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Comparison of Some Semi-parametric Methods in Partial Linear Single-Index Model

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Abstract:

The research dealt with a comparative study between some semi-parametric estimation methods to the Partial linear Single Index Model using simulation. There are two approaches to model estimation two-stage procedure and MADE to estimate this model. Simulations were used to study the finite sample performance of estimating methods based on different Single Index models, error variances, and different sample sizes , and the mean average squared errors were used as a comparison criterion between the methods were used. The results showed a preference for the two-stage procedure depending on all the cases that were used.

Keywords: PLSIM, Two-Stage Procedure, MADE, Single Index Model.

1- Introduction:

Semi-parametric regression models have received wide attention recently, due to their flexibility in combining traditional linear models with nonparametric regression models. Although there are many advantages of both models, we note that the nonparametric model suffers from the problem of the curse of dimensionality. So, to avoid this problem, a Single Index Model can be used to reduce dimensions, assuming that the effect of the explanatory variables X can be combined into a single index $X^T B$ by using an unknown link function g. [17]

In order to obtain accurate predictions for this model, it requires estimation of both the vector of parameters θ, β and the link function g at the same time in an iterative manner where the nonparametric part is estimated first after making initial assumptions about the value of the unknown parameters θ, β and then estimating the vector of the unknown parameters after estimating the nonparametric part then the resulting estimator according to this method is called the semi parametric estimator. [21]

Further Wand and Carroll, 2003 studied the nonparametric components that suffer from the curse of dimensionality and can only accommodate low dimensional covariates X. So, to remedy this, a dimension reduction model which assumes that the influence of the covariate X can be collapsed to a single index, $X^T\beta$, through a nonparametric link function g. [17]

Wang, Xue, Zhu & Chong, 2010 studied partial linear single-index model estimation and they proposed a two-stage estimation to estimate the link function and the parameters in the single index. [17]

Su,L., and Zhang,Y, 2013 highlighted the recent developments on estimate the variable selection for nonparametric and semi-parametric regression models; they explained SCAD and LASSO methods. [15]

Munaf Y. Hmood, 2015 studied the characteristics of the single index model. Local linear regression and Nadaraya-Watson estimators were applied to estimate the nonparametric part of this model, then he made a comparison between those methods based on several selecting smoothing parameter methods including the rule of the thumb and proposed golden ratio methods; his results showed a preference for Local linear estimator with using ROT as a smoothing Parameter selector as well as Nadaraya -Watson estimator but with using a new proposal smoothing parameter method. [10]

Munaf Y. Hmood & Tariq A. S., 2016, compared (MAVE, LASSO-MAVE, and the proposed method Adaptive LASSO-MAVE). The results show that the best method for estimating and variable selection of single-index model is the proposed method (Adaptive LASSO-MAVE). [7]

Park, Petkova, Tarpey & Ogden, 2020 presented a single-index model with multiple-links (SIMML) that estimate a single linear group of the covariates, with nonparametric link functions. The approach assures a focus on the treatment by covariates interaction effects on the treatment making optimal treatment decisions. Asymptotic results for estimator are obtained under possible model misspecification. A treatment decision rule based on the derived single-index is defined, and it is compared to other methods. [13]

2- Partial Linear Single-Index Models (PLSIM)

This model was first proposed by Carroll, Fan, Gijbels and Wand, 1997 [4]. This model has two parts, a linear part and a non-parametric part. Usually, its variables are continuous, and these linear and non-linear variables affect the response variable, and both parts are linked by an aggregate relationship. [14]

This model has many application fields like Economic, Medical and Environment and can be written in the following form: [19]

$$Y = Z^T\theta_0 + g(X^T\beta_0) + \varepsilon, \quad X \in R^p \text{ and } Z \in R^q \quad \dots (1)$$

Where X and Z are covariates with dimensions p and q respectively.
g(.): an unknown link function for the single index.
ε: is the error term with $E\varepsilon$ =0 and $0 < Var(\varepsilon) < \infty$.
θ: Unknown parameters vector of degree (q×1) for the parametric part.
β: Unknown parameters vector of degree (p×1) for the nonparametric part.
We further assume that $||\beta|| = 1$ and $\beta_1 > 0$ for model identification. [10]

3- Estimation methods:

3-1 A two-stage estimation for a partial linear single-index model

This method was proposed by Wang et al. in 2010 [17] to estimate the link function and parameter vector of the single-index model. Constrained estimating equation leads to an asymptotically more active estimator than found estimators in the sense that it is of a smaller limiting variance, the estimator of the nonparametric link function realizes best convergence rates and the structural error variance is obtained.

In addition, the results ease the construction of confidence regions and hypothesis testing for the unknown parameters. This method does not require any repetition and some indicators are based on X to explain Z.

(Y) response variable, the observations are $\{(X_i, Z_i);\ i = 1, 2, \dots, n\}$ a sequence of independent and identically distributed. Samples from in equation (1), the estimation process takes place in two stages, that is, Z can be obtained from one indicator of X.

$$Z = \emptyset(X^T\beta_z) + \eta \quad \dots (2)$$

$\emptyset$ (.): is an unknown function form.
β_z : is an orthogonal matrix, $||\beta_z|| = 1$, β_z positive first component for model identification
η : has to mean zero and is independent of X with the resulting residuals, $\eta_i = Z_i - \emptyset(X_i^T\beta_z)$

So, to estimate the link function, we need firstly to estimate β_z and then estimate $\emptyset$ to get the residuals, β_z is estimated using general least squares by $\beta_Z = (X^TV^{-1}X)^{-1}X^TV^{-1}Z$, V refers to variance matrix.

