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About Semi-parametric Methodology for Fuzzy Quantile Regression Model Estimation: A Review

Elaf Baha Alwan

Department of Statistics
College of Administration and Economics
University of Baghdad,
Baghdad, Iraq
Leccit2@uowasit.edu.iq

Omar Abdulmohsin Ali

Department of Statistics,
College of Administration and Economics
University of Baghdad,
Baghdad, Iraq
dromar72@coadec.uobaghdad.edu.iq

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Abstract

In this paper, previous studies about Fuzzy regression had been presented. The fuzzy regression is a generalization of the traditional regression model that formulates a fuzzy environment's relationship to independent and dependent variables. All this can be introduced by non-parametric model, as well as a semi-parametric model. Moreover, results obtained from the previous studies and their conclusions were put forward in this context. So, we suggest a novel method of estimation via new weights instead of the old weights and introduce another suggestion based on artificial neural networks.

Paper Type: Review article.

Keywords: Quantile regression, Fuzziness concept, Triangular fuzzy number, artificial intelligence algorithms.

1. Introduction

Crisp data and a crisp relationship between the dependent and independent variables are the foundation of the regression modelling methodology. Nevertheless, in the case of ambiguous phenomena or if the phenomenon being researched exhibits fuzzy variance rather than random variance, this is a better method of establishing a fuzzy relationship. When the underlying regression model's distributional assumptions are not satisfied or cannot be checked (for example because of a limited sample size), a fuzzified regression approach is also a promising option. Additionally, there are several instances in practical applications where measurements of the observations cannot be made as accurate amounts because the data can often be vague, linguistically vague, noisy, qualitative, or imprecise. As a result, fuzzy modelling methods offer suitable methods for handling those diverse sorts of uncertain information.

Since fuzzy modelling techniques for regression analysis have grown significantly in recent years, it is becoming more difficult to keep track of and combine the many results into a coherent whole. In order to assist new researchers in this field and to pinpoint potentially profitable future avenues for the subject, the topic of fuzzy regression analysis has now developed to the point where consolidation is required. To provide a thorough systematic evaluation, this work aims to:

The current study aims to literature in the previously specified field of study. The researchers will outline the following points in this study:

The option of substituting neural network systems for the current regression model and evaluating the outcomes. The researchers also recommend utilizing different functions as opposed to the Kernel functions utilized for the majority of the research.

Finally, the researchers discussed substituting the weights in the non-parametric model with alternative weights.

2. Material And Methods

2.1. Quantile Regression Model (QRM)

The functional linear regression models (FLRMs), which include infinite-dimensional random curves in at least one of the input or output, have been widely used to examine the relationship between the response and predictor variables, such as Greven and Scheipl (2017).

By applying any assumptions about the conditional distribution, quantile regression (QR) measures the relationship between explanatory factors and a conditional quantile of a dependent variable. Therefore, rather than modelled the mean as is done in traditional regression, it models the quantiles. In situations where either the specifications for mean regression (Waldmann, 2018).

The functional linear regression models (FLRMs), which include infinite-dimensional random curves in at least one of the output or explanatory variables, are among many others, which were often used to study the relationship between the input and the output (Cao et al, 2018).

However, in the case of function-on-scalar regression models, when the dependent variables HAVE arbitrary curves and the n dependent are scalar variables, the QR options are used (Yang et al, 2020; Liu et al, 2020)

Zhang and Huang (2020) suggest a Bayesian approach to jointly model the three components connected by random effects for the quantile regression modelling.

In same year Arefi (2020) suggested quantiles of fuzzy data that can present a loss function between fuzzy numbers, the fuzzy response variable, and the fuzzy parameters through the utilization of empirical data. Risk management must vary on the region's climatic conditions Abbas et al (2019). And as a result, it offers a more reliable inference in the presence of outliers than mean regression In contrast to mean regression. QR does not make any assumptions about the distribution of the dependent variables and the error terms. As a result, when the error term has a non-Gaussian heavy-tailed distribution, QR can produce findings that are more effective than mean regression. Unlike mean regression, which is a parametric method, the constant

variance assumption for the dependent variables is not necessary for QR. In the presence of variation, it therefore offers a more effective inference than mean regression.

Beyaztas et al (2022) suggested a functional partial quantile regression method, often known as a functional partial least squares regression equivalent in quantile regression, the functional partial quantile regression basis functions are initially extracted using a partial quantile covariance function. The functional partial quantile regression components and final model estimation are then obtained using the retrieved basis functions.

Saulo et al (2022) suggested offering parametric quantile regression models based on the Degum and Singh-Maddala distributions, two significant asymmetric income distributions. The original distributions are re-fitting parameters in the proposed quantile models by adding a quantile parameter.

