



Analysis of Models (NAGARCH & APGARCH) by Using Simulations

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Abstract

Simulation experiments are a means of solving in many fields, and it is the process of designing a model of the real system in order to follow it and identify its behavior through certain models and formulas written according to a repeating software style with a number of iterations. The aim of this study is to build a model that deals with the behavior suffering from the state of (heteroskedasticity) by studying the models (APGARCH & NAGARCH) using (Gaussian) and (Non-Gaussian) distributions for different sample sizes (500,1000,1500,2000) through the stage of time series analysis (identification , estimation, diagnostic checking and prediction). The data was generated using the estimations of the parameters resulting from the application of these models to the return series for the exchange rates of Iraqi dinar against US dollar (IQ/USD) for the period from (21/7/2011) until (21/07/2021) and then using these estimations in the process of generating data. The identifications were made using the (Ljung-Box and ARCH tests) with (1000 replicates) and the result showed the presence of states (autocorrelation and heteroskedasticity) and this states increased with increasing the sample size and the best result of NAGARCH with Normal distribution and the best result of APGARCH with General error distribution. The Maximum Likelihood Estimation method used to estimate the parameters of the models and the best result with largest sample size (2000) , in the diagnostic checking phase the result showed the ability of the models (NAGARCH & APGARCH) to process the states of (autocorrelation and heteroskedasticity) and the best result with (APGARCH) model when the error distributed (General error distribution) .

Keywords: NAGARCH, APGARCH, Simulation, Asymmetric

1- Introduction[12,3,5,8,16]

Time series occupies wide areas in our lives, especially the economic fields, specifically the financial ones. Hence, interest began in studying financial time series, which are often characterized by the feature of instability or volatility, meaning that there are periods of time fluctuations followed by periods of relative calm. In order to address this, it was necessary to use statistical models that take into account these fluctuations and try to explain them, and these models are non-linear (ARCH) models, which were known as autoregressive models conditioned by the heteroskedasticity of variance, which were proposed by the researcher (Robert. Engle, 1982)^[12] in a study on the estimation of inflation variance in the United Kingdom to fill the shortfall suffered by ARIMA linear models. In 1986, the researcher (Bollerslev)^[3] proposed the generalized nonlinear ARCH model or the conditional autoregressive model of generalized variance heteroskedasticity (GARCH for short) where he applied these models using the (t-student) distribution. Then the researchers continued to apply these models using distributions other than the normal distribution, and we also mention (Zhu & Fokianos, 2011)^[5] who employed the (Negative Binomial) distribution .

Despite the importance of these models, they were subjected to many criticisms by some economists such as (Nelson, 1991)^[8] and (Cao & Tsay, 1992)^[16], especially with regard to determining the relationship between the random error square and the conditional variance. And that relationship was achieved only in cases where the changes of the phenomenon studied in the same direction and the same size of impact, but in cases characterized by volatility in opposite directions, it was impossible for these models to take into consideration these volatility, and all these criticisms led to the emergence of many other models from GARCH that took into account the various positive and negative effects of shocks, including Asymmetric Generalized Autoregressive Conditional heteroskedasticity Models and its acronym (Asymmetric GARCH) , which was the beginning of a major transformation in the field of applied economic .

2- Material and methods of analysis

2.1 Ljung-Box test [1]

This test is used to identify the autocorrelation error in the return series. The statistic is given by:

$$Q_{(m)} = n(n+2) \sum_{k=1}^m \frac{\hat{p}_k^2}{n-k} \sim \chi^2_{(m-p)} \quad (1)$$

Where n is the size of series, k is the number of time lags , \hat{p}_k^2 is the residual autocorrelation and the hypothesis is :

$$H_0 : p_1 = p_2 = \dots p_k \dots = p_m = 0 \quad \forall k = 1, 2, 3, \dots, m \quad (2)$$

$$H_1 : p_k \neq 0 \quad \text{for some value of } k \quad (3)$$

We don't reject H_0 and the residual are no autocorrelations if (p value) greater than α significant .

