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Life Function Analysis of a Two-Parameter Exponential Distribution Using Robust Rank Regression

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Abstract:

In this research, a proposed method is presented to estimate the life function of the two-parameter exponential distribution by using the robust rank regression on the dependent variable, which depends on the robust variance-covariance matrix (the FMCD and OGK method) and the robust mean vector through which the rank regression coefficients are estimated using the method of ordinary least squares and employing it in estimating the scale and location parameters of the exponential distribution and thus analyzing the life function on its basis. A comparison was also made between the proposed method and the classical method of rank regression on the dependent variable of life data at different sample sizes (from 10 to 25) to show the efficiency of the proposed method based on the mean square error of the estimated parameters, through a program prepared by the researcher in MATLAB language dedicated to this purpose. The practical application results concluded that the proposed robust method of rank regression on the dependent variable in estimating the parameters of the exponential distribution was more efficient than the classical method.

Paper type: Research paper.

Keywords: Life function, Two-parameter exponential distribution, Parameter Estimation, Maximum Likelihood Estimation, Rank Regression, and Robust Estimation.

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1. Introduction:

Reliability involves the theoretical and practical methodologies essential for evaluating the capacity and probability of Units, ingredients, machinery, and goods, to execute their designated functions without encountering any malfunctions within specified time intervals, under defined conditions, and with a desired level of certainty. This encompasses tasks like setting reliability criteria, integrating them into designs, making predictions, performing tests, and showcasing successful performance (David, 1972; Sedeeq and Jalal, 2021).

Life Data Analysis in the reliability field involves examining and modeling observed product durations. These durations might represent how long a product functioned successfully or how long it operated before encountering failure. These periods of product life can be quantified using various metrics, including hours, distances, repetitions until failure, tension cycles, or any other suitable measurement for gauging a product's lifespan or exposure. This collective information on product lifespans is typically referred to as life data, or more precisely, product life data. our focus will be primarily on the lifetimes of non-living objects such as machinery, components, and systems, especially within the realm of reliability engineering. Nonetheless, it's important to note that these same principles can also be applied to other domains (Murali, 2016).

The exponential distribution finds widespread application in the realm of reliability engineering as a means to represent the duration until a component or system experiences failure. In these contexts, the parameter λ is referred to as the system's failure rate, and the average of the distribution, denoted as $1/\lambda$, is termed the mean time to failure. This distribution is relatively uncomplicated, which sometimes leads to its application in inappropriate scenarios. In essence, it represents a specific instance of the Weibull distribution with a parameter β equal to 1. (Douglas, 2009).

Parameter estimation involves the procedure of utilizing sample data, typically Data related to either the number of times something fails or succeeds. in the context of reliability engineering, to make estimations regarding the parameters of a chosen distribution. Numerous techniques for parameter estimation are accessible and employed in life data analysis. To be more precise, these encompass the uncomplicated approach of Probability Plotting, as well as the more advanced techniques such as Rank Regression (or Least Squares), MLE, and Statistical techniques based on Bayesian estimation (George, 2020). Rank regression is a technique that engages in replacing the data with their corresponding ranks. While linear regression and least squares are often used interchangeably, rank regression is an alternative to least squares or linear regression since it operates on ranked values. Various techniques have been developed for estimating parameters to match a lifetime distribution to a specific dataset. These parameter estimation methods encompass probability plotting, rank regression on the dependent variable (RRY), and maximum likelihood estimation (MLE). The choice of the most suitable analysis method depends on the dataset and, in certain situations, the chosen lifetime distribution (Murali, 2016; Sedeeq and Meran, 2022).

1.1. Literature review :

Several studies have discussed robust regression the most prominent study is:

Fu et al. (2012) used the application of rank regression techniques when reevaluating a dataset from a study on wheat Fusarium crown rot. Their work emphasized that rank regression models appear to be more suitable and rational for the analysis of non-normally distributed data and data that includes outliers.

Wilcox (2016) suggested a robust method for dealing with which independent variables are most important. The proposed technique is based on a particular version of explanatory power used in conjunction with a modification of the basic percentile method.

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Atkinson et al. (2017) employed the iterative forward search technique within regression analysis, incorporating observation weights. This approach provides a versatile and insightful framework for robust regression when dealing with outliers. They utilized variations of forward plots to visually highlight the presence of multiple outliers in a given data set.

Alsalem and Altaher (2019) applied the Gauss-Newton method for parameter estimation in the context of nonlinear least squares, a commonly used approach for estimating exponential model coefficients. They recognized that the presence of outliers could substantially influence parameter estimates in this context. To mitigate this, they explored robust nonlinear techniques, specifically the (M) method, as an alternative to the traditional least squares method. A comparison between the two methods was conducted, and the simulation results across various sample sizes and pollution rates consistently indicated that the robust (M) method outperformed the classical least squares method.

Nugroho et al. (2020) conducted a comparison between the robust M-estimation method and the Ordinary Least Squares (OLS) method using data with varying levels of significance, specifically 1%, 5%, and 10%. The predictor variables employed in this research included the percentage of impoverished society, population density, and certain health facilities. To assess the methods, they used R-squared (R^2). The findings consistently indicated that the robust M-estimation method is superior for handling models with outliers in the data. This conclusion was supported by the higher R^2 values obtained for M-estimation across all data sets compared to the OLS method.

In their study, Ismail and Rasheed (2021) proposed alternative robust methods that utilize Gastwirth's location estimator instead of the mean in OLS, and instead of the median in various M-estimation methods. They also employed a Monte Carlo simulation to evaluate the performance of different estimation methods based on the MSE of regression coefficients. Additionally, they repeated Huber's M-estimation method until the results converged.

Liu et al, (2021) employed the median-of-means technique to calculate the coefficients of the one-parameter exponential regression model when outliers were present. They compared this approach with the least squares method, and the results showed that the estimator using the median-of-means method (MOM) demonstrated superior efficiency.

Irshayyid and Saleh (2022) used robust techniques to treat outlier values in the one-parameter exponential regression model, aiming to estimate the model's parameters. They utilized three robust methods: Median-of-Means, Forward search, and M-estimation. Through simulation, they compared these estimation methods across various sample sizes and considered four different outlier ratios in the data (10%, 20%, 30%, 40%). To determine the most effective estimation method, they calculated the mean square error (MSE), and the simulation results consistently favored the forward search as the best method, as it consistently yielded the lowest mean error.