Also, the unknown link function in (2) can be estimated by using local linear smoother, so that the resulting estimator is defined as: [11]

$$\hat{\emptyset}(X^T\hat{\beta}_z) = \sum_{i=1}^{n} W_{ni}(X^T\hat{\beta}_z)Z_i$$

W weights.

The residual η hence becomes, $\widehat{\eta}_i = Z_i - \widehat{\emptyset}(X_i^T \widehat{\beta}_z)$, and it is possible to use the least-squares approach to estimate θ_0.

$\widehat{\theta} = (\tilde{Z}^T\tilde{Z})^{-1}\tilde{Z}^T\tilde{Y}$

Where $\tilde{Y} = Z - \widehat{\emptyset}(X^T \widehat{\beta}_z)$ using the definition of residuals from equation (2).
The conditional expectation functions are as follows:
Let $X^T\beta_0 = t$

$g_1(t) = E(Y|X^T\beta_0 = t)$, $g_2(t) = E(Z|X^T\beta_0 = t)$.

so that $\hat{g}_1(t;\, \widehat{\beta}_0) = \sum_{i=1}^{n} W_{ni}(t;\, \widehat{\beta}_0)Y_i$.

$\hat{g}_2(t;\, \widehat{\beta}_0) = \sum_{i=1}^{n} W_{ni}(t;\, \widehat{\beta}_0)Z_i$.

We suppose that $(\widehat{a})$ is a solution to the weighted least square problem. [3, 9]

$$\hat{g}(x) = \hat{a} = \frac{\sum_{i=1}^{n} W_i Y_i}{\sum_{i=1}^{n} W_i} \qquad \dots (3)$$

Assuming that the parameter vector B is known, the nonparametric estimator for the function $W(t;\beta)$ is:

$$W_{ni}(t;\beta) = \frac{K_h(X_i^T\beta - t)[S_{n,2}(t;\beta,h) - (X_i^T\beta - t)S_{n,1}(t;\beta,h)]}{S_{n,0}(t;\beta,h)S_{n,2}(t;\beta,h) - S_{n,1}^2(t;\beta,h)} \qquad \dots (4)$$

$$\widetilde{W}_{ni}(t;\beta) = \frac{K_{h1}(X_i^T\beta - t)[(X_i^T\beta - t)S_{n,0}(t;\beta,h_1) - S_{n,1}(t;\beta,h_1)]}{S_{n,0}(t;\beta,h_1)S_{n,2}(t;\beta,h_1) - S_{n,1}^2(t;\beta,h_1)} \qquad \dots (5)$$

$$S_{n,l}(t;\beta,h) = \frac{1}{n}\sum_{i=1}^{n}(X_i^T\beta - t)^l K_h(X_i^T\beta - t) \quad , \qquad l = 0,1,2.$$

K: Kernel Function

The idea of local linear smoothing through smoothing $Y_i - Z_i^T\widehat{\theta}_0$ versus $X_i^T \widehat{\beta}_0$. $g(.), g'(.)$
Respectively, are estimated according to the following formula:

$$\hat{g}(t;\beta,\theta) = \sum_{i=1}^{n} W_{ni}(t,\beta)(Y_i - Z_i^T\theta) \qquad \dots (6)$$

$$\hat{g}'(t;\beta,\theta) = \sum_{i=1}^{n} \widetilde{W}_{ni}(t,\beta)(Y_i - Z_i^T\theta) \qquad \dots (7)$$

The idea of local linear smoothing to reduce the sum of squares error

$$\sum_{i=1}^{n}[Y_i - Z_i^T\theta - \widehat{g}(X_i^T \widehat{\beta}_0; \widehat{\beta}_0, \theta)]^2 \qquad \dots (8)$$

The estimate for $\widehat{\beta}_0$ is used to update the estimate of $\widehat{\theta}_0^*$ and has been repeated till reaches the desired extent. The resulting partial regression estimator is:

$$\widehat{\theta}_0^* = (\tilde{Z}^T\tilde{Z})^{-1}\tilde{Z}^T Y^{**} \qquad \dots (9)$$

$Y_i^{**} = Y_i - \widehat{g}_1(X_i^T \widehat{\beta}_0;\, \widehat{\beta}_0)$, $\tilde{Z}_i = Z_i - \widehat{g}_2(X_i^T \widehat{\beta}_0;\, \widehat{\beta}_0)$

After updating the estimated value of $\widehat{\theta}_0^*$ and calculating the new residuals $(Y - Z^T\widehat{\theta}^*)$ the estimated value of $\widehat{\beta}_0^*$ is updated, and the obtained estimations $\widehat{\theta}_0^*$, $\widehat{\beta}_0^*$ are used to update the estimate of the link function $\widehat{g}$ according to the following equation:

$$\widehat{g}^*(t) = \sum_{i=1}^{n} W_{ni}(t;\widehat{\beta})(Y_i - Z_i^T\widehat{\theta}) \qquad \dots (10)$$

3-2 Minimum Average Deviance Estimation (MADE)