2.2 ARCH test [13]

It is used to test the ARCH effect in the return series, and the statistic of this test is :

$$ARCHtest = T \times \widehat{R}^2 \sim \chi_p^2 \quad (4)$$

Where T is the total number of observation given by:

$$T = n - lag \quad (5)$$

and \widehat{R}^2 based on Regression with the formula :

$$\widehat{R}^2 = \frac{SSR}{SST} \quad (6)$$

The arch test hypothesis is :

$$\begin{aligned} H_0 = \alpha_i = 0 & \quad \text{No ARCH effect} & (7) \\ H_0 = \alpha_i \neq 0 & \quad \text{ARCH effect} & \quad i = 1, 2, 3 \dots, q \quad (8) \end{aligned}$$

We don't reject H_0 when the (p value) is smaller than α significant .

2.1 NAGARCH model [2,13]

This model was presented by (Engle & Ng) (1993) to show asymmetric effect for volatility, the conditional variance equation of this model is :

$$y_t = \mu + z_t \quad (1) \text{ Mean equation}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i (\varepsilon_{t-1} + b\sigma_{t-1}^2)^2 + \sum_{j=1}^q \beta_j \sigma_{t-1}^2 \quad (2) \text{ Variance equation}$$

Where (σ_t^2) indicates the conditional variance of the previous delay period, (ω) is a constant, (α_i) is the parameter of the ARCH effect and indicates the short-term continuity of the current shock, (β_j) refers to the effect of GARCH and to the continuity of the impact of the previous shock in the long term, (b) refers to the effect of asymmetry, and it is called a shift parameter , finally (ε_{t-1}) was identical independent series that follows normal distribution .

2.2 APGARCH [4,6,7]

This model was presented by (Ding & Granger) (1993) when they added the power instead of the square to allow an effect of the leverage (asymmetry). The conditional variance equation is :

$$y_t = \mu + z_t \quad (3) \text{ mean equation}$$

$$\sigma_t^\delta = \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^q \beta_j \sigma_{t-j}^\delta \quad (4) \text{ variance equation}$$

Where (ε_{t-i}) is an independent identical series follows Normal distribution with zero mean and One variance , (δ) is the leverage power , (γ_i) is the leverage effect and its value range from $(1 > \gamma_i > -1)$ and when its value equal to zero this indicates the absence of the effect of asymmetry . When this value is equal to zero then the positive and the negative shocks are the same effect .

2.8 Distribution assumptions of error term and estimation [1,11,17]

The volatility estimated in this paper depends on NAGARCH and APGARCH models with the lower order $(p=1 \text{ } q=1)$ assuming two distributions of random error (Normal & General error distribution) and the models were estimated using Maximum Likelihood Estimation method , the mathematical formula is :

$$L(\theta) = \sum_{t=1}^n J_t(\theta) \quad (5)$$

$$\log(L\theta) = -\frac{1}{2} \sum_{t=1}^T \ln(2\pi) + \ln\sigma_t + \frac{(\varepsilon_t^2)}{\sigma_t} \quad (6)$$

i – The log likelihood with Normal distribution is :

$$J_t(\theta) = -\frac{1}{2} \log(2\pi) - \frac{1}{2} \log(\sigma_t^2) - \frac{1}{2} \left(\frac{\varepsilon_t^2}{\sigma_t^2} \right) \quad (7)$$

The Log Likelihood with General error distribution is : - ii

$$L(\theta) = n \left[\log\left(\frac{v}{\lambda_v}\right) - \left(1 + \frac{1}{v}\right) \log(2) - \log \Gamma\left(\frac{1}{v}\right) - \frac{1}{2} \sum_{t=1}^n \log(\sigma_t^2) - \frac{1}{2} \sum_{t=1}^n \sigma_t^{-v} \left| \frac{\varepsilon_t}{\lambda_v} \right|^v \right] \quad (8)$$

Where $v < 2$ is the shape parameter controls the tail behavior

3- Experimental procedures [6,9,10,14,15]

The models (NAGARCH & APGARCH) will be analyzed via the stage of time series analysis (Identification, Estimation, Diagnostic checking and prediction) based on the estimated values resulting from transformation the return series of exchange rates of the (Iraqi dinar against the US dollar)¹ for the period from (21/7/2011) until (21/7/2021) to the return series using the logarithmic transformation:

$$(z_t) = \ln(p_t) - \ln(p_{t-1}) \quad (9)$$

Where z_t is the return series, (p_t) is the price of the current day and (p_{t-1}) is the price of the previous day. After the time series is converted into a return series, the parameters of the models are estimated by Maximum Likelihood Estimation method and these estimated parameters are used in the process of data generation with a thousand repetitions for each of the models (NAGARCH & APGARCH) for the four sample sizes (500,1000,1500,2000) assuming two distributions of random errors (Normal distribution and general error distribution).