The problem of the research is in the analysis of lifetime data, study data may contain outliers that lead to inefficient estimation of the scaling parameter and location of the exponential distribution using classical methods of rank regression on the dependent variable.

The research aims to propose a method that is robust against outliers used in estimating the scaling and location parameter for the exponential distribution, based on two robust methods that were employed in estimating the rank regression parameters on the dependent variable to obtain the best efficient estimators for the life function analysis data for the exponential distribution.

2. Material and Methods:

2.1. Life Function Analysis:

Several distributions, such as the exponential, normal, lognormal, and Weibull distributions, are frequently referred to as "lifetime distributions" or "life distributions" because they are more suitable for representing data related to the lifespan of objects or systems. The choice of which life distribution to use for a specific dataset is typically made by analysts based on their prior experience and the results of goodness-of-fit tests. In the field of life data analysis, the outcome of the analysis always involves estimation. The actual values of parameters like the probability of failure, reliability, the average lifespan, or distribution parameters are usually unknown and will likely remain so for practical purposes. It's worth noting that once a product ceases production all its units have failed and their data has been gathered and analyzed, one could theoretically determine the true reliability of the product. However, this is an infrequent event. The primary goal of the analysis of lifetime data is to provide accurate estimates for these true values.

A statistical distribution's pdf perfectly captures its characteristics. Through the use of the PDF definition, we can illustrate the acquisition of several functions that are commonly used in the domains of life data analysis and reliability engineering. The pdf definition can be used to quickly derive the function of reliability, the function of failure rate, the meanwhile function, and the function of median life, among other functions (Shahla et al., 2022).

2.2. Two-parameters exponential distribution:

The exponential distribution is frequently utilized for ingredients or systems that display a constant rate of failure. (Raza et al. 2018). Because of its simplicity, it has been widely employed. In its most comprehensive form, the 2-parameters exponential distribution is characterized by:

$$f(t) = \lambda e^{-\lambda(t-\gamma)}$$
$$f(t) \geq 0\,,\ \lambda > 0\,,\ t \geq 0\ or\ \gamma \qquad (1)$$

Where:

λ : represents the consistent rate of failures.

γ: is the location parameter.

Furthermore, $\lambda = \frac{1}{m}$ where m signifies the average time between failures or the time until failure. If we assume that the parameter of location, γ, is equal to zero, the distribution transforms into a one-parameter exponential distribution, (Murali, 2016; Al-Nasser, 2009):

The equation for the 2-parameters exponential cumulative density function, or cdf, is given by:

$$F(t) = \mathrm{Q(t)} = 1 - e^{-\lambda(t-\gamma)} \qquad (2)$$

The reliability function of the 2-parameters exponential distribution is given by:

$$R(t) = 1 - Q(t) = 1 - \int_0^{t-\gamma} f(x)\,dx \qquad (3)$$

$$R(t) = 1 - \int_0^{t-\gamma} \lambda e^{-\lambda x}\,dx = e^{-\lambda(t-\gamma)} \qquad (4)$$

The failure rate function is given by:

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda(t-\gamma)}}{e^{-\lambda(t-\gamma)}} = \lambda \quad (5)$$

2.3. Parameter Estimation

Estimating the parameters of the exponential distribution through probability plotting closely resembles the process used for the Weibull distribution. It's crucial to keep in mind, though, that there may be some obvious variations in how the probability plotting paper looks and the parameter estimate techniques employed depending on the distribution. Because of the characteristics of the exponential cumulative distribution function (cdf), the exponential probability plot stands out for having a negative slope. This distinction results from the fact that, in the exponential probability plotting paper, dependability is represented by the y-axis, whereas unreliability is represented by the y-axis for the majority of other life distributions. When the cdf is linearized, as is required to create the exponential probability plotting paper, this disparity becomes clear. The cumulative density function is written as (George, 2020):

$$F(t) = 1 - e^{-\lambda(t-\gamma)} \quad (6)$$

By applying logarithm provided earlier, we obtain:

$$Ln[1 - F(t)] = -\lambda(t - \gamma)$$

or:

$$Ln[1 - F(t)] = \lambda\gamma - \lambda t$$

Now, let:

$$y = Ln[1 - F(t)]$$

$$a = \lambda\gamma$$

and:

$$b = -\lambda$$

This results in the linear equation of: (Omar et al. 2020):

$$y = a + bt$$

It's worth noting that when using the probability plotting paper of exponential, the y-axis scale is logarithmic, while the x-axis scale is linear. As a result, the zero value is only present on the x-axis. For a time, the value of t = 0, the reliability R=1, and the cumulative distribution function F(t)=0. Therefore, if we were to plot using F(t) on the y-axis, the point (0,0) would need to be plotted. However, due to the logarithmic nature of the y-axis, there is no place to represent this point on the exponential paper. Furthermore, the failure rate, denoted as λ, is the negative slope of the line on the probability plot. However, there is a more straightforward method for determining the value of λ from the probability plot (Murali, 2016).

2.3.1. Maximum Likelihood Estimation (MLE)

Maximum likelihood estimation operates by constructing a likelihood function based on available data and then identifying parameter values that maximize this likelihood function. Estimation of Maximum Likelihood (MLE) is an approach used to find parameter values for a model. It involves statistically estimating the parameters of a probability distribution by optimizing the likelihood function. The parameter value that yields the highest likelihood is referred to as the maximum likelihood estimate. (Ali et al 2023). Maximum likelihood estimates (MLE) are obtained using log-likelihood functions and their corresponding partial derivatives when applying the MLE method to the exponential distribution. The value γ is equal to that of the first failure time that occurred at 5 hours, thus $\gamma = 5\ hours$ (Murali, 2016; George, 2020)

2.3.2. Rank Regression (Least Squares):

Conducting a rank regression on Y involves the process of fitting a straight line to the provided data points in a way that minimizes the vertical deviation sum of the squares from the points to the line (Ali, 2023). We have already discussed regression analysis, also referred to as the least squares parameter estimation approach, in the context of parameter estimation. In that discussion, the following equations were constructed to perform rank regression on Y (RRY) were derived as follows: (Murali, 2016)

