



## Estimation of Reliability through the Wiener Degradation Process Based on the Genetic Algorithm to Estimating Parameters

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### Abstract:

In this paper, the researcher suggested using the Genetic algorithm method to estimate the parameters of the Wiener degradation process, where it is based on the Wiener process in order to estimate the reliability of high-efficiency products, due to the difficulty of estimating the reliability of them using traditional techniques that depend only on the failure times of products. Monte Carlo simulation has been applied for the purpose of proving the efficiency of the proposed method in estimating parameters; it was compared with the method of the maximum likelihood estimation. The results were that the Genetic algorithm method is the best based on the AMSE comparison criterion, then the reliability was estimated by an inverse Gaussian distribution according to the characteristics of the Wiener process. It was also applied based on real data taken from an experiment intended to determine the degradation in the light intensity of lamps under specific experimental conditions.

Paper type: Research paper

**Keywords:** Wiener degradation process, Genetic algorithm, Maximum likelihood method, inverse Gaussian distribution, reliability estimation.

## 1. Introduction

Recently, products degradation data has been relied upon and used to estimate reliability (Liu et al., 2020); degradation data is defined as a decline in product performance over time without reaching a state of complete failure and has provided more information about the product (Pan et al., 2020). The phenomenon of deterioration is a kind of stochastic process; this requires mathematical models to represent the deterioration of the products known as deterioration models. The Wiener model is one of the common stochastic processes (Serban and Gebraeel, 2011) and (Si et al., 2014); one of its advantages is that the stochastic effect of the model follows a normal distribution, and this allows working with an inverse Gaussian distribution to be more appropriate with the reliability function (Wang and Xu, 2010). Several literatures mentioned the Wiener degradation process and its importance in estimating reliability, see for example the references (Si et al., 2015), (Wang, 2010), and (Wang et al., 2020). For estimating the parameters of the Wiener model, the researchers introduced several methods. Liu et al. (2018) used Bayesian inference to analyze reliability based on the degradation process. Pan et al.(2018) adopted the expectation-maximization algorithm to estimate the parameters Wiener processes and a reliability estimation. Lin et al.(2019) used Bayesian posterior to estimate parameters of the Wiener degradation process. Pan et al.(2020) used the maximum likelihood method to estimate the parameters of the Wiener degradation process to evaluate the reliability. In this paper, the researcher suggested using the Genetic algorithm to estimate parameter of Wiener process to estimate the reliability. The remainder of this paper is organized as follows: In section 2, methodology description of the degradation model and estimation of reliability. In Section 3, a Monte Carlo simulation was applied to verify the effectiveness of the suggested estimation method. In Section 4, a case study was applied according to what is done in the simulation aspect, while the most important conclusions reached were mentioned in the Fifth Section.

## 2. Methodology description

In this section, the model of degradation with a Wiener process, the reliability estimation based on Wiener process and Parameters estimation are explained.

### 2.1. Modelling of degradation with a Wiener process

The Wiener process can be used to represent degradation data for experimental unit  $i$  if it has a normal distribution as follows (Zhou et al., 2020):

$$X_i(t) = \mu_i t + \sigma_i B(t) \quad , \quad (1)$$

$i = 1, 2, \dots, n$

Where  $X_i(t)$  represents the Wiener process, i.e. the amount of deterioration of the experimental unit  $i$  at time  $t$ ,  $\mu$  is drift parameter,  $\sigma$  is diffusion parameter,  $B(\cdot)$  is Wiener standard process subject to  $N(0, t)$ . The characteristics of the Wiener process  $\{X_i(t); t \geq 0\}$  are as follows:

1.  $(X(t); t \geq 0)$  have independent increments, that is  $((X(t_2) - X(t_1))$  independent from  $(X(t_3) - X(t_4))$  for  $0 \leq t_1 \leq t_2 \leq t_3 \leq t_4$ .
2. Increase in the path of degradation  $\Delta X(t) = X(t + \Delta t) - X(t)$  follows a normal distribution  $\{\Delta X(t) \sim N(\mu \Delta t, \sigma^2 \Delta t)\}$ .