This method was suggested by Kofi P. Adragni et al, 2018 to estimate the parameter vector and the link function at the same time. The MADE method expanded the estimation method with the least rate of variation MAVE of Xia et al (2002). [1]

The regression of likelihood is used to know the shape of the regression function from the data; the advantage of this method lies in estimating the nonparametric link function to achieve better consistency for the parameter estimator with the possibility of its application to a wide range of models with fewer restrictions on the distribution covariates. [20]

Whereas minimizing the deviations is equivalent to maximizing the regression function, the basis on which the derivations are based on the exponential family of their properties that make them distinct in the inference domain. [12]

The response variable (Y) is within the distributions of the exponential family, where $X \in R^p$ is covariate variable, so the distribution of (Y|X) belongs to an exponential family, and the general formula for these distributions is: [1]

$$f\left(Y \middle| \vartheta(X)\right) = f_0(Y, \emptyset)\exp\left\{\frac{[Y\vartheta(X) - b(\vartheta(X))]}{a(\emptyset)}\right\} \quad \dots (11)$$

$f_0(.,.)$, $a(.)$, $b(.)$, refers to the functions.
$\emptyset$: Dispersion coefficient (or Scale Parameter).
$b(.)$, $f_0(\cdot,\cdot)$: The functions on which the shape of the distribution will depend.
$a(\emptyset)$: Scale parameter function.
$\vartheta(X)$: The canonical parameter and it is related to the conditional mean $E(Y \mid X)$ through a link function $g(.)$.
There is a general formula for finding the mean and variance of the single index model, such that: [6]

$$g\left(E(Y|X)\right) = \vartheta(X)$$
$$E(Y|X) = \mu = b'\left(\vartheta(X)\right)$$
$$Var(Y|X) = a(\emptyset)b''(\vartheta(X))$$

$\{(X_i, Y_i)\ , i = 1, 2, \dots, n\}$ denote the independent and identical distributed random variables. In equation (11), $\vartheta(X)$ is a continuous and smooth function. Thus, at each point, X will have a first-order linear expansion admits.

$$\vartheta(X_i) \approx \vartheta(X) + [\nabla\vartheta(X)]^T(X_i - x) \quad \dots (12)$$

Assuming that $\vartheta(X) = \alpha + \beta^T X$, this is similar in form to the general linear model
So, to estimate the parameters in equation (1), we depend on the following relationship: [12]

$$\hat{B} = \arg\min E\{E[Y - E(Y|B^T X)]^2 | B^T X\} \quad \dots (13)$$

We calculate an initial estimate of the parameters vector B using general least squares (GLS).

$$\alpha = \vartheta(X) \quad , \quad \gamma = \nabla\vartheta(X)$$

Such that $\alpha_j, \gamma_j \in R^{d+1}$ for $j = 1, \dots, n$, $B \in R^{p\times d}$

For the distribution (11), we find that the logarithm of the likelihood function is:

$$L_X(\alpha,\gamma,B) = \sum_{i=1}^{n} W_{0i}(X) \log f(Y_i|\alpha+\gamma^T B^T(X_i - X))$$

$$= \sum_{i=1}^{n} W_{0i}(X)\left[\frac{Y_i\left(\alpha+\gamma^T B^T(X_i-X)\right) - b\left(\alpha+\gamma^T B^T(X_i-X)\right)}{a_i(\emptyset)} + \log f_0(Y_i,\emptyset)\right]$$

$$W_{0i}(X) = \frac{K_h(X_i - X)}{\sum_{j=1}^{n} K_h(X_j - X)}$$

The weights $W_{01}(X),\dots,W_{0n}(X)$ represent the effect of each observation on the model $L_X(\alpha,\gamma,B)$, while $a_i(\emptyset)$ does not depend on X.

$$Q(\alpha,\gamma,B) = \sum_{j=1}^{n} L_{X_j}(\alpha_j,\gamma_j,B) \quad \dots (14)$$

Maximize the likelihood function

$$= \sum_{j=1}^{n}\sum_{i=1}^{n} W_i\left[\frac{Y_i\left(\alpha_j+\gamma_j^T B^T(X_i-X_j)\right) - b\left(\alpha_j+\gamma_j^T B^T(X_i-X_j)\right)}{a_i(\emptyset)} + \log f_0(Y_i,\emptyset)\right]$$

$$W_i = W_i(B^T X) = \frac{K_h(B^T(X_i - X))}{\sum_{j=1}^{n} K_h(B^T(X_j - X))}$$

$$K(u) = (2\pi)^{-\frac{1}{2}}\exp\left(\frac{-u^2}{2}\right)$$

h was selected by using Rule of Thumb. [7]

$$h_{opt} = C(K)\left[\frac{\sigma^2\int W(x)d(x)}{\sum_{j=1}^{n}(\hat{m}''(X_j))^2 W(x)}\right]^{1/5}$$

C(K): A constant value that depends on the type of function used.

We can use Newton-Raphson approach based on Hessian matrix to estimate the parameters$(\alpha_j,\gamma_j) \in R^{d+1}, j = 1,\dots,n$,.

