tools

1.1. Definition of mathematical model:

The mathematical model is one of the most important steps in the methods of process research where it is known as a representation of the problem with all its components and factors affecting and circumstances surrounding it where the problem is transformed from its real complex world to another easier to understand and solve, where the results reached can solve The model is to be a solution to the real problem. Process research in its study is based on mechanisms and methods that are often specific, starting with studying the problem in question from all its aspects and identifying the objectives of the problem and its decision variables and all the data required and available about the problem. Then begin the stage of drafting and building the appropriate mathematical model for it, which we seek to achieve the best value for the goal whether it is by maximizing profits or reducing losses and costs, after which the results reached and testing the solution are applied (Hassan& et al, 2013), (Ashour & Abaas, 2019), (Hillier 2012).

2.2. Steps to build the mathematical model:

For the purpose of building the mathematical model of the problem under study and research, the system analyst or operations research team should first begin to study the situation and gather all the information in a crisis, and if there is something unclear, they should consult with the decision-makers for the purpose of clarifying any ambiguity. Then begin to analyze the problem accurately with the identification of decision variables and the limitations of the problem, and in the same way, the goals are identified and placed all in the form of terms or formulas containing an accurate description of the objectives of the study with a description and identification of variables whether controlled or variables that we cannot control With them, these formulas also include problem constraints that affect the values of the resolution variables of the target function, after which the problem is formulated (P., 2007) , (Rahi & Laibi, 2018). In general, there are a number of reasons why we use and build models, including (Bakhit, et al, 2015):

1- The problem cannot be transferred under study and research from one place to another.
2- The possibility of limiting the specifications of the problem.
3- The high cost of dealing with the problem.
4- The seriousness of dealing with some problems.
5- Difficulty dealing with the problem directly.

2.3. Some points to consider when building the model (P., 2007):

1- Do not try to build a complex model when we can solve the issue using a simple model.
2- The construction of the model according to the available possibilities and techniques, i.e. we work to build a model that can be solved using certain techniques available and not vice versa, i.e. we work to build a model and then look for the appropriate techniques to solve it.

3- To avoid complications while solving the problem, the wording of the model must be very accurate.
4- The form is used according to the purpose or problem for which it was designed and cannot be used to deal with other different situations.
5- It is not possible to use and apply the model without having a clear picture of the reason for which it was built, preferably consulting specialists in the field of operations research and taking their guidance.
6- The model should be as simple as possible and at the same time very precise.

2.4. The general formula of the linear programming model:

The general version of the linear programming model can be expressed by the following law (Alqussi, 2015), (Bakhit & Hashim, 2016):

A- The target function is either Maximize or Minimize.

Maximum or Minimum $Z = K_1X_1 + K_2X_2 + \dots + K_mX_m$

B- Structural restrictions:

$C_{11}X_1 +$	$C_{12}X_2 +$	..........+	$C_{1m}X_m$	$(\leq, =, \geq)$	B_1
$C_{21}X_1 +$	$C_{22}X_2 +$	..........+	$C_{2m}X_m$	$(\leq, =, \geq)$	B_2
:	:	:	:	:	:
:	:	:	:	:	:
$C_{n1}X1 +$	$C_{n2}X_2 +$	..........+	$C_{nm}X_m$	$(\leq, =, \geq)$	B_n

C- Non-negative: $X_1, X_2, \dots, X_m \geq 0$

Where:

Z= represents the value of the target function and is either maximizing or reducing.

K= represents the variable coefficient of the target function such as unit cost or profit, etc.

X= represents decision variables.

C= The quantity of resources a unit needs is as much as it requires to manufacture a unit of a particular product of raw materials, workers, working hours, etc.

m= the number of decision variables.

n= number of structural constraints of the problem.

B= available or available resources.

2.4.1. Methods used to solve the linear programming model:

There are several methods used to solve mathematical models of linear programming (Alshemerty, 2010):

1. The graphic method.
2. Simplified method.
3. The algebraic method.

The first and second methods are one of the main methods used to solve and process linear programming models.

2.5. The Mathematical formula of the Transport Model:

For the purpose of clarifying the mathematical formula of the transport model, we assume that we have (n) sources (stores, factories, companies, etc.) owns different quantities of certain goods and is required to be transported to (m) destinations (cities, markets, customers, etc.), so we can express the problem mathematically, as follows (Dantzig & Thapa, 2006):

Minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{m} K_{ij} * X_{ij}$

Subject to:

$\sum_{j=1}^{m} X_{ij} \leq M_i \quad , \quad i = 1, 2, \ldots, n$

$\sum_{i=1}^{n} X_{ij} \geq D_j \quad , \quad j = 1, 2, \ldots, m$

$X_{ij} \geq 0, \quad i = 1, 2, \ldots, n \ , \quad j = 1, 2, \ldots, m$

Where:

X_{ij} = the quantity that is transferred from the source (i) to the duty stations of demand (j).

K_{ij} = the cost of transporting the required quantity from its available source (i) to the order area (j).

A_i = the quantities of goods or services available are represented at the source (i).

D_j = represents the required quantities of goods or services in the region (j).

n = represents the number of sources where goods or goods are available (warehouse, factory, company, etc.).

m = represents the number of destinations requesting goods or goods (markets, cities, customers, etc.).

2.5.1. Ways to solve the transport problem:

In order to find a solution to a transport problem, the solution goes through two phases (Alshemerty, 2010):

1. The first stage: to find the basic solution possible.
2. The second stage: is to test the basic solution possible to reach the optimal solution to the problem.

