Journal of Economics and Administrative Sciences
2024; 30(144), pp. 537-548

P-ISSN 2518-5764
E-ISSN 2227-703X

Journal of Economics and Administrative Sciences (JEAS)

Available online at http://jeasiq.uobaghdad.edu.iq
DOI: https://doi.org/10.33095/xg2pbe86

Right Truncated Of Mixed Komal -Weibull Distribution With Properties

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Received:31/8/2024 Accepted:6/10/2024 Published Online First: 1 /12/ 2024

Abstract:
Purpose: Truncated distributions occur in many practical situations and predict real phenomena.
Theoretical framework: this paper proposed a right-truncated mixed Komal-Weibull distribution on [0,1] with three parameters, and derived some of its properties.
Design/methodology/approach: To show the ability and behavior of this distribution, some mathematical properties are given, such as the likelihood distribution function, the cumulative distribution function, the reliability function, the hazard function, the properties of the k^{th} moments, the variance, skewness, and kurtosis coefficients, the moment generating function and the distribution of order statistics. In addition, the maximum likelihood estimate is derived.
Findings: A new distribution with the three parameters.
Research, Practical & Social Implications: The new distribution could be used as a model in studying reliability stress-strength model and survival analysis. It enhances the ability to model and analyze truncated data accurately.
Originality/value: The right-truncated mixed Komal-Weibull distribution can be used in various fields such as agriculture, medicine, engineering, and physics.
Keywords: truncated distribution, mixed distribution, Weibull distribution, Komal distribution, maximum likelihood estimator.
JEL Classification: C10, C13, C15
Authors' individual contribution: the Methodology and Writing —Sairan Hamza Raheem.; Review & Editing — Supervisions Bayda Atiya Kalaf and Erum Rehman

1. Introduction:

In recent years, many new and more flexible probability distributions have been developed using different techniques to represent a data set(Abbas et al., 2023)and(Khan et al., 2023). So theoretical and applied statisticians have worked extensively on mixture and truncated distributions as an important area of probability theory. Truncated distributions are more suitable for modeling lifetime data due to their defined boundaries, which can serve as either upper or lower limits, or both, depending on the specific characteristics of the data, in other words, truncation of a distribution involves limiting the domain of the associated random variable according to specific truncation points, resulting in a modification of the distribution's shape. This phenomenon also occurs when events within or outside a defined range, or those falling below or above a certain threshold, cannot be observed or recorded. (M. J. Mohammed & Hussein, 2019). In 1934, (FISHER, 1934a) proposed the combination of multiple distributions to improve the flexibility of the standard distributions. Mixing distributions is viewed as a technique for addressing the limitations present in univariate distributions The relevance of mixture distributions is highlighted by the ongoing difficulties in addressing significant problems where empirical data do not fit with standard probability models. For instance, the Weibull distribution is often utilized to model datasets that display a monotonic hazard rate function. Nevertheless, it may not always serve as the preferred model due to its capacity to exhibit both negative and positive skewness in its density shapes. Moreover, the Weibull distribution is inadequate for representing phenomena with non-monotonic failure rates, such as those illustrated by a bathtub curve (Aryal & Tsokos, 2011),(Hamed, 2020).This situation leads the authors to propose the amalgamation of the Weibull distribution with various other distributions, thereby facilitating the development of more flexible and innovative distribution models. In the last few years, researchers have focused on investigating different mixture distributions.(M. J. Mohammed & Mohammed, 2021) estimated the new inverse exponential Rayleigh distribution parameters.(Hussein et al., 2023) introduced a new distribution that combines characteristics of exponential and Rayleigh distributions. (Areiby Shamran et al., 2023)compared the Modified Weighted Pareto distribution with other distributions. On the other hand, many other different papers studied to get a life distribution platform that fits mixture Weibull as an important distribution with many applications in lifetime analysis. (Almazah & Ismail, 2021) selected the efficient parameter estimation method for two Weibull distributions. While (Daghestani et al., 2021) introduced one-parameter Lindley and Weibull distributions. (Al-Noor et al., 2021) investigated a new distribution with four parameters called Marshall Olkin Marshall Olkin Weibull. (Kumar et al., 2021) proposed a new distribution, based a new distribution, based on the Weibull Marshall-Olkin Lomax distribution, Finley, (Kim et al., 2024) studied new efficient estimators for the Weibull distribution,

Creating a flexible distribution to represent lifetime data has always been a major challenge for authors. This was one of the reasons that prompted researchers to pay attention to truncated distributions as they are of great importance for testing lifetime data in various fields such as engineering, medicine, insurance, and biology (Altawil, 2021). (Singh et al., 2014) investigated the properties of the truncated versions of these Lindley generalizations,(Al-Marzouki, 2019) derived a new truncated Weibull-Power-Lomax distribution. (Teamah et al., 2020) provided a right-truncated Fréchet-Weibull distribution, and (Gul et al., 2021) provided Weibull-Truncated Exponential distribution. (Khaleel et al., 2022) considered and defined a specific model, named [0,1] Truncated Inverse Weibull Rayleigh distribution. (Abbas et al., 2023b) proposed the truncated Weibull exponential distribution. (Okorie et al., 2023) investigated an upper truncated Weibull distribution. Also, (Kalaf et al., 2023) introduced the truncated inverse generalized Rayleigh distribution. Therefore, a new truncated Komal-Weibull distribution was presented in this paper, and some important statistical properties were considered. This article is organized as follows: Section 2 discusses the truncated Komal-

Weibull distribution, Section 3 finds the reliability and the hazard functions, Section 4 derives some of the statistical properties, and Section 5 presents the maximum likelihood estimation method.