2.2 Fuzzy Modelling

An efficient and credible way to determine the effectiveness of one independent variable (or more) on a response/ dependent variable is statistical regression analysis. Of all statistical techniques, it is most frequently used and has a wide range of practical applications. However, several problems can occur, especially with a limited (very small) sample size. There is doubt about the correlation between the dependent and independent variables, the main assumptions about the distribution will be inconclusive, or when the order of events is unclear. Furthermore, information could be more inaccurate in various ways, and using unsuitable underlying models that frequently due to the complexity of application problems. Numerous academics have modified and expanded statistical regression analysis notions using fuzzy set theory principles in order to loosen these restrictions.

Recently, several studies focus on fuzzy regression modelling and analysis, particularly those that discuss probabilistic, fuzzy least-squares, or machine-learning methods. In addition, logistic, type-2, and joint fuzzy regression techniques have occurred. There are other publications giving case studies in different study fields in addition to those primarily devoted to improvements in methods. In order to organize this variety of articles, proposals, and applications, we provide a thorough systematic review and literature on fuzzy regression analysis in this study. Thus, we aim is to give a hand to those new scholars and researchers in this field, considering the important outstanding topics and suggesting potential research paths. We must reasonably group the different fields because there are an almost insurmountable amount of articles that deal with the topic of fuzzy regression. So, in order to arrange the articles and identify the key fields where regression analysis is regarded in fuzziness settings, we must first define such fields. Additionally, we must divide these fields into subfields in order to organize the many approaches in a useful way. The following findings were achieved when the key topics of fuzzy regression analysis were regarded. There are three basic fields, including:

1-Regression analysis Possibility.

2-Methods using fuzzy least squares and fuzzy least absolutes.

3- Techniques for fuzzy regression analysis using machine learning.

Since there are several articles in each of these fields, we suggest the following subarea classification:

- Interval regression analysis, goal programming techniques, and linear and non-linear programming.
- Methods using fuzzy least squares and fuzzy least absolutes.
- Neural networks, support vector machines, evolutionary algorithms, and other machine learning methods.

Additionally, we identify a few minor elds, including vigorous fuzzy regression analysis,

- Fuzziness in probability theory.
- Fuzzy regression analysis type.
- Additional methods.

In this section, hypothetical regression analysis shows "possibility fuzziness pattern." It's worth mentioning that the following idioms are important used to denote various types of a data variety of input/ or output fashions (CIFO): which means that we have both input and output data as fuzzy, (CICO): which refers to input and output as crisp, and we use (FIFO): to denote other different combinations.

2.2.1 Fuzzy Linear Regression

To estimate the value of the dependent variable correlated to the independent variable(s) as closely as feasible to the observed data is one of the most crucial goals of a regression model. Consider three types of fuzzy regression models It is stated as follows (Liu et al, 2017; İcen and Cattaneo, 2017)

FLR1: Crisp coefficients for fuzzy both information and product.

FLR2: Fuzzy coefficients and fuzzy information and product.

FLR3: Information is crisp, while products are fuzzy.

Deng and Lu (2018) provided a left-right fuzzy regression approach that can be used to analyse various types of observed values.

Based on symmetric (TFN), the researcher Zarei et al (2020) proposed a (FFLPM). Arefi (2020) introduced $F(R)$ to represent the set of all fuzzy numbers with continuous membership functions. Notably, the so-called LR-fuzzy numbers, defined by $\tilde{N} = (a; l_n, r_n)_{LR}$, $l_n, r_n > 0$, are the sort of fuzzy numbers that are most frequently employed in $F(R)$.

Chen and Nien (2020) the association between the response and explanatory variables in fuzzy settings is constructed using a fuzzy regression model. In order to increase explanatory power and account for the formulated model's and parameters' uncertainty, add a new operator known as the fuzzy product core (FPC).

A model was revealed by Hesamian and Akbari (2021). A new functional regression model predictor was developed using functional predictors, an LR-fuzzy response, and fuzzy changing coefficients. It was a curve connected to a scalar fuzzy response variable. For many experts, expressing imprecision in a hazy process as an LR-fuzzy number is an easy approach to do so. Additionally, a criteria selection model is provided here using SCAD penalty and absolute error regularization.

Hesamian and Akbari (2021) published a novel robust multiple regression models with fuzzy intercepts and non-fuzzy regression coefficients. This model used fuzzy random variables and also introduced α – value of LR-fuzzy to estimate the model's components.

Naderkhani et al (2021) used Ridge regression using asymmetric trapezoidal fuzzy data for both linear and nonlinear fuzzy models.

Akbari and Hesamian (2022) made an effort to create a unique linear regression model with auto correlated fuzzy error terms, precise predictors, and fuzzy answers. To estimate the unidentified autocorrelation criteria and fuzzy coefficients, a hybrid method using a weighted mean square error and cross-validation criterion is used.

2.2.2 Interval Fuzzy Regression Analysis

Wang et al (2015) provided two outlier identification methods based on normalized upper and lower interval regression models concerned with Tanaka's method's outlier issue. On the other hand, Černý and Hladík (2018) examined two interval regression scenarios: the first case in which crisp input dealt and interval output data, whereas the second case whose interval values of input data dealt with interval output.