We can use one of the following methods to find the first possible basic solution, and these methods are:

1. North-West Corner Method.
2. Least Cost Method.
3. Vogel's Method.
4. Russell's Method.

The solution resulting from the above roads is a possible initial solution and does not necessarily represent the best solution to the transport problem, so you should use methods to test this solution and make sure that there is no other better and better solution than the initial possible solution, and this is done using one of the following methods:

1. Stepping-Stone Method.
2. Modified Distribution Method.

2.6. Comparison of mathematical models of linear programming and transport

First: There is an argument for the similarity between the two models (P., 2007):

1. Possess both models for a goal function.
2. The target function of the two models is linear.
3. Neither model has negative restrictions.

Secondly, the difference is as to the difference between the two methods:

1. The target function in the transport model is of the type of reduction or miniaturization (Min.), either in linear programming is of the max type or of the reduction type (Min.).

2. Resources for which structural constraints are built are in the transport model homogeneous such as a particular resource or commodity and of the same sex, but in the linear programming model the restrictions are different, for example, one of the restrictions for working hours and the other restriction is for raw materials, and so on.
3. The transport problem is solved using the transport algorithm, but the general linear programming problem is solved by simplex.
4. The values of decision variables such as X_{ij} in structural constraints are not specified at a certain value in the general linear programming model, but in the transmission, model is defined as (1.0).

3. Data analysis

The mathematical model data of the research problem (the cost of hajj) was collected through visiting the headquarters of the Authority for Hajj and Umrah and meeting with the officials of the Administrative and Financial Department, the Planning and Follow-up Department, the Service Department, the Department of Religious Guidance, Studies and Research, in addition to the directors of the departments of Transport, Information Technology Centre and Financial department in the Authority, where the cooperation of the brothers in the Authority had a great effect in obtaining all the data necessary to build the model as required, and the data included both the cost of transport the pilgrims by the air or land ports, in addition to the cost of housing and food in Medina and Mecca, as well as data on the mechanism for distributing Iraq's share of pilgrims to the provinces and institutions and the preparation of administrative cadres accompanying the convoys.

The researcher turned the problem of research into a transfer problem to find the optimal values for the numbers traveling through both air and land ports and this was the first mathematical model, the second mathematical model used the method of linear programming to find the optimal number of days the pilgrim spends in the K.S.A., for the purpose of reducing the total cost as much as possible.

After the researcher turned the research problem into a transport problem, he built table 3.1 in the form of a transportation problem table, which covers all probability for transporting pilgrims from their areas of residence to Saudi Arabia (K.S.A.) using all the air and land ports available in Iraq.

Table 3.1: The cost of pilgrim transport from Iraqi provinces to K.S.A.

	International Airport Baghdad		International Airport Basra		International Airport Najaf		International Airport Erbil		International Airport Sulimaniyah		Arar port		Safwan port		Formation share
Baghdad I	1461	X_{11}	1476	X_{12}	1470	X_{13}	1536	X_{14}	1536	X_{15}	794	X_{16}	799	X_{17}	1478
Baghdad II	1461	X_{21}	1476	X_{22}	1470	X_{23}	1536	X_{24}	1536	X_{25}	794	X_{26}	799	X_{27}	2077
Baghdad III	1461	X_{31}	1476	X_{32}	1470	X_{33}	1536	X_{34}	1536	X_{35}	794	X_{36}	799	X_{37}	2596
Mousel	1476	X_{41}	1496	X_{42}	1486	X_{43}	1516	X_{44}	1521	X_{45}	798	X_{46}	803	X_{47}	1988
Gazera	1476	X_{51}	1496	X_{52}	1486	X_{53}	1516	X_{54}	1521	X_{55}	798	X_{56}	803	X_{57}	735
Kirkuk	1476	X_{61}	1496	X_{62}	1486	X_{63}	1516	X_{64}	1517	X_{65}	797	X_{66}	802	X_{67}	1171
Diyala	1466	X_{71}	1486	X_{72}	1476	X_{73}	1531	X_{74}	1526	X_{75}	795	X_{76}	800	X_{77}	1190
Anbar	1466	X_{81}	1486	X_{82}	1476	X_{83}	1536	X_{84}	1536	X_{85}	792	X_{86}	797	X_{87}	1288
Karbala	1466	X_{91}	1486	X_{92}	1466	X_{93}	1536	X_{94}	1535	X_{95}	792	X_{96}	797	X_{97}	892
Wasit	1473	X_{101}	1486	X_{102}	1481	X_{103}	1546	X_{104}	1544	X_{105}	795	X_{106}	798	X_{107}	1006
Salah-Alden	1471	X_{111}	1486	X_{112}	1481	X_{113}	1521	X_{114}	1521	X_{115}	795	X_{116}	800	X_{117}	1158
Najaf	1470	X_{121}	1476	X_{122}	1461	X_{123}	1536	X_{124}	1536	X_{125}	793	X_{126}	800	X_{127}	1078
Babylon	1466	X_{131}	1486	X_{132}	1466	X_{133}	1536	X_{134}	1536	X_{135}	793	X_{136}	800	X_{137}	1501
Qadisiyah	1471	X_{141}	1476	X_{142}	1466	X_{143}	1536	X_{144}	1536	X_{145}	793	X_{146}	800	X_{147}	941
Muthanna	1476	X_{151}	1474	X_{152}	1471	X_{153}	1546	X_{154}	1545	X_{155}	794	X_{156}	799	X_{157}	591
Dhi-Qar	1486	X_{161}	1471	X_{162}	1476	X_{163}	1551	X_{164}	1549	X_{165}	796	X_{166}	794	X_{167}	1531