## 2.2. Reliability estimation

Product failure occurs when the amount of degradation  $X_i(t)$  of product  $i$  reaches the failure threshold  $W$  (the first time the product reaches failure) at time  $t$ ,  $X_i(t)$  is here expressed by the Wiener process, thus, the reliability function can be obtained as follows (Pan et al., 2017):

The failure time  $T$  is defined as:

$$T = \inf\{t \geq 0; X(t) \geq w\} \quad (2)$$

The  $T$  can follow an inverse Gaussian (IG) distribution according to the features of the Wiener process:

$$T \sim IG(w/\mu, w^2/\sigma^2) \quad (3)$$

Thus, the probability density function (PDF) of  $T$  as:

$$f_T(t) = \frac{w}{\sqrt{2\pi\sigma^2 t^3}} \exp\left(-\frac{(w - \mu t)^2}{2\sigma^2 t}\right) \quad (4)$$

And the reliability function  $R(t)$  is given as:

$$R(t) = \Phi\left(\frac{w - \mu t}{\sigma\sqrt{t}}\right) - \exp\left(\frac{2\mu w}{\sigma^2}\right) \Phi\left(\frac{-\mu t - w}{\sigma\sqrt{t}}\right) \quad (5)$$

## 2.3. Parameter estimation

Assuming there are  $n$  products, the performance of each product is measured  $k$  times at a specific time for all products, each time an increase in degradation occurs; according to the features of the Wiener process the increase in deterioration  $\Delta X_{ij}$  for each product follows a normal distribution (Panal.,2020):

$$\Delta X_{ij} \sim N(\mu_i \Delta t_{ij}, \sigma_i^2 \Delta t_{ij})$$

(6)

Where  $i = 1, 2, 3, \dots, n$  and  $j = 1, 2, 3, \dots, k$ .

And that the PDF of  $\Delta X_{ij}$  as follows:

$$f(\Delta X_{ij}) = \frac{1}{\sqrt{2\pi\sigma_i^2 \Delta t_{ij}}} \exp\left(-\frac{(\Delta X_{ij} - \mu_i \Delta t_{ij})^2}{2\sigma_i^2 \Delta t_{ij}}\right) \quad (7)$$

Where

$$\Delta X_{ij} = X_{ij} - X_{ij-1} \quad (8)$$

And

$$\Delta t_{ij} = t_{ij} - t_{ij-1} \quad (9)$$

Now, we need to estimate the parameters  $(\mu, \sigma^2)$ , here the methods were used : maximum likelihood and Genetic algorithm as shown in the next subsections.

### 2.3.1. Maximum likelihood method

The maximum likelihood (ML) method is considered one of the most important estimation methods (Al-Aameri and AL.Doori, 2022); it can be used to estimate parameters  $\mu$  and  $\sigma^2$  through the PDF of the  $\Delta X_{ij}$ , the maximum likelihood function of "Eq. (7)" is as follows (Lin et al., 2019):

$$L(\mu_i, \sigma_i^2) = \prod_{j=1}^K \frac{1}{\sqrt{2\pi\sigma_i^2\Delta t_{ij}}} \exp\left(\frac{-(\Delta X_{ij} - \mu_i\Delta t_{ij})^2}{2\sigma_i^2\Delta t_{ij}}\right)$$

$$= (2\pi\sigma_i^2\Delta t_{ij})^{-\frac{K}{2}} \exp\left(-\sum_{j=1}^k \frac{(\Delta X_{ij} - \mu_i\Delta t_{ij})^2}{2\sigma_i^2\Delta t_{ij}}\right) \quad (10)$$

Take the logarithm of the "Eq. (10)":

$$\ln L(\mu_i, \sigma_i^2) = -\frac{k}{2} \ln(2\pi\sigma_i^2\Delta t_{ij}) - \sum_{j=1}^k \frac{(\Delta X_{ij} - \mu_i\Delta t_{ij})^2}{2\sigma_i^2\Delta t_{ij}} \quad (11)$$

And find the partial derivatives of  $\mu_i$  and  $\sigma_i^2$  respectively as:

$$\frac{\partial \ln L(\mu_i, \sigma_i^2)}{\partial \mu_i} = 0 + \sum_{j=1}^k \frac{2(\Delta X_{ij} - \mu_i\Delta t_{ij}) * \Delta t_{ij}}{2\sigma_i^2\Delta t_{ij}} \quad (12)$$

$$\frac{\partial \ln L(\mu_i, \sigma_i^2)}{\partial \sigma_i^2} = -\frac{k}{2\sigma_i^2} + \sum_{j=1}^k \frac{(\Delta X_{ij} - \mu_i\Delta t_{ij})^2 * 2\Delta t_{ij}}{2\sigma_i^4\Delta t_{ij}} \quad (13)$$