3.1 Results and Discussion

Tables (1,2) below describe the results of Ljung-Box test (autocorrelation test) and ARCH test (heteroskedasticity test) respectively for the four sample sizes (500, 1000, 1500, 2000) assuming that the random error is distributed (Normal distribution and General error distribution) with (1000) iterations for each experiment. The tests were conducted for (30 Lags), the success of the test and the emergence of high iterations indicate the rejection of the null hypothesis (H_0) and don't reject the alternative hypothesis (H_1) and thus the existence of the state of autocorrelation and heteroskedasticity in the square return series the null hypothesis will not be rejected when the (p value) is smaller than α ($\alpha = 0.05$).

Table (1) represents the frequency of autocorrelation and the rejection of the null hypothesis

Distribution of errors	Sample size	NAGARCH	APGARCH
Normal	500	973	961
	1000	1000	965
	1500	1000	981
	2000	1000	1000
General error distribution	500	866	991
	1000	991	993
	1500	999	994
	2000	1000	999

¹ The data was obtained from the website <https://m.investing.com/>

From table (1) the result represents the number of times the null hypothesis is rejected and the presence of autocorrelation for the square residual of the return series, for NAGARCH model we note that increasing the number of iterations when increasing the sample size the best result with normal distribution. We note also for APGARCH model that the iterations are increasing when increasing the sample size and the best result with General error distribution.

Table (2) Represents the frequency of heteroskedasticity and the rejection of the null hypothesis

Distribution of errors	Sample size	NAGARCH	APGARCH
Normal	500	913	854
	1000	976	990
	1500	993	996
	2000	996	999
General error distribution	500	756	883
	1000	878	878
	1500	977	999
	2000	996	999

From table (2) the result represents the number of times the null hypothesis is rejected and the presence of heteroskedasticity , as we note increasing the number of iterations when increasing the sample size for the two models (NAGARCH & APGARCH). This indicates an increase in the effect of heteroskedasticity with an increase in the sample size , the best result of NAGARCH with normal distribution and the best result of APGARCH with General error distribution .

Table (3) shows the result of parameters estimation for NAGARCH model by Maximum Likelihood Estimation method from the return series of (IQ/USD) exchange rate (real data) that will be used to generate simulation data.

Table (3) Parameters estimation from real data for (NAGARCH) model

Parameters	μ	ω	α	β	b	V(shape)
Estimated Value	0	0.000001	0.05	0.9	0.05	2

Tables (4,5) show the result of parameters estimation from generating data for (NAGARCH) model with lower order ($p=1,q=1$) when the error distributed (Normal distribution & General error distribution) .

Table (4) Parameter estimation from generating data for NAGARCH(1,1) with Normal distribution

Sample size	Coefficient of NAGARCH(1,1)					MAE μ	MAE α_1	MAE β_1	MAE b	MAE ω
	μ	α_1	β_1	b	ω					
500	0.0000006	0.057	0.915	-0.552	7.24e-02	3.315e-04	0.027	0.034	0.15	0.00031
1000	-0.00032	0.0623	0.8834	-0.1517	3.61e-02	8.748e-04	0.024	0.014	0.0318	0.00002
1500	0.000482	0.0743	0.9138	-0.022	5.28e-02	5.947e-04	0.018	0.0135	0.02342	0.00001
2000	0.000001	0.081	0.911	0.009	3.015e-04	1.682e-04	0.0155	0.0105	0.01525	0.00001

From table (4), we note that the best estimates were at the sample size of (2000) and this was proven by the result of the mean absolute error for most parameters, where we notice that the result of (MSE) decreases with increasing sample size.