Maysan	**1486**	X_{171}	**1471**	X_{172}	**1486**	X_{173}	**1551**	X_{174}	**1549**	X_{175}	**796**	X_{176}	**794**	X_{177}	**816**
Martyrs of Regime	**1466**	X_{181}	**1476**	X_{182}	**1470**	X_{183}	**1536**	X_{184}	**1536**	X_{185}	**794**	X_{186}	**799**	X_{187}	**1422**
Basra	**1496**	X_{191}	**1461**	X_{192}	**1486**	X_{193}	**1556**	X_{194}	**1546**	X_{195}	**798**	X_{196}	**792**	X_{197}	**2138**
Martyrs of Terrorism	**1466**	X_{201}	**1476**	X_{202}	**1470**	X_{203}	**1536**	X_{204}	**1536**	X_{205}	**794**	X_{206}	**799**	X_{207}	**1422**
Martyrs of Crowd	**1466**	X_{211}	**1476**	X_{212}	**1470**	X_{213}	**1536**	X_{214}	**1536**	X_{215}	**794**	X_{216}	**799**	X_{217}	**1422**
Prisoners Politicians	**1466**	X_{221}	**1476**	X_{222}	**1470**	X_{223}	**1536**	X_{224}	**1536**	X_{225}	**794**	X_{226}	**799**	X_{227}	**550**
Erbil	**1486**	X_{231}	**1496**	X_{232}	**1496**	X_{233}	**1511**	X_{234}	**1521**	X_{235}	**797**	X_{236}	**802**	X_{237}	**1642**
Duhok	**1496**	X_{241}	**1496**	X_{242}	**1496**	X_{243}	**1516**	X_{244}	**1524**	X_{245}	**798**	X_{246}	**804**	X_{247}	**1143**
Sulimaniyah	**1486**	X_{251}	**1496**	X_{252}	**1496**	X_{253}	**1518**	X_{254}	**1511**	X_{255}	**797**	X_{256}	**802**	X_{257}	**1915**
Port absorption	**12000**		**5000**		**3000**		**4500**		**3000**		**20000**		**7000**		

The researcher adopted in calculating the cost of transport in table 4.1 on the data received from the Department of Transport in The Authority and then worked on the following equation:

Cost of transport (C_{ij}) through airports = cost of transportation to the airport + cost of transportation from the airport to the K.S.A. + cost of the K.S.A. institution of the air travellers + cost of the K.S.A. office unified agents for air travellers.

Thus, the cost of transportation for pilgrim by air-ports form any city is:

$$C_{ij} = C_5 + C_7 + C_1 + C_2$$

To calculate the cost of transport (C_{ij}) for pilgrim travelling through land ports, the following formula was used:

Cost of transport (C_{ij}) through land ports = cost of transportation from the lively city of a pilgrim to the K.S.A. + Cost of the K.S.A. Motawfin Foundation for land travellers + cost of the K.S.A. office unified agents for pilgrim travel by land outlet.

Thus, the cost of transportation for passengers travelling through land ports from any city is:

$$C_{ij} = C6 + C3 + C4$$

The cost of transport (C_6) varies from province to province, depending on the distance from the province to the land border crossing. The definition of cost constants above is described in paragraph 4.3.1.2 of this research.

We note that the transport schedule prepared by the researcher is unbalanced because the amount of demand (the number of pilgrims transferred) is lower than the available possibilities (the capacity of the border ports), so the researcher worked on the use of signals (≤, ≥) in building model restrictions instead of adding a fictitious row to complete the balance of the transport schedule to ensure that the results of the solution in the winQSB program are not affected. Note that the restrictions of the model were formulated according to the circumstances of each province and the desire of the decision-maker in the commission.

The target function of the first mathematical model was reduced the cost of transportation, with a model of the transfer problem converted into a mathematical program in the linear programming model to calculate the lowest cost of pilgrim's transportation, as follows:

Minimize $Z = 1461 X_{11} + 1476 X_{12} + 1470 X_{13} + 1536 X_{14} + 1536 X_{15} + 794 X_{16} + 799 X_{17} +$
$1461 X_{21} + 1476 X_{22} + 1470 X_{23} + 1536 X_{24} + 1536 X_{25} + 794 X_{26} + 799 X_{27} +$
$1461 X_{31} + 1476 X_{32} + 1470 X_{33} + 1536 X_{34} + 1536 X_{35} + 794 X_{36} + 799 X_{37} +$
$1476 X_{41} + 1496 X_{42} + 1486 X_{43} + 1516 X_{44} + 1521 X_{45} + 798 X_{46} + 803 X_{47} +$
$1476 X_{51} + 1496 X_{52} + 1486 X_{53} + 1516 X_{54} + 1521 X_{55} + 798 X_{56} + 803 X_{57} +$
$1476 X_{61} + 1496 X_{62} + 1486 X_{63} + 1516 X_{64} + 1517 X_{65} + 797 X_{66} + 802 X_{67} +$
$1466 X_{71} + 1486 X_{72} + 1476 X_{73} + 1531 X_{74} + 1526 X_{75} + 795 X_{76} + 800 X_{77} +$
$1466 X_{81} + 1486 X_{82} + 1476 X_{83} + 1536 X_{84} + 1536 X_{85} + 792 X_{86} + 797 X_{87} +$
$1466 X_{91} + 1486 X_{92} + 1466 X_{93} + 1536 X_{94} + 1535 X_{95} + 792 X_{96} + 797 X_{97} +$
$1473 X_{101} + 1486 X_{102} + 1481 X_{103} + 1546 X_{104} + 1544 X_{105} + 795 X_{106} + 798 X_{107} +$
$1471 X_{111} + 1486 X_{112} + 1481 X_{113} + 1521 X_{114} + 1521 X_{115} + 795 X_{116} + 800 X_{117} +$
$1470 X_{121} + 1476 X_{122} + 1461 X_{123} + 1536 X_{124} + 1536 X_{125} + 793 X_{126} + 800 X_{127} +$
$1466 X131 + 1486 X132 + 1466 X133 + 1536 X134 + 1536 X135 + 793 X136 + 800 X137 +$
$1471 X141 + 1476 X142 + 1466 X143 + 1536 X144 + 1536 X145 + 793 X146 + 800 X147 +$
$1476 X151 + 1474 X152 + 1471 X153 + 1546 X154 + 1545 X155 + 794 X156 + 799 X157 +$
$1486 X161 + 1471 X162 + 1476 X163 + 1551 X164 + 1549 X165 + 796 X166 + 794 X167 +$
$1486 X171 + 1471 X172 + 1486 X173 + 1551 X174 + 1549 X175 + 796 X176 + 794 X177 +$
$1496 X181 + 1461 X182 + 1486 X183 + 1556 X184 + 1546 X185 + 798 X186 + 792 X187 +$
$1466 X191 + 1476 X192 + 1470 X193 + 1536 X194 + 1536 X195 + 794 X196 + 799 X197 +$