Now, by making "Eq. (12)" and "Eq. (13)" equal to zero and simplifying them, we get estimators  $\hat{\mu}_i$  and  $\hat{\sigma}_i^2$  respectively:

$$\hat{\mu}_i = \frac{\sum_{j=1}^K \Delta X_{ij}}{\sum_{j=1}^K \Delta t_{ij}} \quad (14)$$

$$\hat{\sigma}_i^2 = \frac{1}{K} \left[ \sum_{j=1}^K \frac{(\Delta X_{ij})^2}{\Delta t_{ij}} - \frac{(\sum_{j=1}^K \Delta X_{ij})^2}{\sum_{j=1}^K \Delta t_{ij}} \right] \quad (15)$$

### 2.3.2. Genetic algorithm

The Genetic algorithm (GA) is one of the methods of artificial intelligence to reach optimal solutions in mathematical problems. The most important features of GA are the following (Kim et al., 2015), (Magalhães-Mendes, 2013):

1. Shortening the effort and time of the solving issues, taking into consideration the specificity of each issue, the restrictions imposed, the type of data used and the nature of the objective function.
2. It is more effective than traditional methods in solving optimal problems or solving problems that traditional methods are unable to solve and reach optimal solutions.
3. The algorithm can be used without the need for derivatives of the function under study.
4. The algorithm generates number of solutions to the problem under study, and then it is evaluate solutions to reach the optimal solution.

The following are the steps for applying the GA to estimate the parameters:

1: Define the objective function, here it is the logarithm of the maximum likelihood function of the "Eq.(10)" as:

$$S(A) = LnL(f(A)) \quad , \quad where \ A = (\mu, \sigma^2) \quad (16)$$

2: Initialization: It is called the process of creating the primary generation of chromosomes; this step begins with guessing the initial combinations in the boundaries of the parameters to be estimated that make up the chromosomes values, that may or may not include optimal values, a random regular function is used to generate the initial values.

3: Selection: To make a new generation of chromosomes, this is done by selecting suitable chromosomes from the old generation based on the highest values of the differentiation function, where the roulette wheel was used to select chromosomes.

4: Encoding: It is the process of representing chromosomes with a real symbol or number. In this paper, binary coding was used, which is one of the most famous methods of encoding chromosomes, and its fame is due to being the first method through which chromosomes (solutions) were encoded.

5: Crossover: After selecting the chromosomes (parents), the crossover occurs due to the pairing of two original chromosomes. The new chromosome that is produced after the crossing-over process is called the "offspring" and that mating takes place between the parent chromosomes.

6: Mutation: They are random changes within the same son chromosome by changing one or more genes, and this leads us to maintain good traits between genes within one chromosome and to reach the optimal solution faster. In the absence of a mutation, the cloning process is carried out directly without the occurrence of the crossbreeding process.

7: Stop criterion: If any of the chromosomes produces an appropriate value that fulfills the objective function, the iterative process is stopped and the result is the optimal solution to the problem. Otherwise, the process from step 2 is repeated.

### 3. The simulation

To prove whether the ML or GA method is the best, simulation was performed Monte Carlo type. Sample sizes were selected to be  $n = 5$  and  $n = 10$  from the tested units, each unit was measured at different times, five times were selected as  $k = 0, 50, 100, 150, 200, 250$ . The random observations were generated from the Wiener random-effect degradation model under the two parameters  $(\mu, \sigma^2)$ . The methods referred to in section (2) were applied to obtain the estimation of parameters  $(\mu, \sigma^2)$ , and then, the average mean square error (AMSE) of the PDF was calculated for comparison of estimation methods using 1000 replicated. All computational procedures were performed, using Matlab program.

#### 3.1 Discuss the simulation results

Table 1 below show the values of the estimated parameters using the methods of ML and GA of sample size  $n = 5$  and  $n = 10$  units.