Let:

$$S = \begin{pmatrix} S_x & S_{xy} \\ S_{yx} & S_y \end{pmatrix}$$

That is, the sample variances on the main diagonal of the matrix S, and sample covariances on the off-diagonal of the matrix S, therefore then the correlation is:

$$\hat{\rho} = \frac{S_{xy}}{\sqrt{S_x \times S_y}} \qquad (7)$$

Regression coefficients ($\hat{a}$ and $\hat{b}$) are computed as (Kareem et al 2019)

$$\hat{a} = \bar{y} - \hat{b}\bar{x} \qquad (8)$$

$$\hat{b} = \hat{\rho}\sqrt{\frac{S_y}{S_x}} \qquad (9)$$

In our case, the equations representing y and x are as follows:

$$\boldsymbol{y} = Ln[1 - F(t)] \ \ \& \ \ x = t$$

The $F(t_i)$ is estimated in the median ranks table (Appendix). $\hat{\lambda}$ and $\hat{\gamma}$ are obtained:

$$\hat{\lambda} = -\hat{b} \qquad (10)$$

$$\hat{\gamma} = \frac{\hat{a}}{\hat{\lambda}} \qquad (11)$$

2.3.3. Robust Rank Regression Estimation (RRRE) (proposed method):

Robust Rank Regression Estimation (RRRE) is proposed to use the first and second robust methods in estimating the rank regression coefficients, through which the scale and location parameters of the exponential distribution will be estimated. The first robust method depending on the algorithm Rousseuw and Leroy, the FMCD (Fast Minimum Covariance Determinant) method, this method takes i observations out of n (where n/2 < i <= n) which includes the least determinant of the variance-covariance matrix. To obtain consistency at the multivariate normal distribution (MND) and to account for bias at small samples, the estimate is thus the variance-covariance matrix of the i points specified above multiplied by a consistency value and by a correction value. On this basis, we obtain the estimates of the independent, and dependent variable mean (RMx and RMy), and the variance-covariance matrix (RS) that is robust to outliers is as the following (Ali, 2017):

$$R\,S = \begin{pmatrix} RS_x & RS_{xy} \\ RS_{yx} & RS_y \end{pmatrix} \qquad (12)$$

That is, the sample robust variances on the main diagonal of the matrix RS, and sample robust covariances on the off-diagonal of the matrix RS, then the robust correlation is (Ali and Saleh, 2022):

$$r\hat{\rho} = \frac{RS_{xy}}{\sqrt{RS_x \times RS_y}} \qquad (13)$$

Regression coefficients ($r\hat{a}$ and $r\hat{b}$) are computed as (Ali and Saleh, 2021)

$$r\hat{a} = RM_y - r\hat{b}RM_x \qquad (14)$$

$$r\hat{b} = r\hat{\rho}\sqrt{\frac{RS_y}{RS_x}} \qquad (15)$$

In our case, the equations for y and x are:

$$y = Ln[1 - F(t)] \qquad \& \qquad x = t \qquad (16)$$

The F(t) is estimated from the median ranks (Appendix). Once ($r\hat{a}$ & $r\hat{b}$) are obtained, then $\hat{\lambda}$ and $\hat{\gamma}$ for two parameters exponential distribution can easily be determined from the following equations.

$$\hat{\lambda} = -r\hat{b} \qquad (17)$$

$$\hat{\gamma} = r\frac{\hat{a}}{\hat{\lambda}} \qquad (18)$$

The second robust method depends on the algorithm Maronna and Zamar by the Orthogonalized Gnanadesikan-Kettenring (OGK) estimate. Starting with the Gnanadesikan and Kettering (GK) estimator, a pairwise robust scatter matrix that may not be a positive definite matrix, this estimate is a positive definite matrix estimate of scatter. The estimate applies an orthogonalization iteration, a type of principal component analysis, on the pairwise scatter matrix, replacing its potentially negative eigenvalues with robust variances. For better results, this process can be repeated; convergence is often attained after two or three repetitions (Ali and Qadir, 2022). On this basis, we obtain the estimates of the independent, and dependent variable mean (RM_x and RM_y), and the variance-covariance matrix (RS) that is robust to outliers as in the formula (12). By repeating formulas (13-18), we obtain the scale and location parameters of the exponential distribution depending on the second robust method.

2.4. Application Aspect:

For (10, 11, …, 25) units were being reliability tested and the resulting life test data can be found in Table 1. (Murali, 2016). Under the assumption that the data follows a two-parameter exponential distribution, you can estimate the parameters, including the scale parameter estimation error and location (Error1 & 2), mean square error (MSE), and determine the mean time to failure (MTTF), and correlation coefficient, using the classical and proposed methods for rank regression on Y (RRY) and robust (FMCD and OGK) rank regression on Y (RRRY). The median rank values ($F(t_i)$) can be found in rank tables (Appendix). The results are summarized in Tables .2 and 3.

Table 1: Life Test Data

Data point index	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Time-to-failure	5	10	15	20	25	30	35	40	50	60	70	80	90
Data point index	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	
Time-to-failure	100	110	120	130	140	150	160	170	180	190	200	210	

Table 2: Analysis of Life Test Data (N = 10, 11, …, 17)