1466 X201 + 1476 X202 + 1470 X203 + 1536 X204 + 1536 X205 + 794 X206 + 799 X207 +
1466 X211 + 1476 X212 + 1470 X213 + 1536 X214 + 1536 X215 + 794 X216 + 799 X217 +
1466 X221 + 1476 X222 + 1470 X223 + 1536 X224 + 1536 X225 + 794 X226 + 799 X227 +
1486 X231 + 1496 X232 + 1496 X233 + 1511 X234 + 1521 X235 + 797 X236 + 802 X237 +
1496 X241 + 1496 X242 + 1496 X243 + 1516 X244 + 1524 X245 + 798 X246 + 804 X247 +
1486 X251 + 1496 X252 + 1496 X253 + 1518 X254 + 1511 X255 + 797 X256 + 802 X257

Subject to:

Restrictions of not exceeding the province's quota, and not to exceed the province's quota from the pilgrim that transportation via air ports.

$X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} + X_{17} = 1478$
$X_{11} + X_{12} + X_{13} + X_{14} + X_{15} \geq 443$
$X_{21} + X_{22} + X_{23} + X_{24} + X_{25} + X_{26} + X_{27} = 2077$
$X_{21} + X_{22} + X_{23} + X_{24} + X_{25} \geq 623$
$X_{31} + X_{32} + X_{33} + X_{34} + X_{35} + X_{36} + X_{37} = 2596$
$X_{31} + X_{32} + X_{33} + X_{34} + X_{35} \geq 779$
$X_{41} + X_{42} + X_{43} + X_{44} + X_{45} + X_{46} + X_{47} = 1988$
$X_{41} + X_{42} + X_{43} + X_{44} + X_{45} \geq 596$
$X_{51} + X_{52} + X_{53} + X_{54} + X_{55} + X_{56} + X_{57} = 735$
$X_{51} + X_{52} + X_{53} + X_{54} + X_{55} \geq 221$
$X_{61} + X_{62} + X_{63} + X_{64} + X_{65} + X_{66} + X_{67} = 1171$
$X_{61} + X_{62} + X_{63} + X_{64} + X_{65} \geq 351$
$X_{71} + X_{72} + X_{73} + X_{74} + X_{75} + X_{76} + X_{77} = 1190$
$X_{71} + X_{72} + X_{73} + X_{74} + X_{75} \geq 357$
$X_{81} + X_{82} + X_{83} + X_{84} + X_{85} + X_{86} + X_{87} = 1288$
$X_{81} + X_{82} + X_{83} + X_{84} + X_{85} \geq 90$
$X_{91} + X_{92} + X_{93} + X_{94} + X_{95} + X_{96} + X_{97} = 892$
$X_{91} + X_{92} + X_{93} + X_{94} + X_{95} \geq 267$
$X_{101} + X_{102} + X_{103} + X_{104} + X_{105} + X_{106} + X_{107} = 1006$
$X_{101} + X_{102} + X_{103} + X_{104} + X_{105} \geq 302$
$X_{111} + X_{112} + X_{113} + X_{114} + X_{115} + X_{116} + X_{117} = 1158$
$X_{111} + X_{112} + X_{113} + X_{114} + X_{115} \geq 90$
$X_{121} + X_{122} + X_{123} + X_{124} + X_{125} + X_{126} + X_{127} = 1078$
$X_{121} + X_{122} + X_{123} + X_{124} + X_{125} \geq 323$
$X_{131} + X_{132} + X_{133} + X_{134} + X_{135} + X_{136} + X_{137} = 1501$
$X_{131} + X_{132} + X_{133} + X_{134} + X_{135} \geq 450$
$X_{141} + X_{142} + X_{143} + X_{144} + X_{145} + X_{146} + X_{147} = 941$
$X_{141} + X_{142} + X_{143} + X_{144} + X_{145} \geq 282$
$X_{151} + X_{152} + X_{153} + X_{154} + X_{155} + X_{156} + X_{157} = 591$
$X_{151} + X_{152} + X_{153} + X_{154} + X_{155} \geq 177$
$X_{161} + X_{162} + X_{163} + X_{164} + X_{165} + X_{166} + X_{167} = 1531$
$X_{161} + X_{162} + X_{163} + X_{164} + X_{165} \geq 459$

