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Seemingly Unrelated Regression Model to Measure the Profitability of Some Iraqi Private Commercial Banks with Presence of Outliers

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Abstract

A seemingly uncorrelated regression (SUR) model is a special case of multivariate models, in which the error terms in these equations are contemporaneous related. The method estimator (GLS) is efficient because it takes into account the covariance structure of errors, but it is also very sensitive to outliers. The robust SUR estimator can dealing outliers. We propose two robust methods for calculating the estimator, which are (S-Estimations, FastSUR). We find that it significantly improved the quality of SUR model estimates. In addition, the results gave the FastSUR method superiority over the S method in dealing with outliers contained in the data set, as it has lower (MSE and RMSE) and higher (R-Squared and R-Square Adjusted) values.

Keywords: Seemingly uncorrelated regression (SUR); Outliers; Contemporaneous Correlation; GLS Estimation Method; Robust SUR Estimators; FastSUR Estimator.

1. Introduction

The seemingly uncorrelated regression equations model is a special case of multivariate models. It has a special importance in the science of statistics and is considered one of the important topics for its use in wide applications in most fields and in various sciences. Including investment demand equations for many companies and companies in the same industry. Consumer demand function, equations involved in utility maximization behavior. Input demand function, the equations involved in the firm's behavior to minimize cost and maximize profit. Insurance and financing models in the same for-profit activities. And the study is based on a regression model consisting of several equations; often there is an effect of unnoticed factors on the random error in one of the equations also affects the random error in the other equations. Estimating the equations separately using the Least Squares (OLS) method leads to inefficient estimates. (Zellner, 1962) came to propose for the first time the SUR model taking advantage of the fact that the equations appear to be unrelated but are in fact linked together by contemporary random error between the equations of the model. This correlation between the equations make up the SUR model. Using a method (GLS) that estimates a system of equations simultaneously and that considers the covariance of random errors provides additional information about what is available when considering the model equations for each equation separately. Zellner found that the estimates obtained from the SUR method are, at least asymptotically, more efficient than applying the ordinary least squares (OLS) method to each equation, which motivated Zellner's original work (Dwivedi & Srivastava, 1978) and the fact that the increase in efficiency could be larger if the exogenous variables in the different equations were not significantly correlated. Contemporary random errors in different equations are highly correlated. (Zellner, 1963) The presence of outliers in the data set is common. Since the Zellner method based primarily on the method of least squares, the estimator is subject to outlier's values, which are very harmful to the traditional method, which is ideal under normal or linear conditions. These outliers can have obvious disturbing effects on the estimates produced by the traditional method. In this case, estimating model parameters is useless, as the results are misleading, untrustworthy, and give us an unreliable conclusion. Accordingly, it is necessary to move towards robust methods that can counteract outliers in parameter estimation(Saheb & Hussein, 2021). In (Bilodeau & Duchesne, 2000) presented an adaptation of robust S-Estimations that have a high breakdown point after modifying the Ruppert algorithm. In (Hubert et al., 2014) presented a robust method for estimators that can accommodate outliers in the data, as they developed S-Estimations into the FastSUR algorithm, which applies the ideas of the FastS algorithm in SUR models. (Saraceno et al., 2021) suggest a new robust estimation method for the SUR model, in the presence of outliers in rows or cells with increasing number of equations. After identifying the aim of the study, we discussed in the third section the concept of the SUR model, its hypotheses and its estimation method, in the fourth section the robust methods that we used in the estimation (S and FastSUR) and the fifth the studied model and finally the results of the study.

The aim is to find robust estimators to model SUR equations such as (S-estimators and FastSUR-estimators) in the presence of outliers in the data set. In addition, the comparison between these two methods and the extent of their ability to deal with outliers, as well as the comparison between these two robust methods and of the traditional methods (OLS, GLS).

2. Materials and Methods

2.1 SUR model

The SUR system of equations presented by Zellner (1962) consists of a number of M linear regression equations where M≥2. Each regression equation gives us a specific causal relationship or a statistical regression relationship between the endogenous variable and one or more exogenous variables.

The number of equations in the system model is equal to the number of endogenous variables. The system of equations comprising M of the regression equations is as follows:

$$y_i = X_i\beta_i + u_i \quad \dots (1)$$

where $i = 1, \dots, M$; vector of y_i $(Tx1)$ endogenous variable observations, $y_i = (y_{1i}, \dots, y_{Ti})'$. X_i: (TxP$_i$) A matrix whose columns represent the observations of the exogenous variable in the i^{th} equation. β_i: $(p_i x1)$ A vector whose elements are β_{ij} are the parameters of the model, $\beta_i = (\beta_{1i}, \dots, \beta_{p_i i})'$. u_i: $(Tx1)$ vector of the random error in the model, $u_i = (u_{1i}, \dots, u_{Ti})'$. (Srivastava & Giles, 2020)

It can be noticed that each equation in the SUR model is a linear regression model, and it seems unrelated at first sight, but they are related through random error terms. The equations (1) can write in form as follows: (AL-Bermani & Dr.A.H.Kadhim, 2002)

$$\begin{aligned} y_{1t} &= \beta_{10} + \beta_{11}x_{1t,1} + \beta_{12}x_{1t,2} + \cdots + \beta_{1p}x_{1t,p} + u_{1t} \\ y_{2t} &= \beta_{20} + \beta_{21}x_{1t,1} + \beta_{22}x_{2t,2} + \cdots + \beta_{2p}x_{2t,p} + u_{2t} \quad \dots (2) \\ &\vdots \\ y_{Mt} &= \beta_{M0} + \beta_{M1}x_{Mt,1} + \beta_{M2t}x_{Mt,2} + \cdots + \beta_{Mp}x_{Mt,p} + u_{Mt} \end{aligned}$$