Method	Lambda	Gamma	Error1	Error2	MSE	MTTF	Correlation
When N = 10, Lambda = 0.0417 & Gamma = 5							
RRY	0.0460	8.8009	0.0043	3.8009	7.2233	30.5440	-0.9758
Robust1	0.0317	4.5997	0.0100	**0.4003**	**0.0802**	36.1648	**-0.9902**
Robust2	0.0383	7.4522	**0.0034**	2.4522	3.0066	33.5956	-0.9859
When N = 11, Lambda = 0.0361& Gamma = 5							
RRY	0.0395	9.0551	0.0034	4.0551	8.2220	34.4018	-0.9774
Robust1	0.0298	5.1865	0.0063	**0.1865**	**0.0174**	38.7892	-0.9884
Robust2	0.0330	7.2660	**0.0031**	2.2660	2.5673	37.5858	**-0.9940**
When N = 12, Lambda = 0.0316 & Gamma = 5							
RRY	0.0343	9.3353	**0.0027**	4.3353	9.3976	38.4679	-0.9754
Robust1	0.0264	5.1317	0.0052	**0.1317**	**0.0087**	43.0047	-0.9945
Robust2	0.0282	6.7677	0.0034	1.7677	1.5624	42.1972	**-0.9974**
When N = 13, Lambda = 0.0280 & Gamma = 5							
RRY	0.0303	9.6907	**0.0023**	4.6907	11.0011	42.6874	-0.9719
Robust1	0.0240	5.3876	0.0040	**0.3876**	**0.0751**	47.0806	-0.9973
Robust2	0.0245	6.3175	0.0034	1.3175	0.8679	47.0763	**-0.9981**
When N = 14, Lambda = 0.0250 & Gamma = 5							
RRY	0.0271	10.1353	**0.0021**	5.1353	13.1858	47.0247	-0.9679
Robust1	0.0205	4.3909	0.0045	0.6091	0.1855	53.2745	-0.9985
Robust2	0.0208	5.0562	0.0042	**0.0562**	**0.0016**	53.1745	**-0.9988**
When N = 15, Lambda = 0.0226 & Gamma = 5							
RRY	0.0245	10.6700	**0.0020**	5.6700	16.0746	51.4555	-0.9637
Robust1	0.0179	3.6226	0.0047	1.3774	0.94870	59.4692	**-0.9987**
Robust2	0.0183	4.1017	0.0042	**0.8983**	**0.40350**	58.6619	-0.9986
When N = 16, Lambda = 0.0205 & Gamma = 5							
RRY	0.0224	11.2928	**0.0019**	6.2928	19.7997	55.9600	-0.9596
Robust1	0.0161	3.2203	0.0044	1.7797	1.5837	65.2637	**-0.9989**
Robust2	0.0166	3.9482	0.0039	**1.0518**	**0.5531**	64.0969	-0.9981
When N = 17, Lambda = 0.0188 & Gamma = 5							
RRY	0.0206	11.9951	**0.0018**	6.9951	24.4656	60.5244	-0.9557
Robust1	0.0144	2.7223	0.0044	2.2777	2.5939	72.2093	**-0.9995**
Robust2	0.0147	2.9595	0.0041	**2.0405**	**2.0819**	71.0278	-0.9987

Table 3: Analysis of Life Test Data (N = 18, 19, …, 25)

Method	Lambda	Gamma	Error1	Error2	MSE	MTTF	Correlation
When N = 18, Lambda = 0.0173 & Gamma = 5							
RRY	0.0191	12.7700	**0.0018**	7.7700	30.1864	65.1404	-0.9520
Robust1	0.0132	2.4379	0.0041	2.5621	3.2822	78.2474	**-0.9994**
Robust2	0.0135	2.8482	0.0038	**2.1518**	**2.3151**	76.7341	-0.9982
When N = 19, Lambda = 0.0160 & Gamma = 5							
RRY	0.0178	13.6119	**0.0018**	8.6119	37.0826	69.7967	-0.9485
Robust1	0.0120	1.4863	0.0041	3.5137	6.1732	84.9604	**-0.9986**
Robust2	0.0126	2.8752	0.0035	**2.1248**	**2.2574**	82.3782	-0.9975
When N = 20, Lambda = 0.0149 & Gamma = 5							
RRY	0.0167	14.5124	**0.0017**	9.5124	45.2430	74.4886	-0.9453
Robust1	0.0111	1.2150	0.0038	3.7850	7.1633	91.2737	**-0.9985**
Robust2	0.0118	3.0273	0.0032	**1.9727**	**1.9457**	87.9638	-0.9966
When N = 21, Lambda = 0.0140 & Gamma = 5							
RRY	0.0157	15.4663	**0.0017**	10.4663	54.7721	79.2116	-0.9424
Robust1	0.0104	1.0936	0.0036	3.9064	7.6299	97.4414	**-0.9984**
Robust2	0.0107	1.9870	0.0033	**3.0130**	**4.5392**	95.7236	-0.9975
When N = 22, Lambda = 0.0131 & Gamma = 5							
RRY	0.0148	16.4686	**0.0017**	11.4686	65.7648	83.9605	-0.9396
Robust1	0.0096	0.3938	0.0035	4.6062	10.6084	104.9833	**-0.9984**
Robust2	0.0101	2.1824	0.0030	**2.8176**	**3.9693**	101.4579	-0.9968
When N = 23, Lambda = 0.0123 & Gamma = 5							
RRY	0.0140	17.5141	**0.0017**	12.5141	78.3012	88.7318	-0.9370
Robust1	0.0090	0.3459	0.0033	4.6541	10.8304	111.2470	**-0.9982**
Robust2	0.0096	2.4806	0.0028	**2.5194**	**3.1738**	107.1225	-0.9959
When N = 24, Lambda = & Gamma = 5							
RRY	0.0133	18.5995	**0.0017**	13.5995	92.4732	93.5220	-0.9346
Robust1	0.0084	-0.3193	0.0033	5.3193	14.1476	119.0204	**-0.9982**
Robust2	0.0091	2.8707	0.0025	**2.1293**	**2.2670**	112.7272	-0.9949
When N = 25, Lambda = 0.0110 & Gamma = 5							
RRY	0.0127	19.7192	**0.0017**	14.7192	108.327	98.32230	-0.9325
Robust1	0.0080	-0.2586	0.0031	5.2586	13.8266	125.2567	**-0.9980**
Robust2	0.0087	3.3578	0.0023	**1.6422**	**1.34850**	118.2472	-0.9939