Table 1: Initial and estimated parameter values of the Wiener process of  $n = 5$  and  $n = 10$  units

N	Units	The parameters					
		Initial parameters		Estimated parameters by ML method		Estimated parameters by GA method	
		$\mu$	$\sigma^2$	$\hat{\mu}$	$\hat{\sigma}^2$	$\hat{\mu}$	$\hat{\sigma}^2$
n=5	1	9.8	7.6	7.90	4.98	9.79	7.57
	2	9.9	7.9	8.07	5.09	9.89	7.87
	3	10.1	7.7	8.09	5.10	10.09	7.67
	4	10.9	7.8	8.53	5.36	10.89	7.77
	5	12	8.3	9.27	5.82	11.99	8.27
n=10	1	9.8	7.6	7.9	4.98	9.79	7.57
	2	9.9	7.9	8.07	5.09	9.89	7.87
	3	10.1	7.7	8.09	5.1	10.09	7.67
	4	10.9	7.8	8.53	5.36	10.89	7.77
	5	12	8.1	9.19	5.77	11.99	8.07
	6	10.8	8.1	8.59	5.41	10.79	8.07
	7	10.2	7.5	8.06	5.07	10.19	7.47
	8	11	7.8	8.58	5.39	10.99	7.77
	9	8.9	8	7.6	4.84	8.89	8.10
	10	11.1	8.3	8.82	5.56	11.09	8.27

In order to comparing the estimation methods through the value of the probability density function, Table 2 below shows the value of the probability density function when applying the estimation methods referred to above, and Table 3 shows the AMSE's values which were applied of the PDF.

**Table 2: PDF Values for each experiment (n = 5 and n = 10 units), at 6 times of degradation (0,50,150, 200, 250), using ML and GA estimation methods**

N	Time	The PDF Values by Estimation Methods											
		ML						GA					
	Units	0	50	100	150	200	250	0	50	100	150	200	250
n=5	1	0.896	0.875	0.79	0.636	0.583	0.3501	0.856	0.79	0.757	0.481	0.212	0.024
	2	0.914	0.821	0.726	0.715	0.371	0.0652	0.89	0.775	0.775	0.673	0.509	0.49
	3	0.946	0.907	0.826	0.644	0.644	0.0863	0.95	0.9	0.813	0.769	0.404	0.09
	4	0.915	0.774	0.589	0.578	0.525	0.2109	0.89	0.858	0.68	0.218	0.196	0.108
	5	0.881	0.82	0.658	0.272	0.173	0.1007	0.99	0.901	0.633	0.314	0.114	0.008
n=10	1	0.897	0.876	0.79	0.636	0.583	0.35	0.857	0.79	0.758	0.482	0.212	0.025
	2	0.915	0.822	0.726	0.715	0.371	0.065	0.89	0.775	0.775	0.674	0.509	0.49
	3	0.946	0.907	0.827	0.645	0.644	0.086	0.951	0.901	0.814	0.769	0.405	0.09
	4	0.916	0.774	0.589	0.578	0.526	0.211	0.89	0.858	0.681	0.219	0.196	0.108
	5	0.738	0.638	0.47	0.361	0.277	0.194	0.99	0.606	0.398	0.343	0.299	0.036
	6	0.913	0.633	0.625	0.621	0.594	0.151	0.79	0.789	0.472	0.461	0.371	0.361
	7	0.799	0.797	0.713	0.691	0.159	0.057	0.953	0.933	0.696	0.434	0.19	0.087
	8	0.998	0.902	0.812	0.592	0.576	0.038	0.99	0.486	0.352	0.32	0.237	0.004
	9	0.933	0.749	0.605	0.355	0.235	0.093	0.89	0.428	0.382	0.231	0.161	0.073
	10	0.823	0.761	0.509	0.308	0.286	0.04	0.826	0.449	0.215	0.179	0.09	0.038

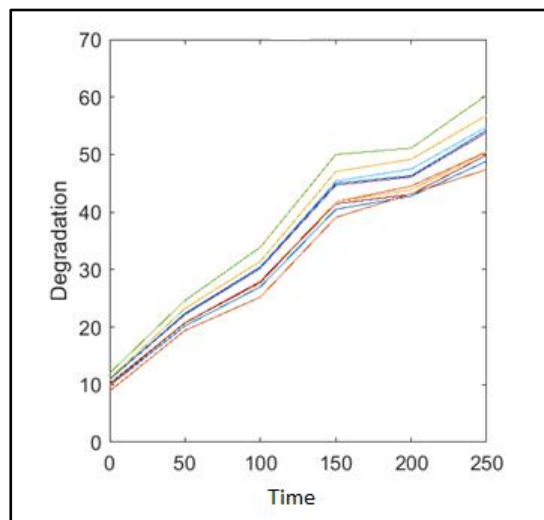
**Table 3: AMSE values for estimating the PDF using methods estimation (ML and GA) of n=5 and n=10 for all indicated experimental times**

n	AMSE values of the PDF	
	ML	GA
n=5	0.4450689	0.4204638
n=10	0.4033365	0.3344066

From Table 3, we noted the decrease in AMSE with the increase in the degradation data, as well as the AMSE value of the GA method compared to the ML method for all sample sizes is the lowest, meaning that GA is the best. Accordingly, the best parameter estimation method for modelling the degradation data will be adopted through the Wiener process “Eq.( 1)”. To clarify this, a sample of size n = 10 will be used, as shown in Table 4 and Figure 1.