The equations in the SUR model are related assuming Contemporaneous correlation, meaning the j^{th} element of the random error in equation i may be related to the j^{th} element of the random error in equation k. With the numbers of observations j and L and the numbers of equations k and I. The variance and covariance of random errors summarized as follows:

$$E(u_{ji}u_{jk}) = \sigma_{ik} \ ; j \in \{1, \dots, T\}, \quad i, k \in \{1, \dots, M\}$$

$$E(u_{ji}u_{Li}) = 0, \quad j \neq L;$$

$$E(u_{ji}u_{Lk}) = 0 \quad , i \neq k, j \neq L .$$

We can combine the equations in the SUR model in two equivalent cases into a single matrix: (Hubert et al., 2014)

The SUR model is in the form of a single linear regression model in the following form:

$$\mathrm{y} = X\beta + u \dots (3)$$

where the y vector of $(TMx1)$ endogenous variable observations, $y = (y'_1, \dots, y'_M)'$, X $(MTxP)$ diagonal matrix where $p = \sum_{i=1}^{m} p_i$ writes $X = bdiag(X_1, \dots, X_M)$, symbolize bdiag ( ) to the factor for which the diagonal matrix is built from its own terms, β $(Px1)$ a vector whose elements are β_{ij} are the parameters of the model, $\beta = (\beta'_1, \dots, \beta'_M)'$, u vector of $(TMx1)$ the random error in the model, $u = (u'_1, \dots, u'_M)'$.

The SUR model is in the form of a multivariate liner regression model as follows: (Hassan & Dr.A.H.Kadhim, 1993)

$$Y = \tilde{X}\beta + U \dots (4)$$

where the y vectors of (TxM) the endogenous variables in the model, $Y = (y_1, \dots, y_M)$, $\tilde{X}$ (TxP) the matrix of exogenous variables in the model, $\tilde{X} = [X_1, \dots, X_M]$, β (PxM) Vertical vectors of model parameters, $\beta = diag(\beta_1, \dots, \beta_M)$, U $(TMx1)$ vertical vector of random errors in the model $u = (u_1, \dots, u_M)$ and writing $u_j = (e_{j1}, \dots, e_{jM})'$. (Hassan & Dr.A.H.Kadhim, 1993)

2.1.1 SUR Model Assumptions

To transact with the equations model (2) as a linear regression model, the following assumptions must be satisfied:(Fiebig, 2007)

$$X_i \text{ is fixed}, \ rank(X_i) = P_i \dots (5)$$

$$\lim_{T\to\infty}\left(\frac{1}{T}X'_iX_i\right) = \Omega_{ii} \dots (6)$$

, $i = 1, \dots, M$, where Ω_{ii} non-singular matrix it contains elements whose values are fixed. We assume that the random errors in the error vector (u_i) follow a multivariate probability distribution where:

$$E(u_i) = 0, (i = 1, \dots, M \dots (7)$$

$$E(u_iu'_i) = E(u_{ti}^2) = \sigma_I^2 I_T = \sigma_{ii} I_T \dots (8)$$

(σ_{ii}) The variance of the random error of equation (i) for each observation in the sample unit matrix(TxT).

The supposed overlap between the equations results in the following:

$$\lim_{T\to\infty}\left(\frac{1}{T}X'_iX_j\right) = \Omega_{ij}, \ i = 1, \dots, M \dots (9)$$

$$E(u_iu'_j) = \sigma_{ij} I_T \dots (10)$$

where Ω_{ij} non-singular matrix; it contains elements whose values are fixed. (σ_{ij}) Represents the covariance of random errors in two equations (i) and (j), which supposed to constant for all observations. In addition, this variance represents the only link between equations (i) and (j) and this correlation is somewhat accurate, as the system of equations M is called SUR. Accordingly, the assumptions of the system of equations (3) of the random error vector are: (Dr.A.H.Kadhim & B.S.Muslim, 2002)

$$E(U) = 0 \dots (11)$$

$$E(U) = \begin{bmatrix} E(u_1) \\ E(u_2) \\ \vdots \\ E(u_T) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \underset{(TMx1)}{0} \dots (12)$$

$$E(UU') = E\left[\begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_M \end{pmatrix}\begin{pmatrix} u'_1 & u'_2 & \dots & u'_M \end{pmatrix}\right]$$

$$= \begin{bmatrix} E(u_1u'_1) & E(u_1u'_1) & \dots & E(u_1u'_M) \\ E(u_2u'_1) & E(u_2u'_2) & \dots & E(u_2u'_M) \\ \vdots & \vdots & \ddots & \vdots \\ E(u_Mu'_1) & E(u_Mu'_2) & \dots & E(u_Mu'_M) \end{bmatrix} = \underset{(TMxTM)}{\begin{bmatrix} \sigma_{11}I_T & \sigma_{12}I_T & \dots & \sigma_{1M}I_T \\ \sigma_{21}I_T & \sigma_{22}I_T & \dots & \sigma_{2M}I_T \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{M1}I_T & \sigma_{M2}I_T & \dots & \sigma_{MM}I_T \end{bmatrix}}$$