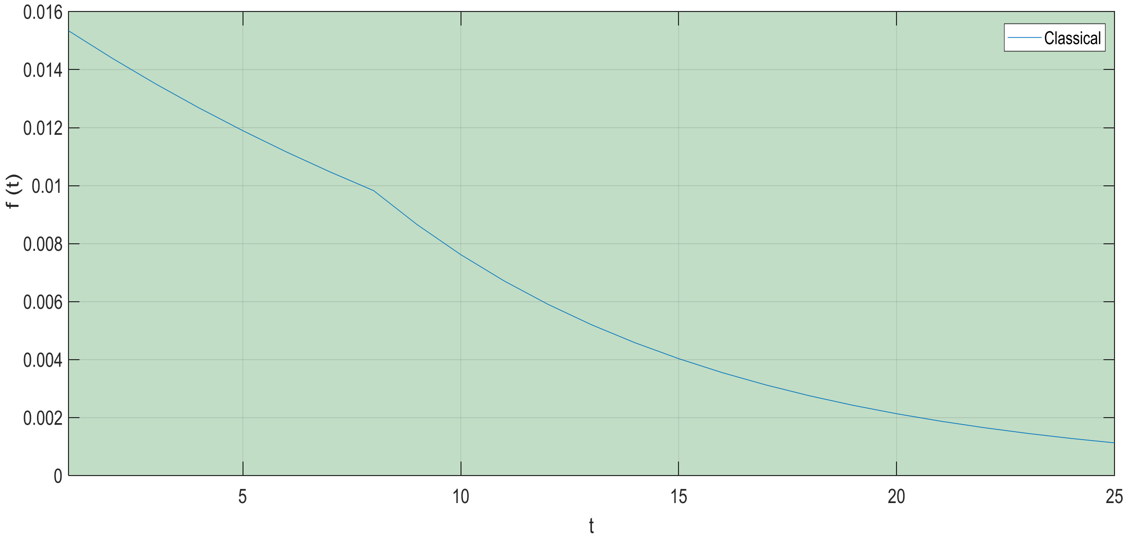

Figure 1: The two-parameter exponential probability density function (Classical)

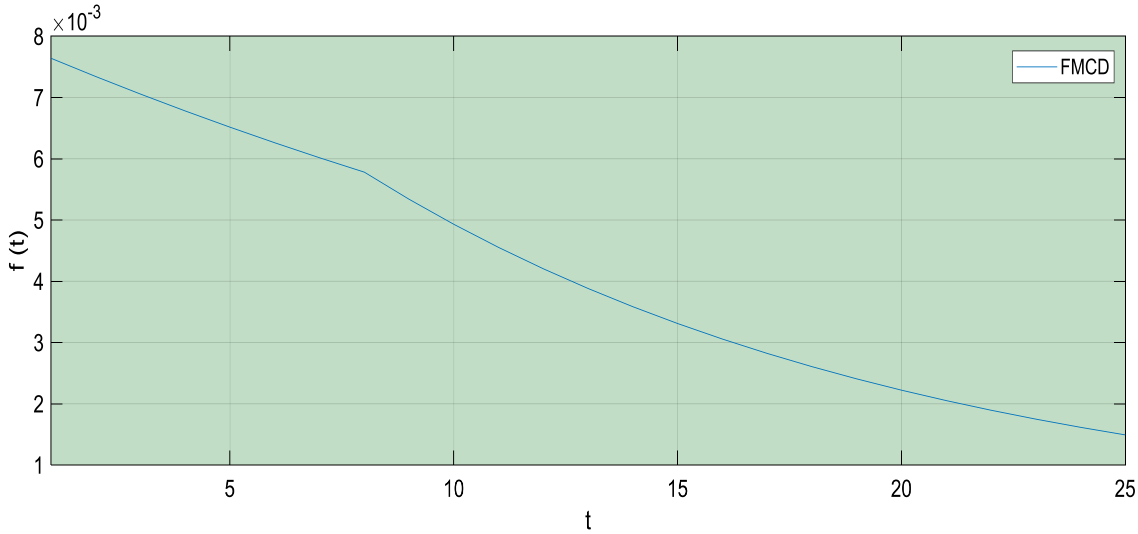

Figure 2: The two-parameter exponential probability density function (FMCD)

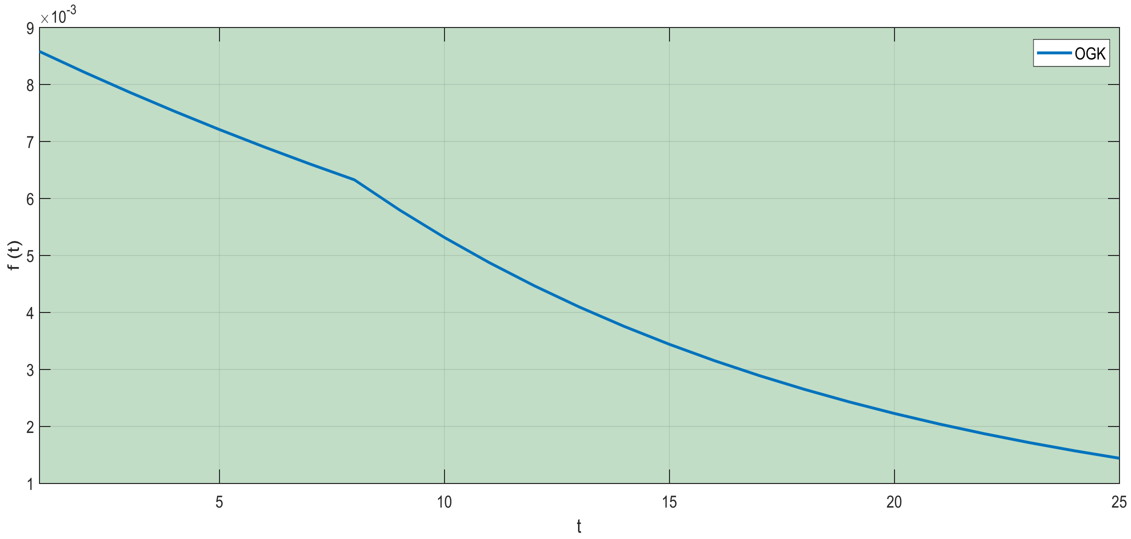

Figure 3: The two-parameter exponential probability density function (OGK)