Table (5) Parameter estimation from generating data for NAGARCH (1,1) with General error distribution

Sample size	Coefficient of NAGARCH (1,1)						MAE μ	MAE α_1	MAE β_1	MAE b	MAE ω	MAE V
	μ	α_1	β_1	b	ω	V						
500	5.315e-05	0.056	0.936	0.035	1.628e-04	2.085	3.621e-05	0.013	0.038	0.212	0.00023	0.181
1000	7.315e-05	0.05889	0.92986	0.08267	1.164e-04	0.13344	2.725e-05	0.0127	0.0305	0.244	0.00081	0.133
1500	1.315e-06	0.05525	0.9335	0.20925	3.925e-04	1.9705	2.954e-05	0.009	0.03375	0.16875	0.00004	0.125
2000	7.315e-06	0.0565	0.9195	0.2635	5.266e-04	1.9075	6.936e-05	0.0075	0.02	0.1415	0.00001	0.118

From table (5), we note that the best estimates were at the sample size of (2000) and this was proven by the result of the mean absolute error where we notice that the result of (MAE) decreases with increasing sample size.

Table (6) shows the result of parameter estimation for APGARCH model by Maximum Likelihood Estimation method from the return series of (IQ/USD) exchange rate (real data) that will be used to generate simulation data.

Table (6) Parameter estimation from real data for APGARCH with General error distribution

Parameters	μ	ω	α	β	δ	γ	V(shape)
Estimated value	0	0.000001	0.05	0.9	2	0.05	2

Tables (7,8) below show the result of parameters estimation from generating data for APGARCH model with lower order ($p=1,q=1$) when the error distributed (Normal distribution & General error distribution), the parameter estimated by Maximum Likelihood Estimation Method (MLE).

Table (7) Parameter estimation for APGARCH(1,1) with Normal distribution

Sample size	Coefficient of APGARCH (1,1)						MAE μ	MAE α_1	MAE β_1	MAE γ_1	MAE δ	MAE ω
	μ	α_1	β_1	γ_1	δ	ω						
500	0.000011	0.054	0.896	0.069	2.368	0.000003	0.00042	0.036	0.158	0.0695	0.4398	0.0086
1000	0.000002	0.0542	0.89615	0.0599	2.3809	0.000017	0.00043	0.030	0.0185	0.0607	0.41065	0.00131
1500	0.000621	0.0545	0.896	0.059	2.3777	0.000041	0.00024	0.0302	0.1622	0.06	0.40733	0.0011
2000	0.000050	0.0548	0.8957	0.0588	2.41	0.000001	0.00001	0.02	0.019	0.05975	0.33975	0.00047

From table (7) we notice that the result of (MAE) for all parameters are small and decreased with increasing sample size except the parameter (δ) where its value is rather high.

Table (8) Estimated parameter for APGARCH(1,1) with General error distribution

Sample size	Coefficient of APGARCH (1,1)							MAE μ	MAE α_1	MAE β_1	MAE γ_1	MAE δ	MAE ω	MAE V
	μ	α_1	β_1	γ_1	δ	ω	V							
500	6.735e-04	0.054	0.896	0.066	2.406	7.641e-07	1.75	2.494e-05	0.0049	0.004	0.025	0.206	0.0011	0.249
1000	1.942e-04	0.05455	0.896	0.073	2.390	3.753e-07	1.73	6.186e-05	0.0045	0.004	0.0285	0.2706	0.0017	0.2464
1500	4.681e-04	0.05425	0.896	0.077	2.388	1.284e-07	1.74	3.852e-05	0.0042	0.004	0.0305	0.178	0.0013	0.2465
2000	7.927e-04	0.05475	0.896	0.08	2.396	2.625e-08	1.73	1.473e-05	0.0042	0.004	0.0317	0.157	0.001	0.2392

From table (8), we note that the result of the mean absolute error for all parameters are small and decreasing with increasing the sample size except the power parameter (δ) where its value is rather high .

Tables (9,10) below show the result of the diagnostic checking tests, the high iteration represents the numbers of times it does not appear the (autocorrelation and heteroskedasticity states) i.e. don't reject (H_0) (The null hypothesis will reject when the (p value) is greater than (0.05) and that means there is no (autocorrelation and heteroskedasticity states) model was suitable), we used Ljung-Box test (autocorrelation test) and ARCH test (heteroskedasticity test) for standard square residual for the return series . These tables show the results from (1000 iterations) for each test .

Table (9) The number of autocorrelation free trails and don't reject the null hypothesis

Distribution of errors	Sample size	NAGARCH	APGARCH
Normal	500	943	934
	1000	943	918
	1500	948	928
	2000	943	945
General error distribution	500	931	937
	1000	949	953
	1500	949	938
	2000	958	947

From table (9), for (NAGARCH & APGARCH) models we note the high iterations for the two distributions (Normal & General error distribution) and these iterations increase with increasing the sample size but the number of iteration of the APGARCH model with (General error distribution) is higher than the number of iteration of NAGARCH model that means the model is more suitable for handling the case of autocorrelation in the standard square residual series .