$$= \underset{(MxM)}{\begin{bmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1M} \\ \sigma_{21} & \sigma_{22} & \dots & \sigma_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{M1} & \sigma_{M2} & \dots & \sigma_{MM} \end{bmatrix}} \otimes I_T \dots (13)$$

$$Var - Cov(U) = \Sigma \otimes I_T = \Omega$$

The symbol ($\otimes$) is Kronnecker Product, and $\Sigma = \Sigma'$ is a symmetric matrix, meaning that $\sigma_{ij} = \sigma_{ji},\ ij = 1, \dots, m$. Also, it is supposed to be a non-singular positive definite matrix. It must satisfy the following assumptions: (Srivastava & Giles, 2020)

i. **Homoskedasticity for each equation in the SUR model, as in Equation No. (8).**
ii. **It is assumed that there is no correlation between the error term for each equation (autocorrelation)**
iii. **It is supposed to that there is no Temporal Covariance between random errors in two different equations and at two different times:**
iv. **Random error term in SUR equations independent of the exogenous variable:**
v. **Contemporaneous covariance between random errors and for the same time period between the equations of the SUR model.**

In Contemporaneous cross-section regression. T represents the time and the matrix (15) indicates the variances and covariance from one period to another constant. In addition, there is no autocorrelation or serial correlation for random error, where $\sigma_{uu'}$ with $u = u'$ represents the variance and with $u \neq u'$ represents the combined variance of the units of random error (or exogenous variable) for any time. (Zellner, 1962)

2.1.2 Estimation of the SUR model

The system of equations (3) looks like a single equation; it has a variance and covariance matrix for random errors (16). Therefore, the estimation of the parameters vector (β) of the system of equations is in the Generalized least Squares (GLS) method, and the estimates we obtain from all equations together are the best linear unbiased estimator (BLUE). According to Atiken's theorem, GLS estimates are better than any other unbiased linear estimate, as it has the least variance. The equation for estimating the parameters of the model is as follows: - (Zellner, 1962)

$$\hat{b}_{SUR(GLS)} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y \dots (14)$$

where Ω matrix (MTxMT) variance and covariance of random errors in SUR. Formula (14) gives an unbiased linear best estimate for the parameters vectors $\underline{\beta}_i$ where $(i = 1, \dots, m)$ is in the case of heterogeneity and this formula knows as the Atiken Estimator and the covariance and covariance matrix for the estimated parameters we get from equation (15):

$Var - Cov(b_{SUR})= (X'\Omega^{-1}X)^{-1}$... (15)

In practical application, the elements of the matrix (Σ) are unknown. We have to find estimates for them and then use them in equation (14). Zellner suggested estimating them by employing the residuals that we get from each of the system equations using the OLS method, and substituting (S_{ij}) into the elements Matrix (Σ). (S_{ij}) Are unbiased estimates consistent with (σ_ij) and the equations are as follows: - (Kadhim & Muslim, 2002)

$$b_{FGLS}^{*} = [X'(S^{-1} \otimes I_T)X]^{-1}X'(S^{-1} \otimes I_T)Y$$

$$Var - Cov(b_{FGLS}^{*}) = [X'(S^{-1} \otimes I_T)X]^{-1}$$

The (GLS) estimate for one-step is almost equivalent to the (MLE) estimate and the iteration of (GLS) for more than one-step leads to minimizing the variance of the random error, and then we get estimates that lead to estimates of the (MLE). (Al-Saadoun & Al-Dulaimi, 2000)

2.2 The Robust Estimation Methods

Zellner's method for the (SUR) model basically depends on its estimation on the (OLS) method, and the estimator is susceptible to outliers, which were defined by (Midi & Jaafar, 2007) "outlier values are a single observation or a group of observations that differ significantly from the bulk of the data or about the pattern identified by the majority of observations. Data collected from different applications often contains one or more outliers, and can greatly harm the traditional statistical method that is ideal under normal or linear conditions. These outliers can have obvious disturbing effects on the estimates produced by these estimates, which makes it necessary to develop many robust regression methods that are not sensitive or affected by outliers. The main goal of these methods is to provide impedance results similar to the results in the conventional method for data free of outliers.

2.2.1 S-Estimation method

(Bilodeau & Duchesne, 2000) adapted Rupper's algorithm where they proposed S-Estimations in the context of the SUR model:

Definition (1)

Let $(X_i, Y_i) \in \mathbb{R}^{T\times(p_i+1)}, j = 1, 2 \ ..., M$, with $T \geqslant p + M$ and let ρ_1 be a function $-\rho$ with parameter c_0.

The function ρ must be available to obtain estimators that resist the outlier values in the model for the following conditions:

Condition (1)

The function ρ is the symmetric and differentiable twice continuously, which$p(\infty) = 1$، $p(0) = 0$.

Condition (2)

The function ρ is strictly incremental for the set of positive real numbers in the period [0,c] and is constant for the period [c,∞] for some c>0.

Condition (3)

The breakdown point is $\partial = \frac{b_1}{P(C)}$, with the constant b_1 given by $b_0 = E_F\{\rho_0(|e|)\}$, as a constant estimator at the assumed error distribution F. When the random errors assumed to follow a standard normal distribution with a mean of, zero which $F \sim N_M(0, I_M)$.