2.2.3 Fuzzy Least Squares

The major issue here is to apply the least squares consideration on fuzzy regression analysis. Objective function has been minimized by reducing the predicted fuzzy values and the given fuzzy data associated with different distances among any two different fuzzy numbers. That implies that the fuzzy least squares (FLS) technique incorporates the notion of goodness of fit and that model correctness may be investigated using the residuals. Chang (2001) discussed a hybrid FLS regression based on weighted fuzzy arithmetic. This technique enables models to be fitted to a variety of data types.

Some least squares estimators' asymptomatic properties are provided by Yoon et al (2016), who were interested in multiple FLR model with triangular fuzzy numbers used to represent FIFO data, and they showed that their LS estimators were consistent and had an asymptotic normality. In their research Khammar et al., (2020) first introduced a new distance from two fuzzy numbers using the kernel function. Subsequently the team used the least squares method to estimate the fuzzy regression model's parameters.

When the response variable and model parameters are fuzzy numbers, researcher Khammar et al., (2021) used a novel general approach to fitting the fuzzy regression model. This approach introduces a new definition of the function that is objective using different loss functions in addition to the mean variation between the α -cuts of the errors.

2.2.4 Interval And Intuitionistic Fuzzy Regression Models

In their study of an intuitionistic FLS regression model, for FIFO data Chachi and Taheri (2016) offer a multiple FLS regression model and use a distance on the space of interval-valued values to estimate the parameters.

The researcher Ahmadini (2022) suggested an intuitive fuzzy logistic regression model to deal with the inaccurate and fuzzy parameters.

2.2.5 Fuzzy Least Absolutes Methods

The generalized Hausdorff-metric or distance can be useful, too. Chachi et al (2016) provided two LAD methods for multiple regression analysis using FIFO data. Li et al (2016) explored several instances with different forms of input and output data as well as regression coefficients when dealing with LAD estimation utilizing a new distance measure for triangular fuzzy integers. More robust strategies are required to handle outliers because both the LS and LP approaches are vulnerable to them. One of these strategies is the least absolute deviations (LAD) based on median instead of mean, and performs better than ordinary least squares (OLS) when the data contains outliers. So, it is a more reliable method. See, for instance, Zeng et al (2017), who applied LAD methods to construct the fuzzy least absolute linear regression model with crisp inputs, fuzzy output and fuzzy parameters, introduced a distance between triangular fuzzy numbers, proposed the least absolute fuzzy linear regression model, and use the similarity measure of triangular fuzzy numbers to evaluate the fitting of the observed and estimated values.

2.2.6 Machine Learning Techniques in Fuzzy Regression Analysis

In order to develop non-linear model architectures, Wieszczy and Grzegorzewski (2016) referred to the learning algorithms to solve problems involving pattern recognition and function estimate. As a special one, support vector machine (SVM) algorithms were referenced, while Chan et al. (2017) presented a fuzzy regression approach based on genetic programming, where the maximization fuzzy criterion will yield the evolution of coefficients.

2.2.6.1 Neural Networks

The fuzzy non-linear regression model created by He et al (2016), they used a random weight network and input/outputs that are triangular and trapezoidal fuzzy numbers, respectively. Pehlivan and Apaydin (2016) used Fuzzy radial basis function networks to estimated fuzzy regression models using (FIFO) data. ~~But~~ However—Razzaghnia (2019) suggested adding an adaptive network fuzzy inference system model to the fuzzy regression model in which outliers already exist. Based on the adaptive neural fuzzy inference system structure fuzzy least squares technique (FLSM) for consequence parameter prediction in the ANFIS method (FWLS).

The researcher Naderkhani et al. (2021) used the adaptive neuro-fuzzy inference system (ANFIS) to analyse and predict a non-parametric fuzzy regression function with non-fuzzy inputs and symmetric trapezoidal fuzzy outputs by proposing two new hybrid algorithms in which (FLS) and linear programming were used to optimize the secondary weights.

When the model does not contain statistical assumptions, the researcher Hu (2022) proposed models to predict using periods to represent uncertainty, and the proposed model was applied using neural networks on non-linear regression analysis to determine the time interval data..

A researcher, Bas (2022), used the Gustafson-Kessel clustering algorithm rather than the fuzzy clustering algorithm, and the planned fuzzy regression functions technique uses this algorithm to derive membership metrics for the supplied data set. As for the researcher, Li (2023) suggested a new method, which is the expansion of the extreme learning machine and the functional correlation of the random vector and merging it into one network.