$X_{171} + X_{172} + X_{173} + X_{174} + X_{175} + X_{176} + X_{177} = 816$
$X_{171} + X_{172} + X_{173} + X_{174} + X_{175} \geq 245$
$X_{181} + X_{182} + X_{183} + X_{184} + X_{185} + X_{186} + X_{187} = 2138$
$X_{181} + X_{182} + X_{183} + X_{184} + X_{185} \geq 641$
$X_{191} + X_{192} + X_{193} + X_{194} + X_{195} + X_{196} + X_{197} = 1422$
$X_{191} + X_{192} + X_{193} + X_{194} + X_{195} \geq 427$
$X_{201} + X_{202} + X_{203} + X_{204} + X_{205} + X_{206} + X_{207} = 1422$
$X_{201} + X_{202} + X_{203} + X_{204} + X_{205} \geq 427$
$X_{211} + X_{212} + X_{213} + X_{214} + X_{215} + X_{216} + X_{217} = 1422$
$X_{211} + X_{212} + X_{213} + X_{214} + X_{215} \geq 427$
$X_{221} + X_{222} + X_{223} + X_{224} + X_{225} + X_{226} + X_{227} = 550$
$X_{221} + X_{222} + X_{223} + X_{224} + X_{225} \geq 165$
$X_{231} + X_{232} + X_{233} + X_{234} + X_{235} + X_{236} + X_{237} = 1642$
$X_{231} + X_{232} + X_{233} + X_{234} + X_{235} \geq 493$
$X_{241} + X_{242} + X_{243} + X_{244} + X_{245} + X_{246} + X_{247} = 1143$
$X_{241} + X_{242} + X_{243} + X_{244} + X_{245} \geq 343$
$X_{251} + X_{252} + X_{253} + X_{254} + X_{255} + X_{256} + X_{257} = 1915$
$X_{251} + X_{252} + X_{253} + X_{254} + X_{255} \geq 575$

Restrictions of not to exceed the capacity of land and air ports:

$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + X_{61} + X_{71} + X_{81} + X_{91} + X_{101} + X_{111} + X_{121} + X_{131} + X_{141} + X_{151} + X_{161} + X_{171} + X_{181} + X_{191} + X_{201} + X_{211} + X_{221} + X_{231} + X_{241} + X_{251} + X_{261} \leq 12000$

$X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + X_{62} + X_{72} + X_{82} + X_{92} + X_{102} + X_{112} + X_{122} + X_{132} + X_{142} + X_{152} + X_{162} + X_{172} + X_{182} + X_{192} + X_{202} + X_{212} + X_{222} + X_{232} + X_{242} + X_{252} + X_{262} \leq 5000$

$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} + X_{63} + X_{73} + X_{83} + X_{93} + X_{103} + X_{113} + X_{123} + X_{133} + X_{143} + X_{153} + X_{163} + X_{173} + X_{183} + X_{193} + X_{203} + X_{213} + X_{223} + X_{233} + X_{243} + X_{253} + X_{263} \leq 3000$

$X_{14} + X_{24} + X_{34} + X_{44} + X_{54} + X_{64} + X_{74} + X_{84} + X_{94} + X_{104} + X_{114} + X_{124} + X_{134} + X_{144} + X_{154} + X_{164} + X_{174} + X_{184} + X_{194} + X_{204} + X_{214} + X_{224} + X_{234} + X_{244} + X_{254} + X_{264} \leq 4500$

$X_{15} + X_{25} + X_{35} + X_{45} + X_{55} + X_{65} + X_{75} + X_{85} + X_{95} + X_{105} + X_{115} + X_{125} + X_{135} + X_{145} + X_{155} + X_{165} + X_{175} + X_{185} + X_{195} + X_{205} + X_{215} + X_{225} + X_{235} + X_{245} + X_{255} + X_{265} \leq 3000$

$X_{16} + X_{26} + X_{36} + X_{46} + X_{56} + X_{66} + X_{76} + X_{86} + X_{96} + X_{106} + X_{116} + X_{126} + X_{136} + X_{146} + X_{156} + X_{166} + X_{176} + X_{186} + X_{196} + X_{206} + X_{216} + X_{226} + X_{236} + X_{246} + X_{256} + X_{266} \leq 20000$

$X_{17} + X_{27} + X_{37} + X_{47} + X_{57} + X_{67} + X_{77} + X_{87} + X_{97} + X_{107} + X_{117} + X_{127} + X_{137} + X_{147} + X_{157} + X_{167} + X_{177} + X_{187} + X_{197} + X_{207} + X_{217} + X_{227} + X_{237} + X_{247} + X_{257} + X_{267} \leq 7000$

Restrictions to not exceed the capacity of land ports:

$X_{16} + X_{26} + X_{36} + X_{46} + X_{56} + X_{66} + X_{76} + X_{86} + X_{96} + X_{106} + X_{116} + X_{126} + X_{136} + X_{146} + X_{156} + X_{166} + X_{176} + X_{186} + X_{196} + X_{206} + X_{216} + X_{226} + X_{236} + X_{246} + X_{17} + X_{27} + X_{37} + X_{47} + X_{57} + X_{67} + X_{77} + X_{87} + X_{97} + X_{107} + X_{117} + X_{127} + X_{137} + X_{147} + X_{157} + X_{167} + X_{177} + X_{187} + X_{197} + X_{207} + X_{217} + X_{227} + X_{237} + X_{247} + X_{256} + X_{266} + X_{257} + X_{267} \leq 23558$

Constrain of the minimum no. of a passenger at Erbil International Airport

$X_{14} + X_{24} + X_{34} + X_{44} + X_{54} + X_{64} + X_{74} + X_{84} + X_{94} + X_{104} + X_{114} + X_{124} + X_{134} + X_{144} + X_{154} + X_{164} + X_{174} + X_{184} + X_{194} + X_{204} + X_{214} + X_{224} + X_{234} + X_{244} + X_{254} + X_{264} \geq 180$

Constrain of the minimum no. of a passenger at Slemani International Airport

$X_{15} + X_{25} + X_{35} + X_{45} + X_{55} + X_{65} + X_{75} + X_{85} + X_{95} + X_{105} + X_{115} + X_{125} + X_{135} + X_{145} + X_{155} + X_{165} + X_{175} + X_{185} + X_{195} + X_{205} + X_{215} + X_{225} + X_{235} + X_{245} + X_{255} + X_{265} \geq 270$

Constrain of the minimum no. of a passenger at Najaf International Airport.