Therefore, $(\hat{\Sigma}, \hat{\beta})$ the S-Estimator for the SUR model is a solution that reduces $|\Sigma|$ for the optimization problem:

$$min_{(\beta,\Sigma)}|\Sigma|, \ subject\ to\ \frac{1}{T}\sum_{i=1}^{T} p_1[\{(e_i)' \Sigma^{-1}(e_i)\}^{\frac{1}{2}}] = b_1 \ldots (16)$$

Where the minimization is over all

$B = bdiag(B_1, \ldots, B_m) \in (\mathbb{R}p \times M)$ $\sum \in PCD_m$, $(B, \sum)$ initial values. b_1 Is a positive constant that we can get by $b_1 = E_{F_0} P(|r|)$ and $r \sim F_0$ and thus the elliptical distribution $E_q(0, I)$. Using the weight function Tukey, we get b_1. (Srivastava & Giles, 2020)

The common choice for the ρ-function is the Tukey's biweight function, which known to have good and robust properties, according to the following formula:

$$\rho(u) = \begin{cases} \frac{u^2}{2} - \frac{u^4}{2c^2} + \frac{u^6}{6c^4}, & |u| \leqslant c \\ \frac{c^2}{6}, & |u| > c \end{cases} \ldots (17)$$

Since c is a suitable adjustment constant, (Rousseeuw & Yohai, 1984) the derivative of this function known as the Tukey's bisquare function:

$$\psi(u) = \rho'(u) = \begin{cases} u\left[1 - \left(\frac{u}{c}\right)^2\right]^2, & |u| \leqslant c \\ \frac{c^2}{6}, & |u| > c \end{cases} \ldots (18)$$

The value of the constant c determines the value of the breaking point with the case of minimization mentioned in equation (16). And the breakdown point is $\partial = \frac{b_1}{P(C)}$, (Lopuhaa, 1989) this formula is between the estimators of S for regression and estimators of S multivariate, because we have to reduce the parameter of the multivariate measurement in the case of M regression equations. In addition, we obtain the estimations of the robust SUR, which are $(\hat{\sum}, \hat{B})$ that satisfy the following equations:

$$\hat{\beta}_S = \{X'(\Sigma_M^{-1} \otimes D_s)X\}^{-1}X'(\Sigma_M^{-1} \otimes D_s)y \ldots (19)$$

$$\hat{\Sigma}_M = M(Y - \tilde{X}B)'D_s(Y - \tilde{X}B)/\sum_{i=1}^{T} v_1(d_{si}) \ldots (20)$$

where $D_s = diag\{u(d_{si})\}$; $d_{si}^2 = e_i(\hat{\beta})'\Sigma_M^{-1}e_i(\hat{\beta})$; $u(d_s) = \rho_1'(d_s)/d_s$; $v_1(d_s) = \rho_1'(d_s)d_s - p_1(d_s) + b_1$

When $\rho_1(d) = d^2$ and $p = M$, the equations for estimators (19) and (20) are reduced to the MLE estimator equations. In contrariwise to the MLE estimator, the S-Estimator gives weight to the observations in multivariate models as in equation (4). According to the residuals of e_i is multivariate and therefore it is suitable for detecting outliers not only in univariate but also in multivariate. (Bilodeau & Duchesne, 2000) In addition, the factor $u(d_{si})$ is the observed weight i included in the estimator's estimate, where the small distance of the residuals leads to a large weight, and contrary to, the large distance of the residuals leads to a small weight, which the observational weight contributes to the estimates of the

SUR model appropriately. (Peremans & Aelst, 2018) S-Estimations meet the first-order conditions for M-Estimations, so they are asymptotically normal. In addition, it is seen that the S-estimator enjoys an n1/2 rate and is asymptotically normal. The estimators $(\widetilde{\boldsymbol{\beta}},\widetilde{\boldsymbol{\Sigma}})$ are evaluated using the modified Ruppert algorithm let $p=\max(p_1,\dots,p_m)$. For the local improvement step, let

$$\Delta(\widetilde{\boldsymbol{\beta}},\widetilde{\boldsymbol{\Sigma}})=(\Delta_1(\widetilde{\boldsymbol{\beta}},\widetilde{\boldsymbol{\Sigma}}),\Delta_2(\widetilde{\boldsymbol{\beta}},\widetilde{\boldsymbol{\Sigma}})\dots(21)$$

The functions Δ_1 and Δ_2 are the values of β and Σ after once iteration of the estimation equations (19), and (20), respectively.

Algorithm S-Estimations

Step (1): Let large $\tilde{s}$ (M-estimates or GLS-estimates).

Step (2): We calculate $\widetilde{\boldsymbol{\beta}}$ and $\widetilde{\sum}$ in the following way:

i. **Choose a random sample of size P from the data of size T for each equation in the model.**

ii. **We calculate the initial estimator $\widetilde{\boldsymbol{\beta}}^{(i)}$ where $i=1,2\dots,M$ from the second step, paragraph 1, using the OLS method.**

iii. **Get** $\widetilde{\boldsymbol{\beta}}=\begin{bmatrix}\widetilde{\boldsymbol{\beta}}^{(1)}\\ \vdots\\ \widetilde{\boldsymbol{\beta}}^{(M)}\end{bmatrix}$ **and** $\widetilde{\boldsymbol{\beta}}=\begin{bmatrix}\widetilde{\boldsymbol{\beta}}^{(1)} & 0 & \dots & 0\\ 0 & \widetilde{\boldsymbol{\beta}}^{(2)} & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \widetilde{\boldsymbol{\beta}}^{(M)}\end{bmatrix}$

iv. **Calculate the covariance and covariance matrix** $\widehat{\sum}=(\mathbf{Y}-\widetilde{\boldsymbol{X}}\widetilde{\boldsymbol{\beta}})'(\mathbf{Y}-\widetilde{\mathbf{X}}\widetilde{\boldsymbol{\beta}})/\mathbf{n}$, **where Y and $\widetilde{X}$ as in equation (3).**