2.2.6.2 Support Vector Machines

For problems involving pattern recognition and function estimation, support vector machines are supervised learning models with corresponding learning methods. Support vector machines are used by Hong and Hwang (2003) to study the convex optimization problem of fuzzy multiple linear and non-linear regression models and build support vector fuzzy regression machines (SVFRM). A fuzzy multiple non-linear regression model estimate for FIFO data using an LS support vector machine is also suggested by (Hong and Hwang, 2006). Asymmetric support vector machines are used by Yao and Yu (2006) to assess functional relationships in linear and non-linear fuzzy regression models. In their algorithm, Hao and Chiang (2008), assumed that the parameters in support vector regression machines are fuzzy integers and show how to build various learning machines with various kinds of nonlinear regression functions. On triangular fuzzy number space, Wu and Law (2010) offered an SVFRM that can penalize Gaussian noise. We cite Wieszczy and Grzegorzewski (2016) for a more in-depth analysis of SVFRM and an SVFRM approach with a loss function based on the Trutschnig distance.

When there is an intuitionistic fuzzy number and a twin support vector machine (TSVM) in the model, the researcher Rezvani et al. (2019) proposed an intuitionistic FTSVM (IFTSVM) model that combines the two.

Using a least-squares support vector machine and an autoregressive integrated moving average, researcher Kaytez (2020) suggests a hybrid model.

Researchers Pant and Kumar (2022), Particle swarm optimization and support vector machines are used to suggest a hesitant fuzzy sets-based hybrid time series forecasting technique.

Researcher Pattanayak et al (2023) a unique reluctant FTS forecasting model using a support vector machine (HFTSF-SVM) is suggested. To make the predictions more accurate and minimize the computational complexity in this model,

2.2.6.3 Other Machine Learning Techniques

Fuzzy regression analysis can be improved by using some of machine learning algorithms in addition to evolutionary algorithms, neural networks, and support vector machines: Ramli et al. (2015), suggested an effective real-time (switching) fuzzy regression method created employing a below-beyond algorithm. By creating fuzzy rules, Zuo et al (2016) described a fuzzy regression transfer learning approach. With a Takagi-Sugeno fuzzy regression model, knowledge can be moved from one domain to another. Zuo et al (2017) proposed techniques for granular fuzzy regression domain adaption as an extension of this work on fuzzy Takagi-Sugeno models. 3- Various minor-elds of fuzzy regression analysis.

2.2.7 Other Applications Of Fuzzy Regression Analysis

2.2.7.1 Robust Fuzzy Regression Analysis

The issue of outliers has been taken into consideration in relation to robust estimate techniques and outlier detection criteria, in addition to the LAD methodology. Some of the fuzzy regression techniques covered above focus. Theil's method for fuzzy regression analysis, which is resistant to outliers, is considered by (Choi et al, 2016). Inspired robust estimation by least trimmed squares can be applied for CIFO data, which is described by Chachi and Roozbeh (2017), and aids in the identification and disregard of outliers and other irregular data. The robust LS fuzzy regression model for CIFO data is discussed by Chachi (2018) and has a weighted objective function, which eliminates the problems with OLS estimation in the presence of outliers, on the impact of outliers. The researcher Chakravarty et al. (2020) recommended employing a robust regression fuzzy functions (FRFN) strategy against outliers since the existence of outliers in output or input variables causes predictions utilizing machine learning techniques to be distorted and may result in incorrect findings. The parameters of robust fuzzy regression model were determined using the least squares method (Khammar et al, 2020).

According to Hesamian and Akbari (2021) when the regression model includes parameters and flexible intercepts they suggest the creation of multiple regression models.

2.2.7.2 Fuzziness in Probability Theory

Ferraro (2017) calculated the prediction error by using the bootstrap method, to examine how well a linear regression model generalizes to imprecise random variables. Jiang et al. (2017) used probability density functions to take into account the unpredictability brought on by specific explanatory factors, with the help of a chaotic optimization technique to specify the parameter values.

2.2.7.3 Type Fuzzy Regression Analysis

In the domain of fuzzy set and because type-2 fuzzy sets have more degrees of freedom than type-1 fuzzy sets, they are more suited to scenarios with high levels of uncertainty. In their work, Hosseinzadeh et al (2015) presented a weighted GPA in FLR analysis using CIFO data using type-2 fuzzy sets to model output data). In order to further the strategy outlined by Darwish et al. (2016) created an affinity measure for two intervals type-2 fuzzy sets. Generalizing their prior work, Wei and Watada (2016) took into account a type-2 fuzzy regression model based on credibility theory. Contrarily, Bajestani et al, (2016) dealt with a linear and a piecewise type-2 fuzzy regression model in a possibility framework. Additionally, based on type-2 fuzzy time series notions, Bajestani et al (2017) presented a type-2 fuzzy regression model.

Bajestani et al (2018) suggested a type-2 interval fuzzy regression model for anticipating diabetic patients' retinopathy, when there is a small sample size of data available in medical research and there is ambiguity and uncertainty about the data.

Chukhrova and Johannsen (2020) proposed the generalized one-tailed hypergeometric test with fuzzy hypotheses for both test methods: significance testing (by controlling the type I error) and hypothesis testing. This was done by applying the Arnold (1996, 1998) methodology for one-tailed hypothesis testing (by controlling both error types).