Let $\xi = (\alpha,\gamma^T)^T$, $Z_i = (1,(X_i - X)^T B)^T$, $Z = (Z_1,\dots,Z_n)^T$, $W = diag(w_1,\dots,w_n)$

We find the estimator of the likelihood according to the following equation.

$$L_X(\alpha,\gamma,B) = \sum_{i=1}^{n} W_i\left[\frac{Y_i.Z_i^T\xi - b(Z_i^T\xi)}{a_i(\emptyset)} + \log f_0(Y_i,\emptyset)\right]$$

The first derivative at ξ is then

$$\frac{\partial}{\partial\xi} L_X(\alpha,\gamma,B) = \sum_{i=1}^{n} W_i \frac{Y_i - b'(Z_i^T\xi)}{a_i(\emptyset)} Z_i = Z^T W H(\xi) \quad \dots (15)$$

$$H(\xi) = [Y_i - b'(Z_i^T\xi)]/a_i(\emptyset) \quad for \quad i = 1,\dots,n$$

H: Hessian Matrix is the matrix of the second derivatives of the logarithm of the likelihood functions with respect to (ξ).

$$J_{H(\xi)} = \left(\frac{\partial}{\partial \xi_j}\frac{[Y_i - \acute{b}(Z_i^T\xi)]}{a_i(\emptyset)}\right) = -\frac{1}{a_i(\emptyset)}\begin{pmatrix} b''(z_1^T\xi)Z_{1,1} & \cdots & b''(z_1^T\xi)Z_{1,d+1} \\ \vdots & \ddots & \vdots \\ b''(z_n^T\xi)Z_{n,1} & \cdots & b''(z_n^T\xi)Z_{n,d+1}\end{pmatrix}$$

$H(\xi)$: From $n \times (d+1)$

After finding the values of the estimator (α_j , γ_j) , j = 1, . . . , n , B is estimated by the formula:

$$Q(B) = \sum_{j=1}^{n}\sum_{i=1}^{n} W_i(B^T X_j)\frac{1}{a_i(\emptyset)}\{Y_i(\alpha_j + \gamma_j{}^T B^T(X_i - X_j)) - b(\alpha_j + \gamma_j{}^T B^T(X_i - X_j))\} \quad \dots (16)$$

Using the Stiefel manifolds [16] algorithm, we get the final estimate for MADE. Whereas, G(.)is another weight function that controls the contribution of (X_j , Z_j , Y_j) to the estimation of (β , θ).

4-Simulation

In this section, our purpose is to compare the methods of estimating the partial linear single-index model, for purpose of describing simulation experiments; it should be noted that the assumptions were made as follows:

a- Sample size: n=50, 150,200.

b- X_1, X_2, X_3 are independent $U \sim (0, 1)$.

c- $\varepsilon \sim N(0, \sigma^2)$, Three error variance values have been assumed (0.5, 1, 1.5).

d- The values of the initial parameter vector are assumed to be equal to:

$$\underline{\beta_0} = \left(\frac{1}{\sqrt{3}}\right)(1, 1, 1)^T \text{ ,with the condition } \|\beta_0\| = 1$$

e- Iterations of the experiment were 400 repetitions (for faster arithmetic).

f- different link functions were assumed:

$g_1(X^T\beta) = 3.2(X^T\beta)^T(X^T\beta) - 1.$ [18]

$g_2(X^T\beta) = \sin(X^T\beta) + \exp(X^T\beta).$ [10]

$g_3(X^T\beta) = \{1 + (X^T\beta)^T(X^T\beta)\}\exp\{-(X^T\beta)^T(X^T\beta)\}$. [5]

g- The Gaussian Kernel function used

$K(.) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{u^2}{2}\right)$ **(The best function compared to the residue).** [2]

$W_{ij}^{\beta} = \frac{K_{h,i}^{\beta}(\beta^T X_j)}{\sum_{l=1}^{n} K_{h,l}^{\beta}(\beta^T X_j)}$. [9]

$$\hat{h}_{opt} = C(K)\left[\frac{\sigma^2 \int W(x)d(x)}{\sum_{j=1}^{n}(\hat{m}''(X_J))^2 W(x)}\right]^{1/5}$$

The results in the Tables (1, 2, 3) respectively indicate the values of the model by changing the functions and sample sizes.

Table1: refer to the MASE values of the model $g_1(X^T\beta)$

n	σ^2	MADE	Two- stage	Best Method
50	0.5	16.63	1.83	Two- stage
	1	16.16	1.12	Two- stage
	1.5	15.4	1.74	Two- stage
150	0.5	0.1175	0.0139	Two- stage
	1	0.1172	0.0132	Two- stage
	1.5	0.1164	0.0128	Two- stage
200	0.5	0.1161	0.0020	Two- stage
	1	0.1174	0.0023	Two- stage
	1.5	0.1185	0.0036	Two- stage

Table2: refer to the MASE values of the model $g_2(X^T\beta)$

n	σ^2	MADE	Two- stage	Best Method
50	0.5	7.791	1.419	Two- stage
	1	9.274	1.213	Two- stage
	1.5	10.322	0.308	Two- stage
150	0.5	0.0445	0.0002	Two- stage
	1	0.0496	0.00011	Two- stage
	1.5	0.0532	0.00049	Two- stage
200	0.5	0.0181	0.00048	Two- stage
	1	0.0183	0.00039	Two- stage
	1.5	0.0193	0.00029	Two- stage