Step (3) we calculate $\widehat{\boldsymbol{\beta}}_{J,0}$ and $\widehat{\boldsymbol{\Sigma}}_{J,0}\tilde{}$ in the following way:

i. **Choose a random sample of size P from the data of size T for each equation in the model.**

ii. **We calculate the initial estimator $\widehat{\boldsymbol{\beta}}_{J,0}^{(i)}$ where $i=1,2\dots,M$ from the second step, paragraph 1, using the OLS method.**

iii. **Get** $\boldsymbol{\beta}_{J,0}=\begin{pmatrix}\boldsymbol{\beta}_{J,0}^{(1)}\\ \vdots\\ \boldsymbol{\beta}_{J,0}^{(M)}\end{pmatrix}$ **and** $\boldsymbol{\beta}_{J,0}=\begin{bmatrix}\boldsymbol{\beta}_{J,0}^{(1)} & 0 & \dots & 0\\ 0 & \boldsymbol{\beta}_{J,0}^{(1)} & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \boldsymbol{\beta}_{J,0}^{(M)}\end{bmatrix}$

iv. **Calculate the covariance and covariance matrix** $\boldsymbol{\Sigma}_{J,0}=(\boldsymbol{Y}-\widetilde{\boldsymbol{X}}\boldsymbol{\beta}_{J,0})'(\boldsymbol{Y}-\widetilde{\boldsymbol{X}}\boldsymbol{\beta}_{J,0})/\boldsymbol{n}$, **where Y and $\widetilde{X}$ as in equation (3).**

Step (4) let $\boldsymbol{\beta}_{J,j}=\begin{bmatrix}\widetilde{\boldsymbol{\beta}}_{J,j}^{(1)}\\ \vdots\\ \widetilde{\boldsymbol{\beta}}_{J,j}^{(M)}\end{bmatrix}$, $\boldsymbol{\Sigma}_{J,j}=\begin{pmatrix}\widetilde{\boldsymbol{\Sigma}}_{J,j}^{(1)}\\ \vdots\\ \widetilde{\boldsymbol{\Sigma}}_{J,j}^{(M)}\end{pmatrix}$ $j=1,\dots,T$

Points recommended by Bilodeau and Duchesne (2000) on a straight line continuous between $(\widetilde{\boldsymbol{\beta}},\boldsymbol{\beta}_{J,0})$ and $(\widetilde{\boldsymbol{\Sigma}},\boldsymbol{\Sigma}_{J,0})$.

Step (5) For $j=0,1,\dots,Tr$; $C_{J,j}\leftarrow|\boldsymbol{\Sigma}_{J,j}|^{-1/q}\boldsymbol{\Sigma}_{J,j}$

Step (6) For $j=0,1,\dots,Tr$ **if**

$$\frac{1}{T}\sum_{i=1}^{T}p\{(\frac{e_i' C_{J,j}^{-1}e_i)^{\frac{1}{2}}}{\tilde{s}}\}\geq b$$

Iterating until we get to:

$$\frac{1}{T}\sum_{i=1}^{T} p\left\{\left(\frac{(e_i' C_{J,j}^{-1} e_i)^{\frac{1}{2}}}{\tilde{s}}\right\} < b\right.$$

Then the following steps:

1. $\widetilde{\beta} \leftarrow \beta_{J,j}$

$\tilde{s} \leftarrow s(\widetilde{\beta}, C_{J,j})$ **with** $s(\widetilde{\beta}, C_{J,j})$ **Solve the equation** $\frac{1}{T}\sum_{i=1}^{T} \rho\left[\frac{(e_i' C_{J,j}^{-1} e_i)^{\frac{1}{2}}}{s(\widetilde{\beta}, C_{J,j})}\right] = b_1$ **… (22)**

To obtain the smallest value $s(\widetilde{\beta}, C_{J,j})$ **that satisfies equation (22) above, we use the Newton-Raphsen method with the introduction of an initial value:**

$$(s(\widetilde{\beta}, C_{J,j}))_{k+1} = (s(\widetilde{\beta}, C_{J,j}))_k - \frac{f(s(\widetilde{\beta}, C_{J,j}))_k}{f'(s(\widetilde{\beta}, C_{J,j}))_k}$$

$$f(s(\widetilde{\beta}, C_{J,j}))_k = \frac{1}{T}\sum_{i=1}^{T} \rho\left[\frac{(e_i' C_{J,j}^{-1} e_i)^{\frac{1}{2}}}{s(\widetilde{\beta}, C_{J,j})}\right] = b_1$$

$$f'(s(\widetilde{\beta}, C_{J,j}))_k = \frac{\partial f((s(\widetilde{\beta}, C_{J,j}))_k)}{\partial((s(\widetilde{\beta}, C_{J,j}))_k)}$$