2.2.8. Further Approaches

For FLR models with symmetric and asymmetric triangular fuzzy coefficients, Liu et al. (2015), the h-level fuzzy regression model for CIFO/FIFO data is discussed by Jung et al. (2015) who used the rank transform method to build the model with the aid of the resolution identity. In order to identify significant repressors.

Chen et al. (2016) provided systematic ways to optimize the h-value. The focus of Chan and Engelke (2016) is a fuzzy regression approach that simulates non-linear and non-symmetrical fuzziness by adjusting a spread based on a third-order polynomial. A fuzzy regression is introduced by Alfonso et al. (2017), a technique that utilizes so-called finite fuzziness numbers. A new possibility for the equality of two fuzzy integers was applied to the problem by Shakouri et al. (2017) An FLR model's objective function is to calculate the regression coefficients. Hose and Hanss (2019) proposed for that inferring membership functions of fuzzy parameters in parameter affine models is based on a clear-cut and consistent data-driven approach.

Additionally, the researcher (Shijina, 2020) suggested a new distance measure when the sets are multi-fuzzy.

2.2.8.1. Fuzzy Entropy Approaches

Shi and Yuan (2015) researcher introduced several novel ideas, such as the interval entropy, the interval similarity measure, the interval distance measure, and the interval inclusion measure of fuzzy sets and used The weight in this model is interval entropy.

To construct new entropies of HFSs, we need to determine the distance between an HFS and its counterpart using the Hausdorff metric. Ciavolino and Calcagni (2016) suggested an estimation method for FLR models based on generalized maximum entropy.

Regarding the study's authors, Abbas and Mohammed (2017), the result was dependent on the position and entropy of these numbers, and they employed an adaptive fuzzy weighted linear regression model with quadruple fuzzy numbers.

New entropy measures were proposed for hesitant fuzzy sets (HFSs) (Hussain and Yong, 2018). The amount of difference between an HFS and its complement is used to calculate the uncertainty for an HFS.

By offering a new, acceptable metric that accounts for the different ways that various imprecisions can affect results

The researcher Zhou et al. (2019) suggested the semi-entropy as risk indicators, as well as the lower and upper semi-entropies.

2.2.8.2. Non-parametric Fuzzy Regression Analysis

Inference for the coefficients of fuzzy regression models with CIFO data for each h-level was proposed by (Lee et al., 2015) and is based on FLS estimation and bootstrapping.

A fuzzy regression model for CIFO data was presented by Chachi et al (2016) based on the (non-parametric) Multivariate Adaptive Regression Splines (MARS) method.

In order to estimate the fuzzy parameters in FLR models, İcen and Demirhan (2016) and İcen and Cattaneo (2017) have considered several distance measures for Monte Carlo approaches. Additionally, İcen and Günay (2019) enhance the use of fuzzy expert systems for parameter estimation using Monte Carlo techniques in FLR models.

Hesamian and Akbari (2020) suggested brand-new varying coefficient model with precise predictors and fuzzy responses that can be used when the data set contains outliers. In order to estimate unknown fuzzy (nonparametric) varying coefficients, a widely used M-estimator and the locally weighted approximation idea were combined.

Hesamian and Akbari (2021) suggested model which is a fuzzy nonlinear univariate regression with non-fuzzy predictors and fuzzy responses. Regarding estimating the centre of a fuzzy smooth function, two strategies were recommended and contrasted: (1) A non-parametric approach (similar to the left and right spreads) and (2) A parametric approach using the truncated spline regression nonlinear regression model. Topuz et al (2022), also used bootstrap techniques.

2.2.8.3. Kernel Smoothing (K-S)

If the function is continuous, symmetrical, and limited it is a kernel function, often described as (K) kernels with bandwidth α by (Hidayat et al, 2019).

A kernel smoother is a statistical method for estimating a positive real function as the weighted average of local observed data, where $f: \mathbb{R}^P \rightarrow \mathbb{R}$ is the real value of the function to be estimated. The kernel sets the weight to gives closer points a higher weight. The smoothness of the approximated function is determined by a single parameter. A kind of weighted moving average is kernel smoothing Naderkhani et al (2021).

2.2.8.4. Regression Analysis Based On Fuzzy Prior Information

Arnold and Stahlecker (2010) deal with a linear regression analysis' uniformly best estimator when given fuzzy prior knowledge.

2.2.8.5. Surveys

Regarding other sources on surveys and fuzzy regression analysis, see: Azadeh et al (2008), D'Urso (2017) and Chukhrova (2019) for a comprehensive overview study on methods for exploratory multivariate analysis utilizing fuzzy information, which also includes a section on fuzzy regression analysis.