Table3: refer to the MASE values of the model $g_3(X^T\beta)$

n	σ^2	MADE	Two- stage	Best Method
50	0.5	50.54	4.39	Two- stage
	1	54.49	3.84	Two- stage
	1.5	56.83	1.72	Two- stage
150	0.5	0.286	0.0047	Two- stage
	1	0.297	0.0073	Two- stage
	1.5	0.303	0.0092	Two- stage
200	0.5	0.162	0.0039	Two- stage
	1	0.159	0.0035	Two- stage
	1.5	0.157	0.0032	Two- stage

Figures for semi-parametric single-index models and estimators for different methods explained as follows:

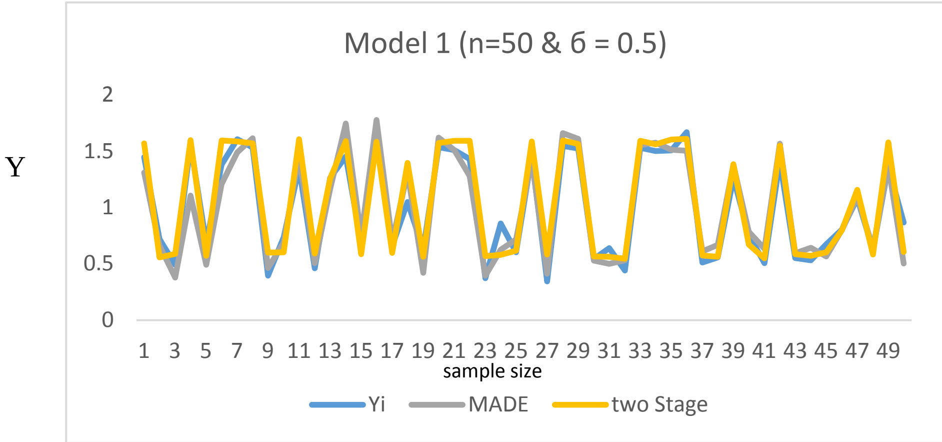

Figure 1: Refers to a partial linear single-index model with methods estimation ($n = 50$, $\sigma = 0.5$, $g_1(X^T\beta)$).

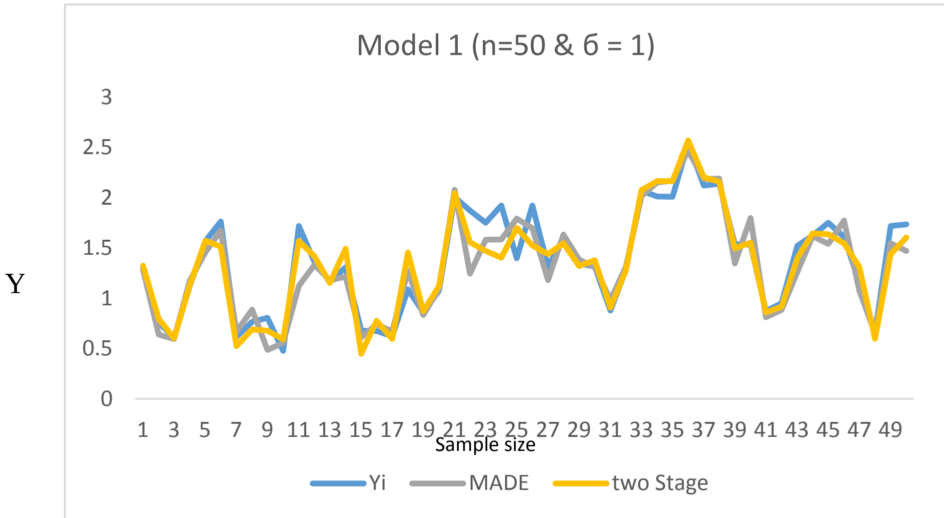

Figure 2: Refers to a partial linear single-index model with methods estimation ($n = 50$, $\sigma = 1$, $g_1(X^T\beta)$).

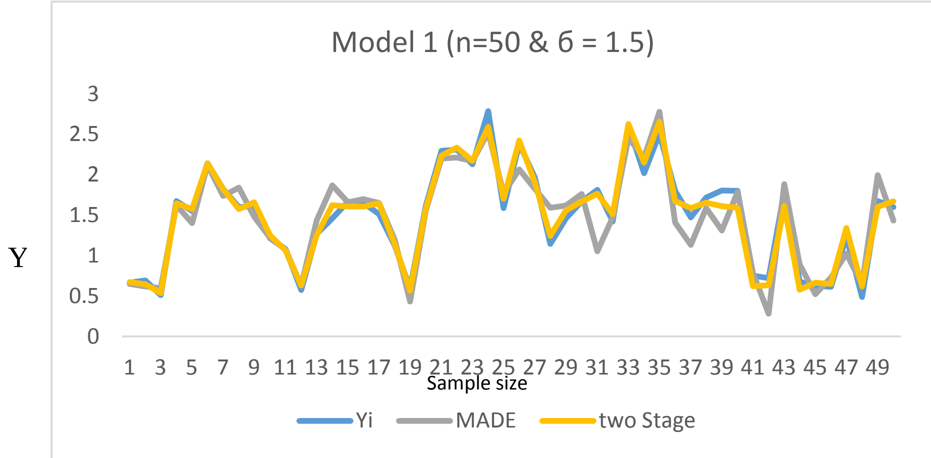

Figure 3: Refers to a partial linear single-index model with methods estimation ($\mathbf{n = 50\ ,\sigma = 1.5\ ,g_1(X^T\beta)}$).