$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} + X_{63} + X_{73} + X_{83} + X_{93} + X_{103} + X_{113} + X_{123} + X_{133} + X_{143} + X_{153} + X_{163} + X_{173} + X_{183} + X_{193} + X_{203} + X_{213} + X_{223} + X_{233} + X_{243} + X_{253} + X_{263} \leq 900$

Constrain of the minimum no. of a passenger at Basra International Airport.

$X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + X_{62} + X_{72} + X_{82} + X_{92} + X_{102} + X_{112} + X_{122} + X_{132} + X_{142} + X_{152} + X_{162} + X_{172} + X_{182} + X_{192} + X_{202} + X_{212} + X_{222} + X_{232} + X_{242} + X_{252} + X_{262} \leq 1350$

In addition to non-negative restrictions

$X_{ij} \geq 0$, **for all i & j**

The variables in the mathematical model were represented by symbols such as (X_{11}, X_{12}, ..., X_{ij}) in both the target function and the specific limitations of the model and the limitations of non-negative. In the current research model, the number of decision variables was (175) variables included all the expected possibilities of travel of pilgrims from the provinces of their pilgrim residence to the K.S.A., plus (63) constraint included all the parameters approved by the decision-maker in the commission. The following is a statement of the details of these variables for the Baghdad (I) city just to be clear, and for the rest of the provinces, the definition will be in the same way, as follows:

X_{ij} = represents the number of pilgrims transferred from the city (i) to the K.S.A. via port (j).

X_{11}= represents the number of pilgrims transported from Baghdad(I) city to the K.S.A. via Baghdad International Airport.

As for the constants in the first mathematical model, the costs of transportation from the pilgrim provinces to the K.S.A., were as follows:
C_{ij} = cost of transportation per pilgrim from his province (i) to the K.S.A. via port (j).
C_{11}= represents the cost of transporting the pilgrim from Baghdad(I) city to the K.S.A. via Baghdad International Airport.
With regard to the other costs used in the model, its representation was as follows:
C_1 = cost of the K.S.A. office unified agents for pilgrim travel by airport for the hajj season.
C_2 = cost of the K.S.A. Motawfin Foundation for the pilgrims travels by airport for the hajj season.
C_3 = cost of the K.S.A. office unified agents for pilgrim travel by land port for the hajj season.
C_4 = cost of the K.S.A. Motawfin Foundation for the pilgrims travels by land for the hajj season.
C_5 = cost of transporting the pilgrim from the governorate of lives to the port.
C_6 = cost of transporting the pilgrim from the province of lives to the K.S.A. via the land ports.
C_7 = cost of transporting hajj from the province of lives to the K.S.A. via airports at northern airports (Erbil, Slemani) is 600$ and at central and southern airports (Baghdad, Najaf, Basra) is 550$.

The method of linear programming was adopted in the second mathematical model of the research problem in order to reduce the total cost of hajj by finding the optimal number of survival days of a pilgrim in K.S.A. (Medina and Mecca), so the mathematical formula of the second model becomes as follows:

$$\text{Min. } Z = (Y_{12}*D_{12}) + (Y_{12}*D_{14}) + (Y_{11}*D_{13}) + (Y_{11}*D_{17}) + (Y_{22}*D_{22}) + (Y_{22}*D_{24}) + (Y_{21}*D_{23}) + (Y_{21}*D_{27})$$

Subject to:

$Y_{11} \geq 25$, **constrain of stay duration in Mecca (to pilgrim transport by plane).**
$Y_{21} \geq 18$, **constrain of stay duration in Mecca (to pilgrim transport by bus).**
$Y_{12} \geq 3$, **constrain of stay duration in Medina (to pilgrim transport by plane).**
$Y_{22} \geq 3$, **constrain of stay duration in Medina (to pilgrim transport by bus).**
$Y_{ij} \geq 0$, **for all i,j Non-negative restriction.**

To define the variables of the second mathematical model, they were as follows:
Y_{11} = number of days that pilgrim (transport by plane) spends in Mecca.
Y_{12} = number of days that pilgrim (transport by plane) spends in Medina.
Y_{21} = number of days that pilgrim (transport by bus) spends in Mecca.
Y_{22} = number of days that pilgrim (transport by bus) spends in Medina.

The constants were written in this way for the purpose of taking advantage of the mathematical model in the coming years after changing the values of these constants only in the objective function because they change according to the contracts concluded with Saudi companies for housing, transportation and food, and then the results are obtained by using the program (winQSB), so the constant are:

D_{11} = cost of living pilgrim (transport by plane) in Mecca per day.
D_{12} = cost of living pilgrim (transport by plane) in Medina per day.
D_{13} = cost of feeding the pilgrim (transport by plane) in Mecca per day.
D_{14} = cost of feeding the pilgrim (transport by plane) in Medina per day.
D_{15} = cost of feeding the pilgrim (transport by plane) in the sacred place for the hajj season.
D_{16} = cost of transportation the pilgrim (transport by plane) to the K.S.A.
D_{17} = cost of transfer the pilgrim to the five prayers per day.
D_{21} = cost of housing the pilgrim (transport by bus) in Mecca per day.
D_{22} = cost of housing the pilgrim (transport by bus) in Medina per day.
D_{23} = cost of feeding the pilgrim (transport by bus) in Mecca per day.
D_{24} = cost of feeding the pilgrim (transport by bus) in Medina per day.
D_{25} = cost of feeding the pilgrim (transport by bus) in the sacred place for the hajj season.
D_{26} = cost of transporting the pilgrim (transport by bus) to the K.S.A.
D_{27} = transport for the five prayers of the pilgrim (transport by bus) per day.