2.2.9. Semi-parametric Regression (SPR)

Regarded as a partial model that comprises of a non-parametric component and a parameter vehicle. The key strength of this model is that it incorporates both parametric and non-parametric features. However, using parametric regression estimation methods, such as the least squares method and the maximum likelihood, which asks for the search of alternatives to conventional estimation methods, will not be appropriate because the data frequently contain outliers. Semi-parametric regression has been the subject of numerous academic articles. The year is Focuses on a semi-parametric partly linear regression model for FIFO data.

The semi-parametric model was investigated by the researchers Tian and Yu (2015) in order to explore diet and nutrition improvement. The semi-parametric regression model for longitudinal data was estimated by the researchers Li et al (2017). Fuzzy smooth function, and crisp coefficients Hesamian et al (2017) present a two-phase method based on curve fitting methods and LAD for estimation of the smooth function and the fuzzy parameters. They studied the quasi-parametric model. A fuzzy semi-parametric model was presented by the researchers Hesamian and Akbari (2017) using interval-valued fuzzy product and interval-valued fuzzy coefficients, the space of interval-valued fuzzy numbers with exact information, interval-valued fuzzy numbers product s, interval-valued fuzzy coefficients, and interval-valued fuzzy smooth function.

the semi-parametric sample selection approach with the focus Hesamian and Akbari (2017) fuzzy notion as a hybrid. Using the resources offered by the theory of fuzzy sets to account for uncertainty and ambiguity is the best strategy.

Furthermore, certain scholars, including Zhang et al (2017), are interested in the subject of fuzzy ridge regression.

Akbari and Hesamian (2019) combine kernel smoothing and elastic net penalized methods for the construction of a variable selection approach within a multiple fuzzy regression model .

Riedel and Stulp (2019) used an approach to analyze models, in robotics. They considered variables that had responses and uncertain predictors. When variables are fuzzy replies, and fuzzy predictors, Hesamian and Akbari (2019) used technique with fuzzy smooth function a semi-parametric kernel-based approach to extend the quantile regression model for use in fuzzy environments.

Semi-parametric partial linear regression models with fuzzy predictors and fuzzy responses were applied by Akbari and Hesamian (2019) when outliers appeared in the data set and predictors had multiple linearity. The precise coefficients based on the ridge methodology are proposed, and a kernel-based weighted criterion is proposed.

The semi-parametric regression model was estimated utilizing robust non-parametric approaches by Bahez and Rasheed (2022), and these methods were then contrasted using the MSE comparison criterion.

2.3. Fuzzy Quantile Regression Model (FQRM)

Linear regression analysis, we attempt to construct a linear regression model between the input and output variables using several approaches, including quantile and least-squares methods. The quality of the data, the homogeneity and uncorrelated distribution of errors, among other assumptions, are all included in these approaches. However, there are some constraints and observations in the real world that keep these assumptions from being true. Thus we are unable to apply them there. Some of these problems are amenable to being fixed using fuzzy linear regression models. Researchers have recently looked during certain fuzzy methods based on the quantile. The following characteristics of the quantile function are present. The works of some researchers in this study on fuzzy regression are reviewed in the section that follows.

Hesamian and Akbari (2019) suggested a fuzzy quantile regression model based on signed-rank distance measurements with fuzzy predictors and fuzzy response. A fuzzy quantile regression model with precise predictors and triangular fuzzy answers has been utilized as a strategy. The introduction of a random variable with precise values and its empirical estimation. Then, using a hybrid algorithm, a fuzzy empirical kernel-based quantile regression approach was created to assess the unknown bandwidth and quantile level.

The modeling of a quantile regression model using the fuzzy response variable and fuzzy parameters was done using a novel method based on the loss function and fuzzy numbers (Arefi, 2020).

Some studies have concentrated on quantile-based fuzzy regression models. Chachi and Chaji (2021) suggested a method based on the some weights of sorted residuals, to estimate the parameters involved in the fuzzy quantile regression model by obtaining the precise of the prediction with its response (y). While by Khammar et al. (2021) presented a local estimation for the coefficient of dependent fuzzy variable that concerning to the regression model

The researcher Khammar (2021) proposed the quantile cross-validation approach, we choose the value of h at which the following function is minimized.

A sign distance metric for triangular fuzzy numbers was developed for this reason. A well-known sign distance, that is often applied in fuzzy settings, was compared to the suggested sign distance metric (Khammar, 2021).

When the response variables are represented by triangular fuzzy numbers and the predictors are accurate data, the researcher Hesamian and Akbari (2022) reviewed a new nonlinear quantile regression model.

- Under several quantiles, the quantile fuzzy regression model is adaptable.
- In of outlier (or irregular) data presence, the quantile fuzzy regression model is strong.
- The quantile fuzzy regression model is more adaptable to skewed output responses than Fuzzy models based on least-squares and least-absolutes techniques.

3. Discussion of Literature review

We give a critical discussion of the approaches presented in section 2.