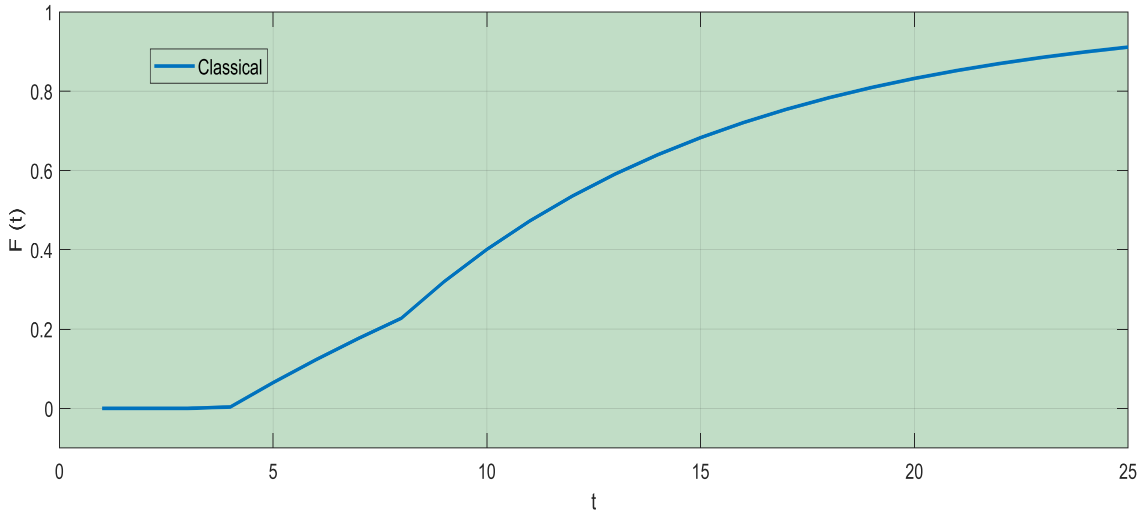

Figure 4: The two-parameter exponential cumulative density function (Classical)

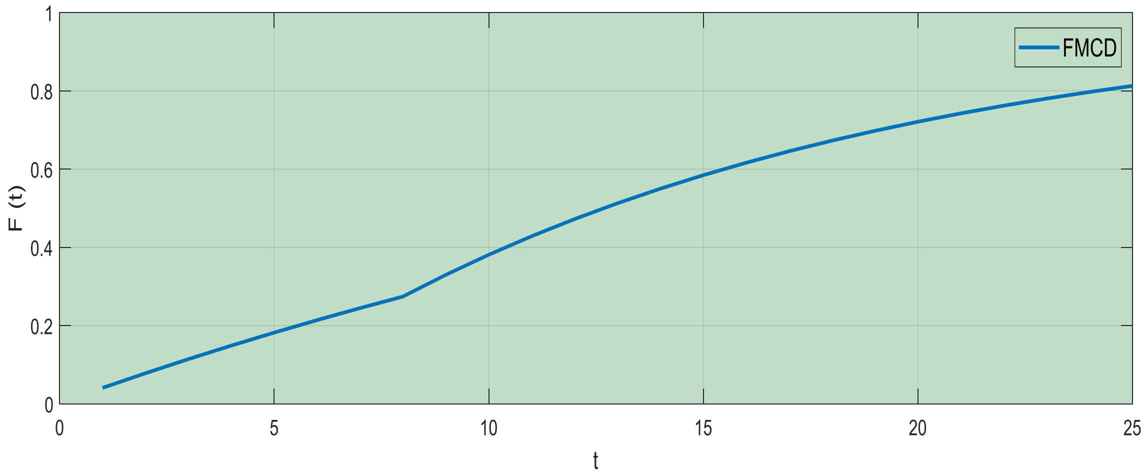

Figure 5: The two-parameter exponential cumulative density function (FMCD)

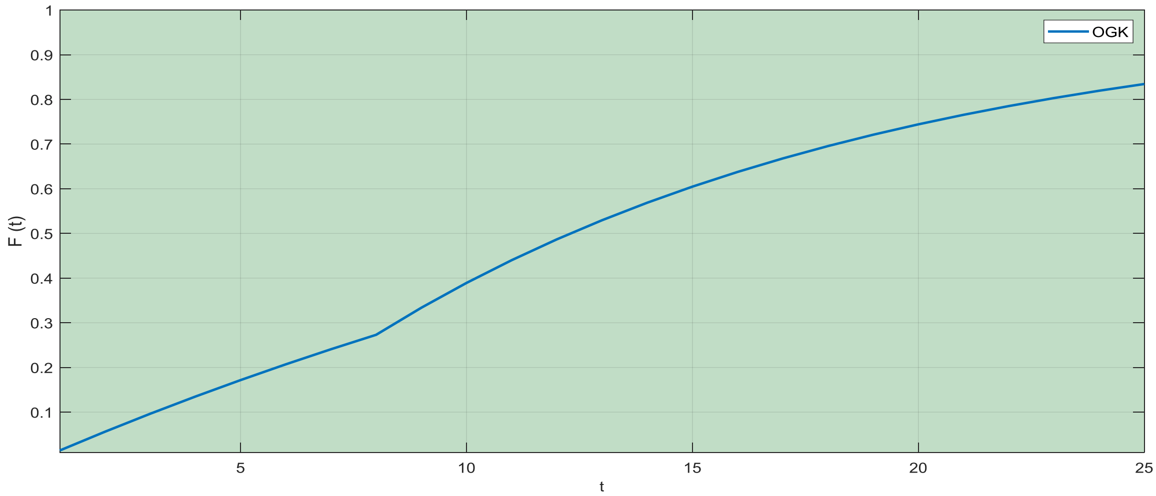

Figure 6: The two-parameter exponential cumulative density function (OGK)

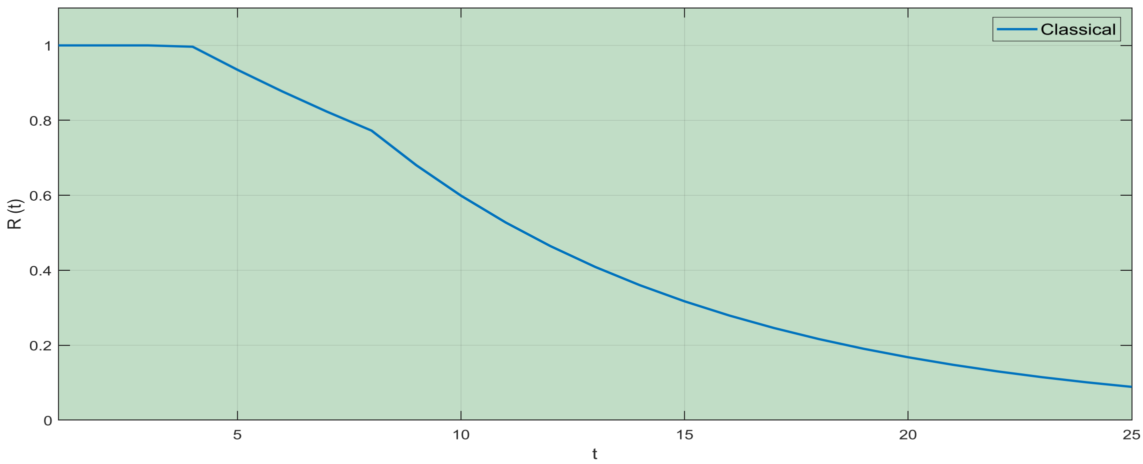

Figure 7: The two-parameter exponential reliability function (Classical)