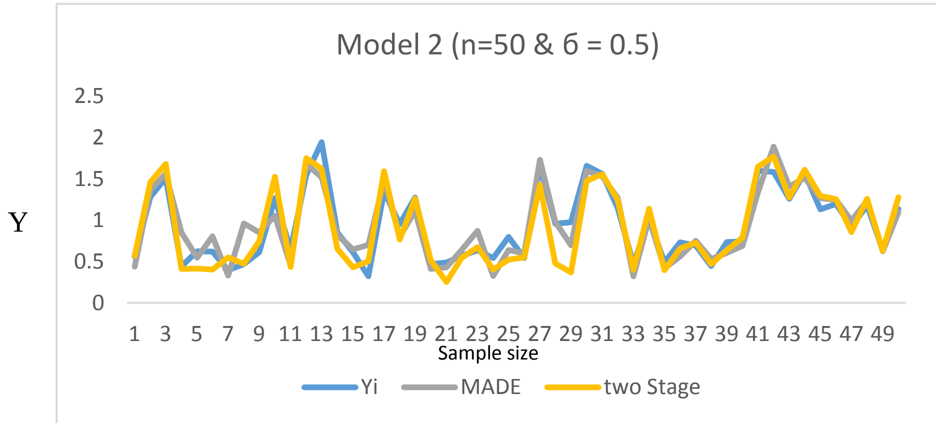

Figure 4: Refers to a partial linear single-index model with methods estimation ($\mathbf{n = 50\ ,\sigma = 0.5\ ,g_2(X^T\beta)}$).

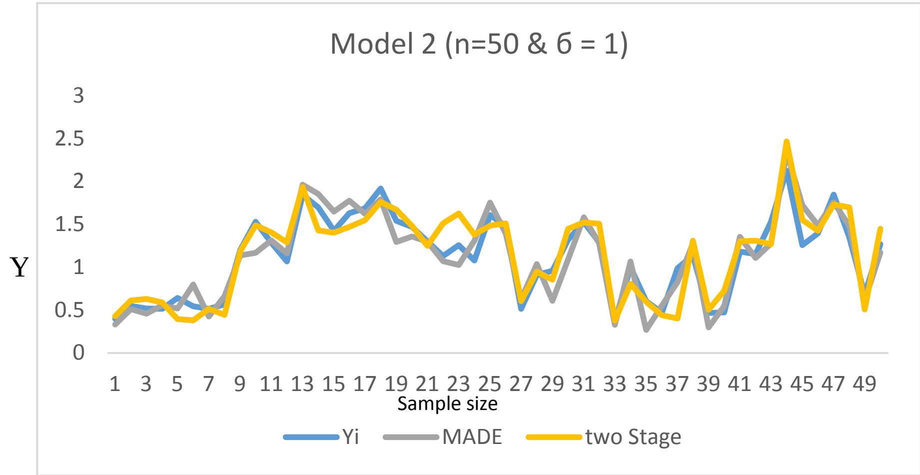

Figure 5: Refers to a partial linear single-index model with methods estimation ($n = 50$, $\sigma = 1$, $g_2(X^T\beta)$).

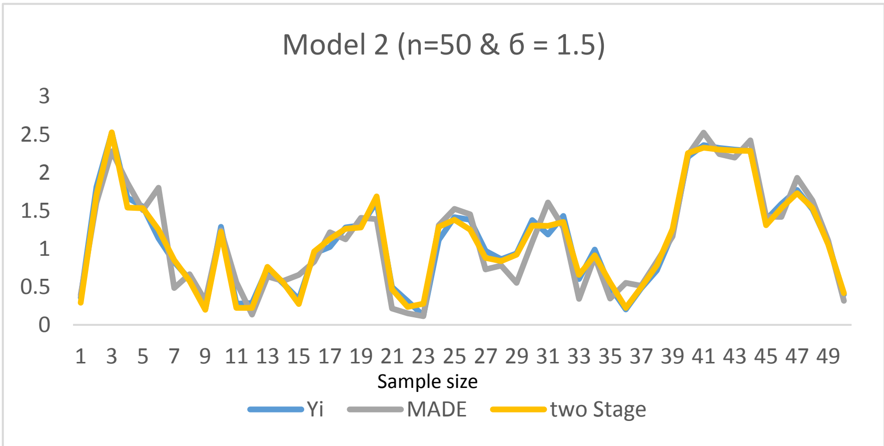

Figure 6: Refers to a partial linear single-index model with methods estimation ($n = 50$, $\sigma = 1.5$, $g_2(X^T\beta)$).