The researchers Chachi and Taheri (2016) explained through their study that the model is characterized by the fact that there are no restrictions on fuzzy observations to get the same kind of membership functions because it is used in the first stage sets α -level. Finally, this method can be applied when the observations contain multiple types of membership functions. However, it cannot be applied in the case where the fuzzy observations are converted into crisp values.

The researchers He et al. (2016) showed by using a simple network that a better performance of a regression model can be obtained due to the effectiveness and advantages of the method used (FNRRWN) compared to the model (FNR) that uses networks BP and RBF. However, the researcher recommended many points that must be addressed in future research. One of them is when the data is more general than the fuzzy data, one resorts to using the (FNR) model based on the RWN.

During the same year, researcher Wieszczy and Grzegorzewski (2016) explained in the numerical examples that they presented that fuzzy regression methods work better when using (SVM) compared to normal fuzzy regression, but it is also clearer. This conclusion must be confirmed in the experiments of other studies. The researcher explained that the model that he reviewed is the most appropriate and better than the previous fuzzy regression models through the numerical examples that he presented. However, in the case of the missing data set, the model was not tested.

The results of the researchers Zuo et al. (2017) showed that the method described improves significantly on the performance of the current models in estimating the values of the data under study, and the researcher recommended in future studies to use addressing cross-domain adaptation problems.

İçen and Cattaneo (2017) used the Monte Carlo (MC) method to determine the best types of distance measures between two fuzzy numbers to estimate the parameters of the fuzzy linear regression model and explained that the real number was used to calculate the distance between two fuzzy numbers for the distance measures used in his study. Therefore, it is necessary to develop future studies using of a fuzzy distance scale and Applying different types of regression, such as exponential regression or non-parametric regression.

The researchers Hesamian and Akbari (2019) that one of the benefits of the model is that it meets all tenable criteria, including robustness in the space of fuzzy numbers, and performs better in situations where the data contains a skewness or when the assumption of normality about the data fails, as opposed to regression methods, the proposed model can only be used when the data are predictors and the responses are fuzzy numbers.

And the researcher, Razzaghnia (2019), proved during a study that the model increases the accuracy of prediction when the dependent variable, and that the outliers do not affect the estimated values. Through the numerical examples that were applied and according to the error criterion used across the network, the estimates of the model were less errors than other methods. However, the results cannot be generalized to a model when the independent variable is fuzzy.

Hesamian and Akbari (2020) explained the higher performance of the aforementioned strategy on the fuzzy multiple regression model. He also said that future research should concentrate on situations where predictors are crisp numbers. The method's expansion to nonlinear changing coefficient regression models may.

Chen and Nien (2020) association between the response and explanatory variables in fuzzy settings is constructed using a fuzzy regression model. In order to increase explanatory power and account for the formulated model's and parameters' uncertainty, add a new operator known as the fuzzy product core (FPC). The suggested method improves model performance by reducing the quantity of extraneous or irrelevant information resulting from ambiguous observations and determining the sign of model parameters. The relevant techniques' weakness of having fuzzy model parameters that need to be predetermined during formulation processes is strengthened by this. Even with sharp explanatory factors, the suggested method beats current models in terms of the measures of distance, mean similarity, and credibility.

The researchers Hesamian and Akbari (2020) discussed the benefits of the approach in situations where it is impossible to define a parametric functional form for the centres when using the non-parametric approach (used with right and foot spreads). All fuzzy numbers can also be calculated using the approach outlined. Finally, the method offers a quick and efficient way to handle the applied regression model. The response variables are ambiguous, future research could concentrate on adapting the suggested strategy to multivariate cases or to cases where outliers appear in the data set.

The researchers Hesamian and Akbari (2021), based on the absolute error and the SCAD loss function to evaluate the unknown components of the model. The punishment method was proposed, and then, in order to estimate the fuzzy value function, a fuzzy large number notion was proposed also, the proposed method can be applied to any LR-fuzzy response variable except it the above method cannot be applied when the predictors are also fuzzy-valued function.

When the independent variables and predictors are fuzzy trigonometric variables, the approach proposed by Hesamian and Akbari (2021) is an efficient simple method for a multivariate fuzzy regression model in the presence of outliers. Therefore, the proposed technique can only be used with unimodal and bounded support LR.-FNs.

In the same year, the researchers Naderkhani et al. (2021) developed two new hybrid algorithms when the outputs are symmetrical fuzzy, trapezoidal numbers, and non-fuzzy inputs, by using fuzzy least squares and linear programming to improve the secondary weights. By studying the non-parametric fuzzy inference system Clearer, the accuracy increases when the number of observations increases. Therefore, the presented method reduces fuzziness and has a speed of adaptation. However, it loses these advantages in the case of using another regression model.

Chachi and Chaji a proposed (2021), the model fitting approach will be very robust to the presence of outliers. It will accurately identify outlier points and counteract the detrimental effects of outliers on the estimating process. However, the method cannot be broadly applied to the situation of observations with fuzzy inputs and fuzzy outputs, nor can it be used to study fuzzy time series modelling.