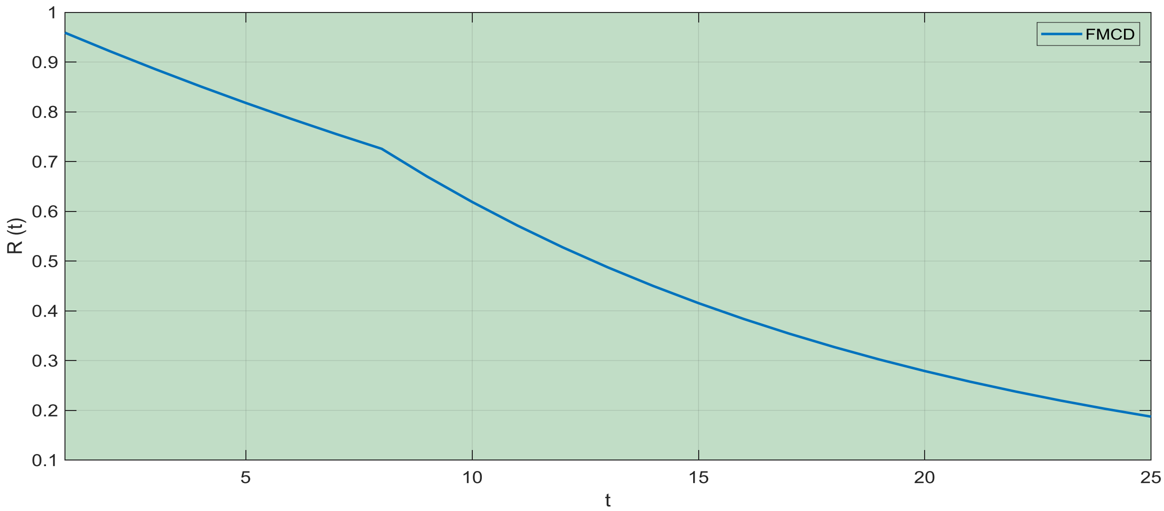

Figure 8: The two-parameter exponential reliability function (FMCD)

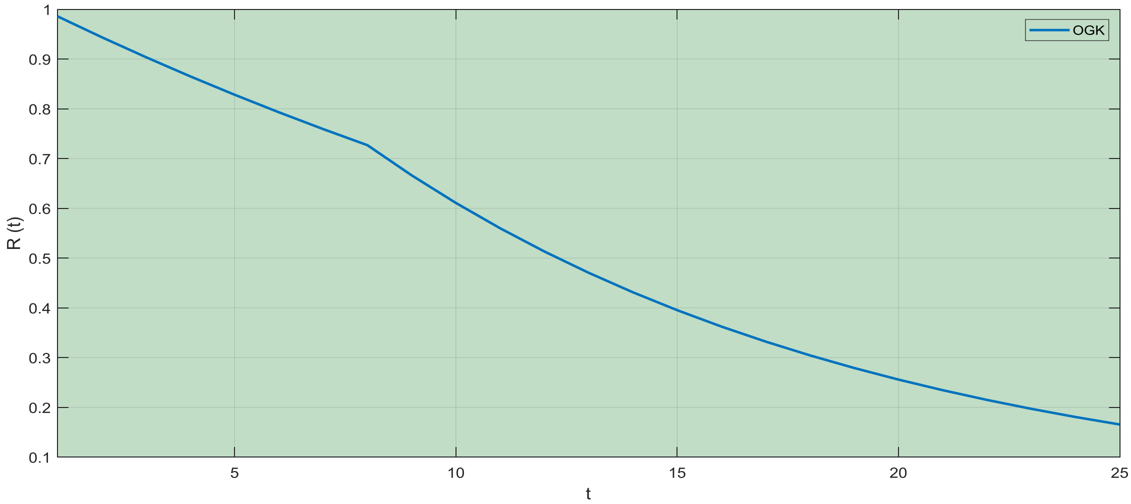

Figure 9: The two-parameter exponential reliability function (OGK)

3. Discussion of Results :

3.1. The results of Tables 3 and 4 show the following:

1. The robust methods (RRRY) outperformed the classical method, depending on MSE.

2. The primary resilient technique (FMCD) excels in comparison to the secondary robust method (OGK) when evaluating sample sizes of 10, 11, 12, and 13. However, the second robust method was better at the rest of the larger sample sizes, depending on MSE.

3. The second suggested robust technique demonstrated superior performance in estimating the scale parameter for sample sizes 10 and 11. Conversely, the classical method (RRY) outperformed the remaining larger sample sizes, contingent upon the error in scale parameter estimation (Error1).

4. The suggested robust method outperformed the classical method in estimating the location parameter, as determined by Mean Squared Error (MSE).

5. The first proposed robust method was better in estimating the location parameter at sample sizes (10, 11, 12, and 13), while the second proposed method was better at the rest of the larger sample sizes, depending on location parameter estimation error (Error2).

6. The robust methods have a stronger correlation coefficient than the classical method and the first proposed robust method was better.

7. The mean time-to-failure of the robust methods was greater than the classical method while the mean time-to-failure of the first robust method was greater than the second for all samples.

8. Figures (1-9) show the probability, cumulative, and reliability functions of the classical and proposed methods, respectively. When N=25.

4. Conclusions

1. The proposed robust method was better than the classical method.

2. The first robust method (FMCD) is better than the second robust method (OGK) at sample sizes (10, 11, 12, and 13), while the second robust method was better at the rest of the larger sample sizes.

3. The second proposed robust method was better in estimating the scale parameter at sample size (10 and 11), while the classical method was better at the rest of the larger sample sizes.