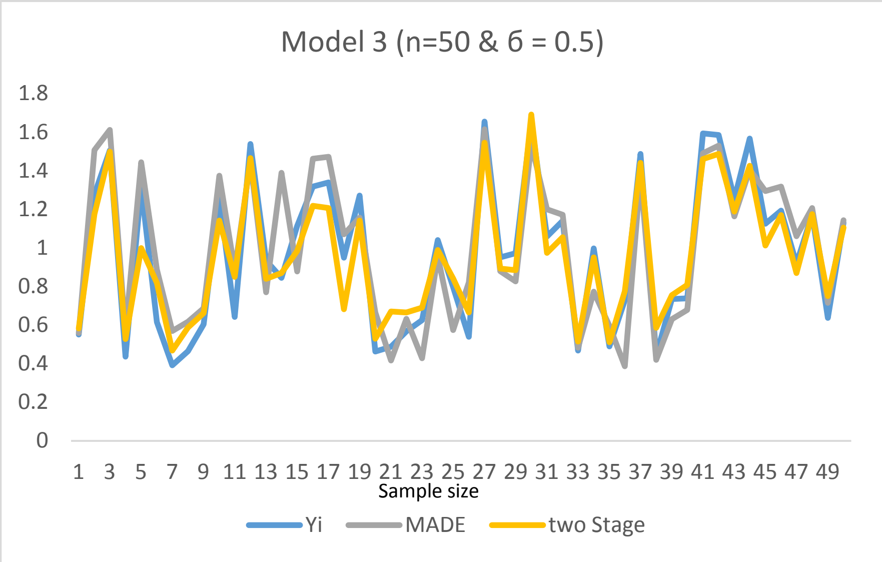

Figure 7: Refers to a partial linear single-index model with methods estimation $(n = 50\ ,\sigma = 0.5\ ,g_3(X^T\beta))$.

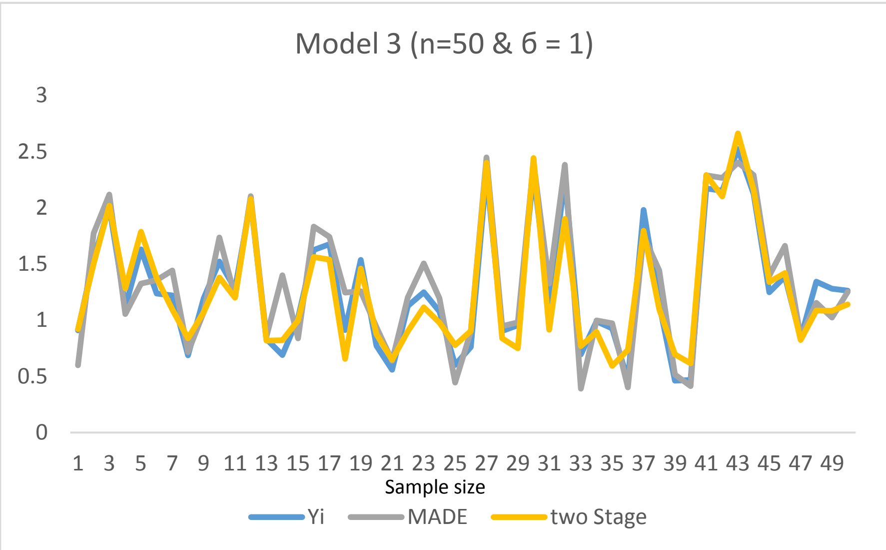

Figure 8: Refers to a partial linear single-index model with methods estimation $(n = 50\ ,\sigma = 1\ ,g_3(X^T\beta))$.

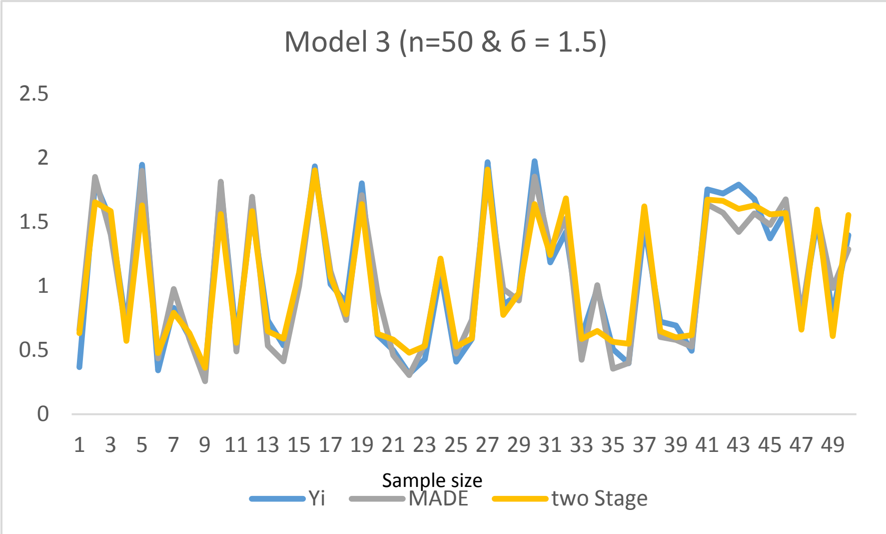

Figure 9: Refers to a partial linear single-index model with methods estimation ($n = 50$, $\sigma = 1.5$, $g_3(X^T\beta)$).

5- Conclusions:

From the tables and figures presented, we find the following:

Through Tables 1-2-3, the results indicated that the (two-stage) method is the best estimation method for the model because it gives the lowest value for the mean squared error (MASE) for different simulation experiments and at different sample sizes and error variances. The results also showed that the mean squares mean values the error decreases with increasing sample size and increasing the variance value.

Through the figures, it is clear that the estimated values of the vector y using the two-stage method is almost identical, smoother, and have less dispersion due to the small value of the mean error squares, and this proves that the two-stage method is better than the (MADE) method in estimating the model.