The benefits of the suggested were discussed by the researcher Khammar et al (2021). It is effective when there is outlier data present. The regression model used in the study performs better when based on various kernel functions, including the Gaussian, Epanechnikov, and triweight kernel functions, in order to determine the quantile CV smoothing parameter. The suggested model is, therefore, not used when the response variable and the model's parameters are fuzzy integers unless there is a quantile loss function and kernel function.

By adjusting the estimate of fuzzy coefficient values in the presence of the autocorrelation coefficient, Akbari and Hesamian's (2022) fuzzy regression technique was successful. Investigation of non-zero relationship between error situations employed the standard test. When comparing the findings with those of certain well-known approaches, the proposed method significantly improved some common quality parameters, such as the measure of similarity, mean squared error, and coefficient of selection criteria. Nevertheless, only LR symmetric fuzzy number replies can be employed with the suggested approach.

As the researcher Topuz et al (2022) explained in their proposal bootstrap fuzzy least squares regression technique and bootstrap ordinary least squares technique are, It is used when the size of the samples is small, as it is used in the case of not knowing the type of distribution. It is preferable to use the Bootstrap reshaping technique in fuzzy least squares regression analysis and ordinary least squares regression techniques. Finally, it can be used in the case of random sample selection. In the case of large samples and other regression techniques, it cannot be used this method.

4. Final Findings (Research needs and goals)

The key purposes of the research were to review the most areas in which fuzzy statistics techniques have been used to improve the analysis of traditional regression model, to simplify the subject to give a hand to the junior scholars in this field, and to propose potential research possibilities. We have organized the diversity based on these goals. Regression analysis in fuzzy settings has been carefully examined, and a bibliography has been included in this study. It may be said that fuzzy modelling techniques for regression analysis have grown significantly and have emerged as the most significant topic of study for fuzzy statistics most often used are fuzzy least squares, fuzzy least absolutes, and machine learning-based fuzzy regression analysis, with intuitionistic fuzzy regression analysis coming in second and third one. Therefore, the majority of articles dealt with fuzziness in regression model are concerning these three main categories. Many studies also discuss robust, probabilistic, and time series analysis based fuzzy regression techniques. In addition, we review fuzziness concept that involved into regression methodology, as well as its improvements, expansions, and changes, including several more approaches, such as association of entropy concept with fuzziness, Monte-Carlo simulator, or Boot-strapping methods, among many others. Additionally, we have developed a list of the various suggested real-world applications and case studies that use fuzzy regression modelling. That may introduce a solid and wide foundation, which could be useful for scholars and academics who are interested in regression modelling and analysis under fuzzy fashion and to encourage them to expand their fields that still need to be studied In order to support the observations at certain specific levels.

Following a thorough analysis of the body of literature in the previously described field of study, the following gaps in the subject to assist new scholars in this field and to identify potential areas for future research. Based on these goals, we have organized the many fuzzy regression analysis methodologies as well as their improvements, extensions, and adjustments. We have also compiled a list of the various suggested real-world uses and case studies based on fuzzy regression modelling.

The researcher will present some ideas:

- Substituting the weight function presented by previous studies with a new weight function.
- Entropy input with semi-parametric model.
- Using alternative functions for kernel functions.
- The possibility of using neural network systems instead of the used regression model and comparing the results.

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حول المنهجية شبه المعلمية لتقدير الانموذج التجزيئي الضبابي: مراجعة

عمر عبدالمحسن علي
جامعة بغداد / كلية الادارة والاقتصاد/ قسم الاحصاء
بغداد, العراق
dromar72@coadec.uobaghdad.edu.iq

إيلاف بهاء علوان
جامعة بغداد / كلية الادارة والاقتصاد/ قسم الاحصاء
بغداد, العراق
Leccit2@uowasit.edu.iq

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مستخلص البحث

في هذه البحث، تم تقديم مراجعة لدراسات سابقة حول الانحدار الضبابي. إذ يظهر أنموذج الانحدار الضبابي كتعميم لانموذج الانحدار التقليدي الذي يصوغ علاقة بين البيئة الضبابية والمتغيرات المستقلة والمعتمدة. كل هذا يمكن تقديمه من خلال انموذج غير معلمي، وكذلك من خلال انموذج شبه معلمي. علاوة على ذلك، تم طرح النتائج التي تم الحصول عليها من الدراسات السابقة في هذا السياق. لذلك نقترح طريقة جديدة في التقدير عبر أوزان جديدة بدلاً من الأوزان القديمة، ونقدم اقتراحاً آخر يستند إلى الشبكات العصبية الاصطناعية.

نوع البحث: بحث مراجعة.

المصطلحات الرئيسية للبحث: الانحدار التجزيئي، مفهوم الضبابية، الرقم الضبابي المثلث، الأوزان، خوارزميات الذكاء الاصطناعي.