4. The proposed robust method was better at estimating the location parameter than the classical method.

5. The mean time-to-failure of the robust methods exceeded that of the classical method while the average time until failure of the first robust method was greater than the second for all samples.

Authors Declaration:

Conflicts of Interest: None

-We Hereby Confirm That All The Figures and Tables In The Manuscript Are Mine and Ours. Besides, The Figures and Images, Which are Not Mine, Have Been Permitted Republication and Attached to The Manuscript.

- Ethical Clearance: The Research Was Approved By The Local Ethical Committee in The University.

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Appendix

Table a: MEDIAN RANK TABLES (50%), N = 10, 11, …, 17

O/N	10	11	12	13	14	15	16	17
1	0.06697	0.06107	0.05613	0.0519	0.04830	0.04520	0.04240	0.04000
2	0.16226	0.14796	0.13598	0.12579	0.11702	0.10940	0.10270	0.09680
3	0.25857	0.23579	0.21669	0.20045	0.18647	0.17432	0.16365	0.15422
4	0.35510	0.32380	0.29758	0.27528	0.25608	0.23939	0.22474	0.21179
5	0.45169	0.41189	0.37853	0.35016	0.32575	0.30452	0.28589	0.26940
6	0.54831	0.50000	0.45951	0.42508	0.39544	0.36967	0.34705	0.32704
7	0.64490	0.58811	0.54049	0.50000	0.46515	0.43483	0.40823	0.38469
8	0.74143	0.67620	0.62147	0.57492	0.53485	0.50000	0.46941	0.44234
9	0.83774	0.76421	0.70242	0.64984	0.60456	0.56517	0.53059	0.50000
10	0.93303	0.85204	0.78331	0.72472	0.67425	0.63033	0.59177	0.55766
11		0.93893	0.86402	0.79955	0.74392	0.69548	0.65295	0.61531
12			0.94387	0.87421	0.81353	0.76061	0.71411	0.67296
13				0.94808	0.88298	0.82568	0.77526	0.73060
14					0.95170	0.89060	0.83635	0.78822
15						0.95484	0.89730	0.84578
16							0.95760	0.90322
17								0.96005

Table b: MEDIAN RANK TABLES (50%), N = 18, 19, …, 25

O/N	18	19	20	21	22	23	24	25
1	0.03780	0.03580	0.03410	0.03250	0.03100	0.02970	0.02850	0.02730
2	0.09150	0.08680	0.08250	0.07860	0.07510	0.07190	0.06900	0.06620
3	0.14581	0.13827	0.13147	0.12531	0.11970	0.11458	0.10987	0.10553
4	0.20024	0.18989	0.18055	0.17209	0.16439	0.15734	0.15088	0.14492
5	0.25471	0.24154	0.22967	0.21891	0.20911	0.20015	0.19192	0.18435
6	0.30921	0.29322	0.27880	0.26574	0.25384	0.24297	0.23299	0.22379
7	0.36371	0.34491	0.32795	0.31258	0.29859	0.28580	0.27406	0.26324
8	0.41823	0.39660	0.37711	0.35943	0.34335	0.32863	0.31513	0.30270
9	0.47274	0.44830	0.42626	0.40629	0.38810	0.37147	0.35621	0.34215
10	0.52726	0.50000	0.47542	0.45314	0.43286	0.41431	0.39729	0.38161
11	0.58177	0.55170	0.52458	0.50000	0.47762	0.45716	0.43837	0.42108
12	0.63629	0.60340	0.57374	0.54686	0.52238	0.50000	0.47946	0.46054
13	0.69079	0.65509	0.62289	0.59371	0.56714	0.54284	0.52054	0.50000
14	0.74529	0.70678	0.67205	0.64057	0.61190	0.58569	0.56163	0.53946
15	0.79976	0.75846	0.72120	0.68742	0.65665	0.62853	0.60271	0.57892
16	0.85419	0.81011	0.77033	0.73426	0.70141	0.67137	0.64379	0.61839
17	0.90849	0.86173	0.81945	0.78109	0.74616	0.71420	0.68487	0.65875
18	0.96222	0.91322	0.86853	0.82791	0.79089	0.75703	0.72594	0.69730
19		0.96418	0.91749	0.87469	0.83561	0.79985	0.76701	0.73676
20			0.96594	0.92136	0.88030	0.84266	0.80808	0.77621
21				0.96753	0.92488	0.88542	0.84912	0.81565
22					0.96898	0.92809	0.89013	0.85508
23						0.97031	0.93105	0.89447
24							0.97153	0.93377
25								0.97265

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تحليل دالة الحياة للتوزيع الأسي ذو المعلمتين باستخدام انحدار الرتبة الحصين

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مستخلص البحث:
تم في هذا البحث تقديم طريقة مقترحة في تقدير دالة الحياة للتوزيع الاسي ذو المعلمتين باستخدام انحدار الرتبة الحصين على المتغير التابع، الذي يعتمد على مصفوفة التباين والتباين المشترك الحصينة (طريقة FMCD وOGK) ومتجه المتوسطات الحصينة التي يتم من خلالها تقدير معاملات انحدار الرتبة بأسلوب طريقة المربعات الصغرى الاعتيادية وتوظيفها في تقدير معلمة القياس والتموضع للتوزيع الأسي وبالتالي تحليل دالة الحياة على أساسها. تم أيضاً عمل مقارنة بين الطريقة المقترحة الحصينة والطريقة التقليدية لانحدار الرتبة على المتغير التابع لبيانات الحياة عند أحجام عينات مختلفة (من 10 الى 25) مشاهدة لتبيان كفاءة الطريقة المقترحة اعتمادا على متوسط الخطأ التربيعي للمعلمات المقدرة وذلك من خلال برنامج أعده الباحث بلغة ماتلاب مخصص لهذا الغرض. توصلت نتائج التطبيق العملي الى أن الطريقة المقترحة الحصينة لانحدار الرتبة على المتغير التابع في تقدير معلمات التوزيع الاسي كانت ذات كفاءة أعلى من الطريقة التقليدية.

نوع البحث: ورقة بحثية

المصطلحات الرئيسة للبحث: دالة الحياة؛ التوزيع الاسي ذو المعلمتين، تقدير المعلمات، تقدير الامكان الأعظم، انحدار الرتبة، التقدير الحصين.