With regard to the cost of staying in Medina and Mecca, it is collected from the housing and food department in the Authority.

4. Discussion the results:

After complete the formulation of mathematical models in the form of transport problem, initiated by the researcher to data entry on the program of the quantitative system of business (winQSB) that provide the results shown in table 4.1.

Table 4.1: the result of the first model.

Id	Decision Variable	Solution Value	Id	Decision Variable	Solution Value	Id	Decision Variable	Solution Value
1	X_{11}	405	20	X_{106}	358	39	X_{186}	1,017.00
2	X_{16}	1,073.00	21	X_{107}	378	40	X_{192}	630
3	X_{21}	1,763.00	22	X_{111}	90	41	X_{197}	1,508.00
4	X_{26}	314	23	X_{116}	1,068.00	42	X_{201}	405
5	X_{31}	765	24	X_{123}	315	43	X_{206}	1,017.00
6	X_{36}	1,831.00	25	X_{126}	763	44	X_{211}	405
7	X_{41}	585	26	X_{131}	225	45	X_{216}	1,017.00
8	X_{46}	1,403.00	27	X_{133}	225	46	X_{221}	135
9	X_{51}	180	28	X_{136}	1,051.00	47	X_{226}	415
10	X_{56}	555	29	X_{143}	270	48	X_{231}	450
11	X_{61}	315	30	X_{146}	671	49	X_{236}	1,192.00
12	X_{66}	856	31	X_{152}	45	50	X_{241}	135

13	X_{71}	315	32	X_{153}	90	51	X_{244}	180
14	X_{76}	875	33	X_{156}	456	52	X_{246}	828
15	X_{81}	90	34	X_{162}	450	53	X_{251}	270
16	X_{86}	1,198.00	35	X_{167}	1,081.00	54	X_{255}	270
17	X_{91}	225	36	X_{172}	225	55	X_{256}	1,375.00
18	X_{96}	667	37	X_{177}	591			
19	X_{101}	270	38	X_{181}	405			

As for the rest of the decision variables, their value was equal to zero. Through the results obtained, we note that the available possibilities (border ports) have not been fully utilized, as there are some ports that have not been used in transport, in order to maintain the lowest cost in addition to ensuring the comfort of the pilgrim, pointed out not use of these ports in the transportation of pilgrims in some provinces, in addition to the increased cost of transportation, an increase in the distance travelled to reach those ports and travel through them.

In the same way that was used in the solution of the first mathematical model, used the program of the quantitative system for works (winQSB) again, the researcher entered the data for the second mathematical model to find the best solution for the number of days that the pilgrim is supposed to spend in the holy homes (K.S.A) to perform rituals, which in turn reduce the total cost of hajj, where possible, so the results were as indicated in Table 4.2.

Table 4.2: the results of the second model.

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Y_{11}	25	15.222	380.55	0	Basic	0	M
2	Y_{12}	3	74.003	222.009	0	Basic	0	M
3	Y_{21}	18	11.592	208.656	0	Basic	0	M
4	Y_{22}	3	80.833	242.499	0	Basic	0	M
	Objective	Function	(Min.) =	1,053.71				

Other data obtained from the results, can be useful for decision-makers (Senior Management of the Authority):

- **The cost of staying in Mecca for each additional day is 15,222 U.S. dollars.**
- **The cost of staying in Medina for each additional day is 74,003 U.S. dollars.**
- **The cost of staying in Mecca for each additional day is 11,592 U.S. dollars.**
- **The cost of staying in Medina for each additional day is 80.8330 U.S. dollars.**
- **The total cost of the optimal duration of the pilgrim's stay (transport by plane) in the K.S.A. is 502,559 U.S. dollars.**

- **The total cost of the optimal duration of the pilgrim's stay (transport by bus) in the K.S.A. is 451.55 U.S. dollars.**

The cost of hajj for the season was adopted for the season 2019 A.D. (the year in which the data was received from the Authority) as a basis for comparing it with the results of the mathematical model prepared by the researcher, where the total cost of hajj for that year was (115,987,358) U.S. dollars equivalent to (139,184,829,600) Iraqi dinars.

After the results obtained from the formulation and solution of the two mathematical models, the researcher calculated the new cost as well as the amount of reduction from the previous cost, and the researcher followed the following steps:

First: Using the results of the second mathematical model, each of the travel through the air or land ports is assigned as follows:

- **The cost for one pilgrim (transport by plane) = (502,559$) cost of the best duration to stay in the K.S.A. + (1845$) The cost of accommodation in Mecca + (1456$) the Cost of transport (C_{ij}) through airports (from the first model) + (27$) the cost of feeding in the sacred place = 3830.559$.**
- **The cost for one pilgrim (transport by bus) = (451.55$) cost of the best duration to stay in the K.S.A. + (1367$) the cost of accommodation in Mecca + (789$) the cost of transportation (C_{ij}) by land port (from the first model) + (27$) the cost of feeding in the sacred place = 2634.55$.**

Second: use of the results of the first mathematical model where the total pilgrims traveling through all the land ports are (23558) pilgrims, and the total pilgrims traveling through all airports is (10133) pilgrims.

Thus, the total cost of hajj is according to the following equation:

The total cost of hajj = (cost of hajj traveling through all land ports) + (cost of hajj traveling through all airports)

= (cost of hajj by bus ∗ number of pilgrims traveling through land ports) + (cost of hajj by plane ∗ number of pilgrims travelling through land ports)

= (2634.55 x 23558) + (3830.559 x 10133) = 100,879,783.2 U.S. dollars.

After that, the researcher worked to calculate the amount of reduction in the total cost of hajj, according to the following equation:

Amount of cost reduction = the total cost of 2019 (comparison year) - the total cost of the research model

=115,987,358 - 100,879,783.2 = 15,107,574.8 U.S. dollars.