Table (10) The number of Arch test free trail and don't reject the null hypothesis

Distribution of errors	Sample size	NAGARCH	APGARCH
Normal	500	944	982
	1000	954	991
	1500	956	993
	2000	956	995
General error distribution	500	947	978
	1000	952	995
	1500	958	998
	2000	965	996

From table (10), for (NAGARCH & APGARCH) models we note the high replicates for the two distributions (Normal distribution and General error distribution) and this iteration increases with increasing the sample size ,but the iteration of APGARCH with General error distribution is higher than NAGARCH, that means the model is suitable for handling the case of heteroskedasticity in the standard square residual series .

4- Conclusion

In the Identifications phase the tests used (Ljung-Box and ARCH test) have proven efficient in the process of identifying the autocorrelation and heteroskedasticity states for the models (NAGARCH & APGARCH) with respect to the residual of the square return series ,we note that the number of iteration increases with increasing the sample size and the best result for NAGARCH model with Normal distribution and the best result for APGARCH model with General error distribution , either in the estimation phase the result shows that the Maximum Likelihood Estimation method is successful in estimation of the two models and the best results are at size (2000) and this has been proven by the result of (MAE) as the test result where decreasing with increasing a sample size. In the diagnostic checking phase, the result shows that the models are suitable for processing with the case of autocorrelation and heteroskedasticity and the best result for APGARCH model when the error distributed (General error distribution) and the best result for NAGARCH model when the error distributed Normal. Finally we conclude that the model APGARCH with General error distribution is superior to the NAGARCH and proves his efficiency in the dealing with the states of (heteroskedasticity and autocorrelation) when the error follows General error distribution and the best result at the largest sample size (2000) .

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تحليل النماذج NAGARCH & APGARCH باستعمال المحاكاة

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مستخلص البحث:

تعد تجارب المحاكاة وسيلة للحل في العديد من المجالات العلمية ، ويمكن تعريفها على انها عملية تصميم نموذج للنظام الحقيقي من أجل متابعته والتعرف على سلوكه من خلال نماذج وصيغ معينة مكتوبة على وفق اسلوب برمجي مكرر بعدد معين من التكرارات ، الهدف من هذه البحث هو دراسة سلوك النماذج غير المتماثلة (APGARCH & NAGARCH) من خلال وبافتراض عدة توزيعات للخطأ العشوائي Gaussian & (Non-Gaussian) و لحجوم عينات مختلفة (2000,1500,1000,500) من خلال مراحل بناء الانموذج (التشخيص ، التقدير ، فحص مدى الملائمة والتنبؤ) ، تم الحصول على البيانات باستعمال المقدرات الناتجة عن تطبيق هذه النماذج على سلسلة العوائد لأسعار صرف الدينار العراقي مقابل الدولار الأمريكي (IQ / USD) للفترة من (2011/7/22) حتى (2021/7/21) ثم يتم استعمال هذه المقدرات في عملية توليد البيانات. في مرحلة التشخيص تم استعمال اختباري (ARCH و Ljung-Box) وأظهرت النتائج تشخيص وجود حالات (الارتباط الذاتي وعدم تجانس التباين) وان عدد هذه الحالات يزداد مع زيادة حجم العينة ، وان أفضل النتائج لأنموذج (NAGARCH) مع توزيع (Normal distribution) و لأنموذج (APGARCH) مع توزيع (General error distribution) ، تمت عملية التقدير باستعمال طريقة الامكان الاعظم الشرطية وان افضل النتائج كانت مع حجوم العينات الكبيرة وهذا ما اثبتته نتائج متوسط مطلق الخطأ (MAE) حيث ان قيمة (MAE) كانت تتناقص بازدياد حجم العينة ، في مرحلة فحص مدى الملائمة تم اعادة استعمال اختباري (ARCH test & Ljung-Box) ولكن لسلسلة البواقى المعيارية وأظهرت النتائج قدرة النماذج (APGARCH) عندما يتوزع الخطأ (General error distribution) .

المصطلحات الرئيسية للبحث : APGARCH , NAGARCH , الغير متناظرة , المحاكاة