**Table 4: Amount of degradation  $X_i(t)$  in the data generated after modeling by the Wiener process whose parameters were estimated by the GA method of  $n=10$  at 6 degradation times (0,50,150,200,250)**

$X_i(t)$	Units									
	1	2	3	4	5	6	7	8	9	10
0	9.79	9.89	10.09	10.89	11.99	10.79	10.19	10.99	8.89	11.09
50	20.21	20.78	20.81	22.2	24.34	22.46	20.7	22.35	19.43	23.22
100	27.02	27.67	27.95	30.22	33.61	30.36	27.95	30.49	25.23	31.45
150	40.48	41.78	41.77	44.68	49.3	45.37	41.43	45	39.07	47.04
200	42.76	44.51	43.9	46.11	50.04	47.47	43.09	46.32	43.16	49.18
250	48.86	50.49	50.4	53.86	59.4	54.79	49.93	54.24	47.38	56.83



**Figure1: Plot of the degradation data of  $n=10$  units at 6 degradation times for each unit after modeling by the Wiener process whose parameters were estimated by the GA method**

After that,  $R(t)$  was evaluated using “Eq. (5)” according to the amount of degradation  $X_i(t)$  in Table 4 and the parameters estimated by GA method, the results of  $R(t)$  are shown in Table 5 and Figure 2.

**Table 5: Reliability function results of  $n=10$  units at 6 degradation times (0,50,150,200,250)**

$R(t)$	Units									
	1	2	3	4	5	6	7	8	9	10
$R(0)$	1	1	1	1	1	1	1	1	1	1
$R(50)$	0.509	0.6655	0.904	0.9529	0.9394	0.7902	0.8392	0.8325	0.7838	0.7968
$R(100)$	0.2666	0.6236	0.8546	0.8902	0.6716	0.6119	0.3599	0.8286	0.6226	0.7921
$R(150)$	0.1231	0.3391	0.6239	0.3056	0.3329	0.4547	0.2928	0.3893	0.5499	0.7541
$R(200)$	0.0272	0.1143	0.5246	0.0946	0.2735	0.251	0.2732	0.3425	0.3184	0.3056
$R(250)$	0.0026	0.114	0.0902	0.0867	0.238	0.2435	0.1902	0.1522	0.1661	0.0902



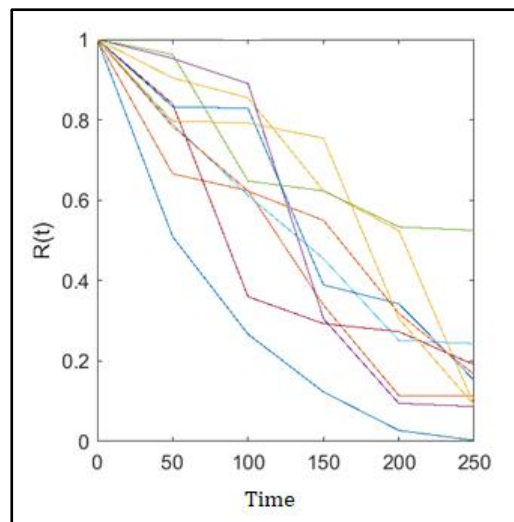


Figure 2: Plot of the reliability function of  $n=10$  units, at 6 degradation times for each unit with parameters estimated by the GA method

From Table 5 and Figure 2, we notice that the reliability of each product decreases over time and this is according to reliability theory. However, when the degradation amount shown in Table 4 reaches the failure threshold here  $w = 30$ , this shows the exact reliability of the products at failure time; Table 6 shows the failure time (after passing the failure threshold) and the corresponding reliability for each product.