2. $\widetilde{\Sigma} \leftarrow \tilde{s}^2 C_{J,j}$

3. We find the smallest integer $0 < z(\widetilde{\beta}, \widetilde{\Sigma}) < 10$ **that satisfies:**

i. $\widetilde{\beta} \leftarrow (\widehat{\beta}, \widehat{\Sigma})(1 - 2^{-z}) + \Delta_1(\widehat{\beta}, \widehat{\Sigma})2^{-z}$ **Where** $\Delta_1(\widehat{\beta}, \widehat{\Sigma})$ **Equation No. (22).**

ii. $\widetilde{\Sigma} \leftarrow (\widehat{\beta}, \widehat{\Sigma})(1 - 2^{-z}) + \Delta_2(\widehat{\beta}, \widehat{\Sigma})2^{-z}$ **Where** $\Delta_2(\widehat{\beta}, \widehat{\Sigma})$ **Equation No. (22).**

iii. $C_{J,j} \leftarrow |\Sigma_{J,j}|^{-1/M}\Sigma_{J,j}$

iv. $\frac{1}{T}\sum_{i=1}^{T} p\{\frac{(e_i' C_{J,j}^{-1} e_i)^{\frac{1}{2}}}{s(\widetilde{\beta}, C_{J,j})}\} = b$

Step (7) we repeat the steps from step four to step six n times until we get the smallest number of z, which is:

$\tilde{s} \leftarrow s(\widetilde{\beta}, C_{J,j}), \widetilde{\Sigma} \leftarrow \tilde{s}^2 C_{J,j}, \widetilde{\beta} \leftarrow (\widehat{\beta}, \widehat{\Sigma})(1 - 2^{-z}) + \Delta_1(\widehat{\beta}, \widehat{\Sigma})2^{-z}$

2.2.2 FastSUR Estimation Method

S-estimations in the SUR models presented by Bilodeau and Duchesne (2000) has good robust properties such as high breakdown point but is computationally expensive.

(Hubert et al., 2014). A robust and fast SUR method according to the ideas of FastS algorithm (Salibian-Barrera & Yohai, 2006) called FastSUR algorithm. We first mention the definition of the S-estimator (β̂, Σ̂) so that:

$$(\widehat{\beta}, \widehat{\Sigma}) = arg \min_{(\beta, C)} |C| \qquad \dots (23)$$

which satisfies the condition of the equation:

$$\frac{1}{n}\sum_{i=1}^{n} p(e_i(\beta)' C^{-1} e_i(\beta)) = b_1 \qquad \dots (24)$$

To obtain the robust estimations capable of resisting outliers, they must satisfy the conditions mentioned in (3.1), which satisfy equation (19) and (20).

We rewrite C above as $C = \sigma^2\Gamma$ where $|\Gamma| = 1$and $\sigma = |\Sigma|^{1/2M}$ to get the robust estimators $\{\widehat{\beta}, (\hat{\Sigma} = \widehat{\sigma}, \hat{\Gamma})\}$ by minimizing σ under the condition of equation No. (26), the minimization is for all (β, σ, Γ) where $\beta \in R^{pxm}$, $\Gamma \in PSD_M$ of dimension (MxM) with |Γ|= 1, σ is a positive standard number..(Hubert et al., 2014)

2.3 Comparison criteria

1. Mean squared error (MSE) = $S_{sur}^2 = \frac{Y'(s^{-1} \otimes I_T)Y - b^* X\prime(s^{-1} \otimes I_T)Y}{MT-P}$

2. Root of mean squared error (RMSE) = $\sqrt{\frac{1}{T}\sum_{t=1}^{T} e_t^2}$

With $e_t^2 = (Y_t - \widehat{Y}_t)^2$ where T is the amount of time (t) being predicted; e_t is the model residual at the t-time; $Y_t$is the observation value at the t-time; and $\widehat{Y}_t$ is the prediction value at the t-time. (Ashari et al., 2020)

3. R-squared $(R^2) = \frac{SSR}{SST} = 1 - \frac{SSE}{SST} = 1 - \frac{U^{*\prime}(s^{-1} \otimes I_T)U^*}{Y\prime(s^{-1} \otimes I_T)Y}$

4. Adjusted R-Squared $(\overline{R}^2) = 1 - \frac{MT-1}{MT-p}(1 - R^2)$ (Hassan & Dr.A.H.Kadhim, 1993)

3. Discussion of Results

The return on equity (ROE) is an indicator for measuring the profitability of banks and evaluating their performance. (Singh, 2010) We used the index (ROE) as an endogenous variable in the SUR regression model in this study. The external variables are

• Li: Liquidity Ratio. • Cr: Credit. • D: Deposits. • S: Bank Size. • I: Financial Investments.

$ROA_{it} = \beta_{i0} + \beta_{i1} Li_{i1t} + \beta_{i2} Cr_{i2t} + \beta_{i3} D_{i3t} + \beta_{i4} S_{i4t} + \beta_{i5} I_{i5t} + u_{it}$; i=1, 2 and 3 ،t=1… 19

We used the R-language program (Version (4.1.2)) in conducting the statistical analysis to achieve the objectives of the study.