References:

1- Adragni, K. P. ,(2018), "Minimum Average Deviance Estimation for Sufficient Dimension Reduction". Journal of Statistical Computation and Simulation, Vol.88, No.3, pp.411-431.

2- Al-Kazaz, Q. N., & Hmood, M. Y. ,(2012), "A proposal method for selecting smoothing parameter with missing values". International Conference on Statistics in Science, Business and Engineering (ICSSBE) , pp. 1-5.

3- Carroll, R. J., Fan, J., Gijbels, I. & Wand, M. P. (1997)." Generalized Partially Linear Single-Index Models". Journal of the American Statistical Association, Vol. 92, No. 438, pp. 477-489.

4- Chen, S. X. (2002). "Local Linear Smoothers Using Asymmetric Kernels". Annals of the Institute of Statistical Mathematics, Vol.54, No.2, pp. 312-323.

5- Dong, C., Gao, J., & Tjøstheim, D. (2016). "Estimation for Single Index and Partially Linear Single Index Integrated Models". The Annals of Statistics, Vol. 44, No.1, pp.425-453.
6- Härdle, W., Müller, M., Sperlich, S. & Werwatz, A., (2004), " Nonparametric and Semi parametric Models: An introduction ", Springer Series in Statistics.
7- Hmood, M. Y. & Stadtmuller, U., (2013). "A New Version of Local Linear Estimators" Chilean journal of statistics, Vol.4, No.2, PP.61-74.
8- Hmood, M. Y., & Salih, T. A., (2016), "Compared Some of Penalized Methods in Analysis the Semi-Parametric Single Index Model with Practical Application", Journal of Economics and Administrative Sciences, Vol.22, No.90, pp. 407-427.
9- Hmood, M. Y., (2000), "Comparing Nonparametric Kernel estimators for Estimating Regression Function", MS.c Thesis, Department of Statistics, College of Administration and Economics, University of Baghdad.
10- Hmood, M. Y., (2015), "On Single Index Semiparametric Model" Journal of Statistical Sciences, No .6, pp. 1-17.
11- Kong, E., & Xia, Y. (2007). "Variable Selection for the Single Index Model". Biometrika, Vol.94, No. 1, pp.217-229.
12- McCullagh, P., & Nelder, J. A. (1989). "Generalized Linear Models", Chapman and Hall.
13- Park, H., Petkova, E., Tarpey, T., & Ogden, R. T. (2020). "A Single Index Model with Multiple links", Journal of Statistical Planning and Inference, Vol. 205, pp.115-128.
14- Speckman, P. (1988). "Kernel Smoothing in Partial Linear Models", Journal of the Royal Statistical Society: Series B (Methodological), Vol.50, No.3, pp.413-436.
15- Su, L., & Zhang, Y. (2013). "Variable Selection in Nonparametric and Semiparametric Regression Model" The Oxford Handbook of Applied Nonparametric and Semiparametric Econometrics and Statistics, Chapter 9.
16- Tagare, H. D. (2011). "Notes on Optimization on Stiefel Manifolds", Technical Report. Yale University.
17- Wang, J. L., Xue, L., Zhu, L., & Chong, Y. S. (2010). "Estimation for a Partial Linear Single Index Model", The Annals of Statistics, Vol.38, No.1, pp. 246-274.
18- Wang, Q., Linton, O., & Härdle, W. (2004). "Semiparametric Regression Analysis with Missing Response at Random". Journal of the American Statistical Association, Vol.99, No.466, pp.334-345.
19- Xia, Y. & Härdle, W. (2006). "Semi parametric Estimation of Partially Linear Single Index Models", Journal of Multivariate Analysis, Vol.97, No.5, PP.1162-1184.
20- Xia, Y., Tong, H., Li, W. K., & Zhu, L. X. (2009). "An adaptive estimation of dimension reduction space". In Exploration of A Nonlinear World: An Appreciation of Howell Tong's Contributions to Statistics. pp. 299-346.
21- Yatchew, A. (2003). "Semiparametric regression for the Applied Econometrician". Cambridge University Press.

مقارنة بعض الطرائق شبه المعلميه للأنموذج الخطي الجزئي أحادي المؤشر

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مستخلص البحث:

تناول البحث دراسة مقارنة بين بعض طرائق التقدير شبه المعلميه للأنموذج الخطي الجزئي أحادي المؤشر بأستعمال المحاكاة ، والتطرق الى طريقتين من طرائق تقدير الأنموذج وهما أجراء ذو مرحلتين وطريقة تقدير بأقل معدل أنحراف .تم أجراء تجارب المحاكاة لبيان أفضلية الطرائق المستعملة لتقدير الأنموذج وبأستعمال نماذج المؤشر الواحد ، وتباينات الأخطاء وحجوم العينات المختلفة ، و الاعتماد على معدل متوسط مربعات الخطأ كمعيار للمقارنة بين الطرائق . أظهرت النتائج أفضلية أجراء ذو المرحلتين أعتمادًا على جميع الحالات التي تم أستعمالها.

المصطلحات الرئيسة للبحث/: الانموذج الخطي الجزئي أحادي المؤشر ، أجراء ذو مرحلتين ، طريقة تقدير بأقل معدل أنحراف ، أنموذج المؤشر الواحد .

*البحث مستل من رسالة ماجستير