Table 6: Units failure time at the failure threshold with the corresponding reliability

Units	Failure time	R(t)
1	150	0.1231
2	150	0.3391
3	150	0.6239
4	100	0.8902
5	100	0.6716
6	100	0.6119
7	150	0.2928
8	100	0.8286
9	150	0.5499
10	100	0.7921

#### 4. Case study

The laboratories of the Physics Department of the College of Education Ibn Al-Haytham for Pure Sciences / University of Baghdad have been taken as a place to conduct an experiment to find out the state of degradation of products over time through measuring the intensity of the light of 10 lamps with a Digital Lux meter with a Lux measurement unit, by operating the lamps at 220 volts for a period of 250 hours. The intensity of the lamp light is measured every 50 hours; the amount

of degradation shown in Table 7 is calculated by taking the difference between the standard light intensity with which the lamp should work and the measured light intensity after the specified operating time.

**Table 7: Luminous intensity degradation data for lamps (units) every 50 hours starting from time 0 to 250 hours**

Time	Units									
	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0
50	5.9	5.4	3.8	5.1	4.4	3.9	3.7	4.9	5.6	5
100	8.6	9.3	8.9	7	8.4	6.3	8.7	6.7	7.1	6.4
150	12.7	13.2	11.5	12.6	12.8	10.9	10.5	11.1	13.1	11.2
200	18.9	15.2	19.7	16.7	15.3	15.5	17	16	16.5	18.7
250	23.6	23.7	21.4	22	21.2	24.9	22.1	23.8	21.5	24.9

A good ness-of-fit test is performed on the degradation data to know if they follow a normal distribution, and this informs whether the data conform to the Wiener process; the Chi-Square test with Matlab program is used to verify the test hypotheses:

$H_0$ : The data follow a normal distribution.

$H_1$ : The data do not follow a normal distribution.

The result of the test was that the calculated Chi-Square value is less than the tabular value at the 0.05 level of significance, and therefore the null hypothesis  $H_0$  is accepted, which means that the data follow a normal distribution and the Wiener process can be used for data modeling. Through the simulation results, it was found that the GA is the best for estimating the parameters, so the GA was used in the real data to estimate the parameters, the results are shown in Table 8.

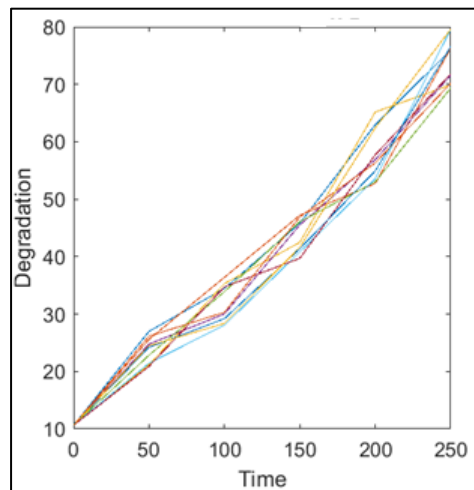
Table8: Values of the estimated parameters ( $\hat{\mu}$ ,  $\hat{\sigma}^2$ ) of the real data by the GA method

Unties	$\hat{\mu}$	$\hat{\sigma}^2$
1	10.64	7.653
2	10.62	7.655
3	10.59	7.666
4	10.63	7.599
5	10.61	7.654
6	10.64	7.662
7	10.58	7.643
8	10.62	7.631
9	10.64	7.666
10	10.6	7.637

Now, the degradation data are modeled by the Weiner process as shown in Table 9 and Figure 3.

**Table 9: Amount of degradation  $X_i(t)$  in the real data after modeling by the Wiener process whose parameters were estimated by the GA method of  $n=10$  at 6 degradation times (0,50,150,200,250).**

$X_i(t)$	Units									
	1	2	3	4	5	6	7	8	9	10
0	10.64	10.64	10.64	10.64	10.64	10.64	10.64	10.64	10.64	10.64
50	26.98	25.59	21.16	24.76	22.82	21.44	20.88	24.21	26.15	24.48
100	34.45	36.39	35.28	30.02	33.90	28.08	34.73	29.19	30.30	28.36
150	45.80	47.19	42.48	45.53	46.08	40.82	39.71	41.37	46.91	41.65
200	62.97	52.73	65.19	56.88	53.00	53.56	57.71	54.94	56.33	62.42
250	75.98	76.26	69.89	71.55	69.34	79.58	71.83	76.54	70.17	79.58