By the data obtained from the Authority's Financial Department was the cost of hajj through land ports was 2,972 U.S. dollars, and 4,125 U.S. dollars for hajj by air ports. The results obtained using the mathematical model of research showed a decrease in the cost of hajj, which is paid by the pilgrim to the Authority by (337.45) U.S. dollars for travellers through land ports and (294,441) U.S. dollars for travellers through air ports.

5. Conclusions

By solving the mathematical model of the research problem, it is important to use the methods of Operations Research in dealing with the problems faced by the decision-maker in the Authority such as the cost of hajj, where the model contributed to the reduction of the total cost of hajj with limit 11% from the total cost of the last year 2019, that's how much (15,107,574.8) U.S. dollars, thus reducing the amount paid by the pilgrim as well as reducing the amounts of hard currency that is transferred out of the country. The model also contributed to the provision of necessary data that the administration can use to transport pilgrims from Iraq to Saudi Arabia in the coming years, in addition to information on the cost of staying pilgrims for additional periods in K.S.A.

References:

1. Alqussi, M. M. 2015. Administrative Mathematics. Academic Book Center. Amman.
2. Alshemerty, Hameed Saad Noor. 2010. Operations Research Understands and Apply. Magdelawy. Baghdad.
3. Ashour, M. A., & Abaas, R. N. (2019). Aggregate production planning using linear programming with practical application. Journall of Economiics and, 25(113), 526- 542.
4. Bakhit, A. K., & Hashim, Z. K. (2016). Building a mathematical model of transportation problem with demand dynamic constraints with practical application. Journal of Economics and Administrative Sciences, 22(91), 353-366.
5. Bakhit, A. B., S. A. AlNuaimi, and A. H. Batikh. 2015. Operations Research Basic Foundations and Scientific Decisions. Alwameth for printing and publishing. Baghdad.
6. Dantzig, George B., and Mukund N. Thapa. 2006. Linear Programming 1: Introduction. Springer Science & Business Media. Stanford.
7. Hassan, D. S., A. S. Jabar, and N. A. I. Alsemery. 2013. Operations Research. Althakera Publishing and Distribution. Baghdad.
8. Hillier, Frederick S. 2012. Introduction to Operations Research. Tata McGraw-Hill Education. New York.
9. Mohammed, B. F. (2015). Linear programming will be used to solve the transport problem and test the ideals of the solution in the modified method. Journal of Madenat Alelem University College, 7(1), 104-119.
10. Noor, H. S., and A. R. Sahad. 2012. "Building Mathematical Models Using Microsoft Excel." Journal Allgrei for Economics and Management 8(23):163–84.
11. P., Rama Murthy. 2007. Operation Research. New Age International (P) Ltd., Publishers. New Delhi.
12. Rahi , A. K., & Laibi, W. M. (2018). Solve the transport problem according to the discount restrictions on the quantities transferred. Journal of Management and Economics(116), 291-298.

بناء أنموذج لتكاليف الحج في الهيئة العليا للحج والعمرة العراقية

الباحث/ حيدر عبد الرضا عبود
جامعة بغداد، كلية الادارة والاقتصاد، قسم الاحصاء، بغداد، العراق

أ.م.د . صباح منفي رضا
جامعة بغداد، كلية الادارة والاقتصاد، قسم الاحصاء، بغداد، العراق

h.abdulretha76@gmail.com drsabah@coadec.uobaghdad.edu.iq

Received: 25/4/2020 Accepted :9/8/2020 Published: November / 2020

مستخلص البحث:

يتلخص البحث في بناء أنموذج رياضي باستخدام أكثر الأساليب شيوعًا في علم بحوث العمليات، وهي كل من نماذج النقل والبرمجة الخطية لإيجاد أفضل حل لمشكلة البحث المتمثلة بالمساهمة في خفض كلفة الحج في العراق، ويتم ذلك من خلال الوصول إلى العدد الأمثل من الحجاج المسافرين عبر كل من المنافذ البرية (الحدودية) او الجوية (المطارات) الموزعة في كافة انحاء العراق، وبدلاً من الاعتماد على الخبرة الشخصية لمتخذي القرار في هيئة الحج والعمرة في تحديد أفضل منفذ (الأقل تكلفة) لسفر الحجاج، والتي يمكن أن تحتمل الصواب او الخطأ في بعض الاحيان، فقد استند الباحث إلى الأساليب العلمية لبحوث العمليات لتوفير آلية علمية لأتخاذ القرار الاداري المناسب، لذا عمد الباحث الى بناء أنموذجين رياضيين، تم صياغة النموذج الأول على شكل مشكلة النقل وهدفه هو تحديد العدد الأمثل من الحجاج المسافرين من خلال الموانئ الجوية أو البرية وكل محافظة مع تحديد أفضل منفذ للسفر ، في حين استخدمت البرمجة الخطية في النموذج الثاني لتحديد الفترة الزمنية المثلى لبقاء الحجاج في المملكة العربية السعودية، مع مراعاة قيود المشكلة، واظهرت النتائج اهمية استخدام اساليب بحوث العمليات في اتخاذ القرارات من خلال تخفيض التكلفة الإجمالية للحج بمقدار (574.8،107،15) دولار أمريكي أو مايقارب الـ 11%، كما تضمنت النتائج إعداد خطة شاملة لتطوير نقل الحجاج يمكن لصناع القرار بالهيئة الاستفادة منها لنقل الحجاج من العراق إلى السعودية في السنوات القادمة.

المصطلحات الرئيسة للبحث : بحوث العمليات، البرمجة الخطية، مشاكل النقل، النماذج الرياضية، نماذج النقل.

***البحث مستل من رسالة ماجستير**