Table (1) The results of estimating regression equations using the classic methods.

banks	Variables	OLS-Estimations				GLS-Estimations			
		Coefficient	Std. Error	t - value	Pr(>|t|)	Coefficient	Std. Error	t - value	Pr(>|t|)
A	Intercept	-0.9364	0.5021	-1.8649	0.0849	-0.8297	0.4070	-2.0380	0.0415*
	X11	0.0063	0.1491	0.0422	0.9670	0.0167	0.1169	0.1430	0.8861
	X12	-0.0437	0.0518	-0.8441	0.4139	-0.0366	0.0408	-0.8950	0.3707
	X13	0.3303	0.2753	1.1995	0.2517	0.3688	0.2207	1.6710	0.0947
	X14	0.1018	0.0537	1.8972	0.0802	0.0862	0.0438	1.9700	0.0488*
	X15	0.0804	0.3104	0.2590	0.7997	-0.0366	0.2529	-0.1450	0.8850

B	Intercept	0.8466	0.7164	1.1817	0.2585	0.0181	0.4343	0.0420	0.9668
	X21	-0.1321	0.0888	-1.4878	0.1607	-0.2933	0.0482	-6.0830	0.0000***
	X22	-0.0700	0.1530	-0.4575	0.6548	-0.2023	0.0849	-2.3820	0.0172**
	X23	0.0731	0.2133	0.3428	0.7372	0.2447	0.1145	2.1380	0.0325*
	X24	-0.0845	0.0808	-1.0458	0.3147	0.0251	0.0506	0.4960	0.6197
	X45	0.0479	0.0765	0.6267	0.5417	0.0054	0.0443	0.1230	0.9022
C	Intercept	-0.1871	1.5961	-0.1172	0.9085	-0.1283	0.9095	-0.1410	0.8878
	X31	-0.4069	0.2364	-1.7213	0.1089	-0.0413	0.1349	-0.3060	0.7596
	X32	-0.3832	0.5424	-0.7065	0.4924	0.3196	0.2733	1.1690	0.2423
	X33	0.2638	0.5100	0.5172	0.6137	0.6621	0.2650	2.4990	0.0125**
	X34	0.0838	0.1880	0.4457	0.6631	-0.0171	0.1102	-0.1550	0.8767
	X35	-0.5125	0.6640	-0.7719	0.4540	-0.1142	0.3324	-0.3430	0.7313

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*, **, *** : Represent the tabulated t-statistics value, which means that the parameter is statistically significant at levels (1%), (5%), and (10%) respectively.

Table (1) presents the results of estimating the model for the banks under study using traditional methods (OLS, GLS). It displays the values of the estimates for the variables, the standard errors, the t-value, and the probabilities for each variable, as some of the model variables were found to be insignificant, which indicates that the model suffers from some measurement problems. We conclude that the model suffers from the problem of outliers.

Table (2) Tests for the model

I. Heteroscedasticity test: Breusch-Pagan-Godfrey (BPG)			
GQ-Statistic	2.8936	p-value	0.03751
II. Normality test: Shapiro-Wilk			
W-Statistic	0.75804	p-value	$2.53\, e^{-08}$
III .Outliers test: Bonferroni			
Bonferroni-Statistic	$2.0283\, e^{-09}$	p-value	$3.5584\, e^{-11}$
IV. Serial Correlation test : Durbin-Watson			
DW-Statistic	1.9396	p-value	0.1358

Table (2) above shows some diagnostic tests for the remaining banks under study, as the following becomes clear:

• The used study model suffer from the problem of heteroscedasticity, where we find that the p-value of the test used Breusch-Pagan-Godfrey (BPG) test is less than 5%, and therefore the null hypothesis (H0) rejected in the presence of the problem of instability of variance.

• We find that the residuals not normally distributed, according to the Shapiro-Wilk test, where we find that the p-value of the test used is less than 5%. Thus, the null hypothesis is rejected, which indicates that the residuals are not normally distributed.

• According to Bonferroni's test, we find that the model contains outlier's problem, where we find that the p-value of the test used is less than 5%. Thus, the null hypothesis is rejected, which indicates that the model contains the problem of outliers.

• We also note that the study model used do not contain the problem of Serial Correlation, where we find that the p-value of the test used Durbin-Watson test is greater than 5%, and therefore the null hypothesis (H_0) is accepted, that there is no autocorrelation problem in the model used.

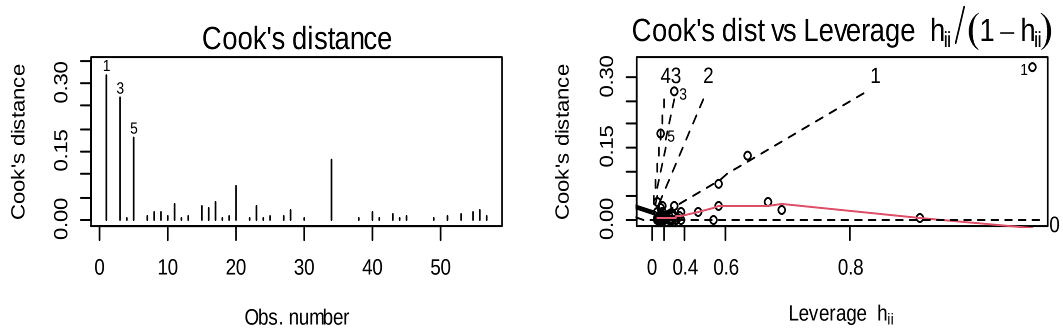

Figure (1) Cooks distance for the data of the banks under study.