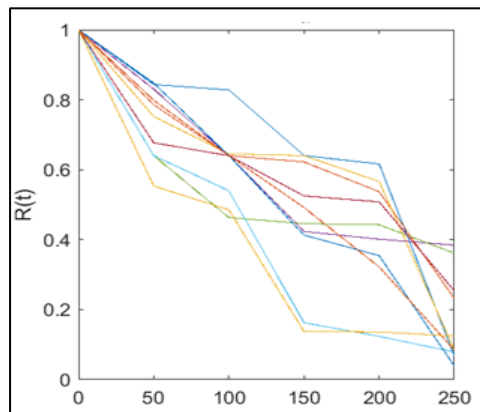


**Figure 3: Plot of the luminous intensity degradation data for lamps after modeling by the Wiener process whose parameters were estimated by the GA method**

The  $R(t)$  is evaluated according to the degradation data  $X_i(t)$  as shown in Table 10 and Figure 4.

**Table 10 : Reliability function results of the luminous intensity degradation data for lamps (units)**

R(t)	Units									
	1	2	3	4	5	6	7	8	9	10
R(0)	1	1	1	1	1	1	1	1	1	1
R(50)	0.8436	0.7984	0.7522	0.8301	0.6402	0.6402	0.6769	0.8476	0.7858	0.5533
R(100)	0.8279	0.6402	0.6451	0.6402	0.4632	0.5398	0.6402	0.6402	0.6402	0.4851
R(150)	0.6402	0.6222	0.6402	0.4233	0.4457	0.1626	0.5250	0.4130	0.4919	0.1371
R(200)	0.6164	0.5371	0.5662	0.4014	0.4426	0.1236	0.5080	0.3543	0.3221	0.1361
R(250)	0.0685	0.2320	0.0846	0.3844	0.3615	0.0788	0.2540	0.0389	0.0846	0.1245



**Figure 4: Plot of the reliability function of the luminous intensity degradation data for lamps**

And when the degradation amount shown in Table 9 reaches the failure threshold here  $w = 30$ , this shows the exact reliability of the lamps at the time failure; Table 11 shows the failure time (after passing the failure threshold) and the corresponding reliability for each product.

**Table 11 : Lamps (units) failure time at the failure threshold with the corresponding reliability estimated R(t)**

Units	Failure time	R(t)
1	100	0.8279
2	100	0.6402
3	100	0.6451
4	100	0.6402
5	100	0.4632
6	150	0.1626
7	100	0.6402
8	150	0.4130
9	100	0.6402
10	150	0.1371

## 5. Conclusions

1. In this paper, the GA method proposed by the researcher here for estimating the Wiener degradation process parameters in order to estimate the reliability is the best according to the AMSE comparison criterion, because the results of the simulation experiments show that the estimators of the proposed method have lower AMSE values compared to the ML method.

2. From the conclusions of the application side, the reliability function shows that it decreases with the amount of degradation of the lamps, and this is consistent with the reliability theory.

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## تقدير المعولية من خلال عملية التدهور Wiener بالاعتماد على الخوارزمية الجينية لتقدير المعلمات

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### مستخلص البحث

في هذا البحث اقترح الباحث استعمال طريقة الخوارزمية الجينية لتقدير معلمات عملية التدهور Wiener ، حيث يتم الاعتماد على عملية Wiener لتقدير المعولية للمنتجات عالية الكفاءة، وذلك لصعوبة تقدير المعولية لها باستخدام التقنيات التقليدية التي تعتمد فقط على اوقات فشل المنتجات . قد تم تطبيق محاكاة Monte Carlo لغرض اثبات كفاءة الطريقة المقترحة في تقدير المعلمات ، واجريت مقارنتها مع طريقة الامكان الاعظم ، وكانت النتائج أن طريقة الخوارزمية الجينية هي الافضل بناءً على معيار المقارنة AMSE ، ثم تم تقدير المعولية بناءً على معكوس توزيع Gaussian وفقاً لخصائص عملية Wiener. كذلك تم التطبيق على بيانات حقيقة مأخوذة من تجربة الغرض منها معرفة التدهور في شدة ضوء المصابيح تحت ظروف تجريبية محددة.

نوع البحث: ورقة بحثية.

المصطلحات الرئيسية للبحث: عملية التدهور Wiener، الخوارزمية الجينية ، طريقة الامكان الاعظم ، تقدير المعولية ، معكوس توزيع Gaussian.

ملاحظة: البحث مستل من رسالة ماجستير