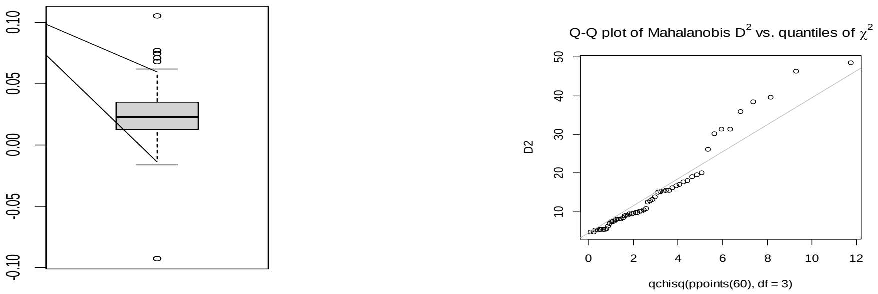

Figure (2) Mahalanobis distance for the data of the banks under study.

In Figures 1 and 2, the standard residuals are plotted to identify the outliers, and the outliers identified by Mahalanobis distance appear to be the same as observed for the leverage values. Maximum outliers can be detected by squared distances. Figure 2 shows that some observations have a very large effect on the regression line. The boxplot also indicates that there are some outliers in the standard residuals of the model. We conclude that outliers have a significant effect on the SUR model. Therefore, we will apply robust estimation methods to obtain consistent and highly efficient estimates.

Table (3) Estimation of SUR model coefficients by robust methods.

Bank	Variables	S-Estimator		Fast SUR-Estimator	
A		Coefficient	Std. Error	Coefficient	Std. Error
	Intercept	-0.1829*	0.0918	-0.1591**	0.0718
	X11	-0.0143	0.0289	-0.0172	0.0295
	X12	-0.0400***	0.0056	-0.0389***	0.0058
	X13	0.1948**	0.0539	0.1799**	0.0555
	X14	0.0027	0.0410	0.0036	0.0403
	X15	-0.1260*	0.0715	-0.1033*	0.0686
B					
	Intercept	0.2017	0.0839	0.1062*	0.0528
	X21	-0.3166**	0.0837	-0.3281***	0.0788
	X22	-0.1778**	0.0458	-0.1822***	0.0420
	X23	0.1538*	0.0672	0.1392**	0.0692
	X24	0.0358*	0.0345	0.0375*	0.0344
	X25	-0.0140	0.0409	-0.0117	0.0409
C					
	Intercept	-0.0732*	0.0519	-0.0928**	0.0392
	X31	0.0083	0.0164	-0.0083	0.0143
	X32	-0.0223	0.0506	-0.0886*	0.0509
	X33	0.4649***	0.0640	0.4129***	0.0548
	X34	-0.0170	0.0198	-0.0093	0.0183
	X35	-0.1498*	0.0870	-0.1617*	0.0869
*, **, ***: The statistical value of T-Statistic means that the parameter is statistically significant, whether at the level of 10%, 5%, or 1%, respectively.					

Table (4) Comparison criteria for SUR methods.

Table (3) presents the estimation of the model parameters using the robust regression methods (FastSUR, S). We note that most of the model variables for the robust methods are statistically significant because the p-values of their variables are less than 5%. These results indicate that the robust estimation methods have standard errors. It is smaller compared to traditional estimators, and in general, the model to both methods is considered statistically significant. Table No. (3) Estimation of SUR model coefficients by robust methods.

Measure	OLS-Estimator	GLS-Estimator	S-Estimator	Fast SUR-Estimator
MSE	0.0210	0.0201	0.0101	0.0063
RMSE	0.1449	0.1417	0.1004	0.0793
R-Squared	0.3820	0.4907	0.8493	0.9165
Adjusted R-Squared	0.1559	0.3043	0.7941	0.8859

The results presented in Table (4) show that the robust methods have the smallest values for all comparison criteria, and the R-Squared values are higher than the traditional methods. We note that the robust estimation methods significantly improved the quality, efficiency and importance of the two SUR models compared to the traditional methods. Since the FastSUR estimation has

minimum MSE and RMSE and higher values of coefficient of determination (R-Squared and R-Squared (Adjusted), we can say that FastSUR estimation is better than the S-Estimator method for the three studied banks data set.

4. Conclusions

The results show that the traditional methods are unable to describe SUR model in a highly accurate manner, and there is no predictive power for them due to the data containing outliers in the model under study. The robust methods significantly improved the quality, efficiency, and significance of the SUR model. The FastSUR-Estimations method is better than the S-Estimations method as it has lower (RMSE and MSE) and higher coefficient of determination values.

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نموذج انحدار غير المرتبط ظاهريا لقياس ربحية بعض المصارف التجارية الخاصة العراقية في ظل وجود قيم شاذة

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مستخلص البحث

نموذج الانحدار غير المرتبط ظاهرياً (SUR) هو حالة خاصة من النماذج متعددة المتغيرات، حيث تكون شروط الخطأ في هذه المعادلات مرتبطة بشكل متزامن. يعتبر مقدر الطريقة (GLS) فعالاً لأنه يأخذ في الاعتبار بنية التباين المشترك للأخطاء، ولكنه أيضًا حساس جدًا للقيم الشاذة. مقدرات SUR الحصينة يمكنها التعامل مع القيم الشاذة . نقترح طريقتين حصينتين لحساب المقدر، وهما (S-Estimations , FastSUR). وجدنا أن المقدرات الحصينة أدى إلى تحسين جودة تقديرات نموذج SUR بشكل كبير .بالإضافة إلى ذلك، أعطت النتائج طريقة FastSUR تفوقًا على طريقة S-Estimations في التعامل مع القيم الشاذة الموجودة في مجموعة البيانات، حيث تمتلك قيم أقل (MSE , RMSE) وقيم أعلى(R-Squared , Adjusted R-Square) .

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*البحث مستل من رسالة ماجستير