2. The Truncated Komal-Weibull Distribution:

The probability density function (pdf) and hazard rate function of the Komal distribution are defined by (Shanker, 2023) as

$$q(x,\Theta)=\frac{\Theta^2}{(\Theta^2+\Theta+1)}(\Theta+x+1)e^{-\Theta x};\ x>0,\Theta>0 \qquad (1)$$

$$h_k(x)=\frac{\Theta^2(\Theta+x+1)}{(\Theta^2+\Theta+\Theta x+1)} \qquad (2)$$

While the two-parameter Weibull density function presented by Weibull (Waloddi Weibull, n.d.) is usually expressed as follows:

$$g(x,\alpha,\beta)=\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}e^{-\left(\frac{x}{\beta}\right)^{\alpha}};x>,\alpha>0,\beta>0 \qquad (3)$$

$$h_w(x)=\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} \qquad (4)$$

With the method of combining the hazard rate functions of two distributions to get a new hazard rate function of mixture distribution see (Almalki & Yuan, 2013) and (TARVİRDİZADE, 2021) hence, the hazard rate function of the new mixture Komal-Weibull distribution KWD has been obtained by adding (2) to (4) as follows:

$$h_{KWD}(x)=\frac{\Theta^2(\Theta+x+1)}{(\Theta^2+\Theta+\Theta x+1)}+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} \qquad (5)$$

So, the reliability function is equal to:

$$R_{kwd}(x)=e^{-\int_0^x h_{KWD}(t)dt}=e^{-\int_0^x\left(\frac{\Theta^2(\Theta+t+1)}{(\Theta^2+\Theta+\Theta t+1)}+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right)dt} \qquad (6)$$

Then

$$R_{kwd}(x,\Theta,\alpha,\beta)=\frac{(\Theta^2+\Theta+\Theta x+1)}{(\Theta^2+\Theta+1)}e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)} \qquad (7)$$

Where x $>0,\alpha>0,\Theta>0$ and $\beta>0$. Hence the corresponding distribution function and the density of the new KWD are obtained respectively by

$$F_{kwd}\big((x,\Theta,\alpha,\beta)\big)=1-\left(\frac{(\Theta^2+\Theta+\Theta x+1)}{(\Theta^2+\Theta+1)}e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}\right) \qquad (8)$$

Thus

$$f_{kwd}\big((x,\Theta,\alpha,\beta)\big)=\frac{\left(\Theta+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right)(\Theta^2+\Theta+\Theta x+1)-\Theta}{(\Theta^2+\Theta+1)}e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)} \qquad (9)$$

Such that $\geq 0,\Theta,\alpha\ and\beta>0$, where α represents the shape parameter while $\Theta and\beta$ represent the scale parameters. Now by assuming that the r.v X is distributed as Komal-Weibull distribution with the positive parameters Θ, $\alpha\ and\beta$, such that X lies within the interval [0, 1] then according to (Aryuyuen & Bodhisuwan, 2019) the probability density function u_{RTIGRD} is:

$$u_{RTIGRD}(t)=\frac{f(x)}{F(1)} \ (10)$$

$$F(1)=1-\left[\frac{(\Theta^2+2\Theta+1)}{(\Theta^2+\Theta+1)}\right]e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)} \qquad (11)$$

Substitute (11) in (10) to get the probability density function of $RTKWD$:

$$u_{RTKWD}(x)=\frac{\left[\frac{\left(\Theta+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right)(\theta^2+\theta+\Theta x+1)-\theta}{(\theta^2+\theta+1)}\right]e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{1-\left[\frac{(\Theta^2+2\Theta+1)}{(\Theta^2+\Theta+1)}\right]e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}}=\frac{\left[\left(\Theta+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right)(\Theta^2+\Theta+\Theta x+1)-\Theta\right]e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} \qquad (12)$$

P-ISSN 2518-5764
E-ISSN 2227-703X

The cumulative distribution function is derived as follows:

$$U_{RTKWD}(x) = \frac{F(x)}{F(1)} \quad (13)$$

Substitute (11) in (13):

$$U_{RTKWD}(x) = \frac{1-\frac{(\theta^2+\theta+\theta x+1)}{(\theta^2+\theta+1)}e^{-\left(\theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{1-\left[\frac{(\theta^2+2\theta+1)}{(\theta^2+\theta+1)}\right]e^{-\left(\theta+\frac{1}{\beta^{\alpha}}\right)}}$$

or

$$U_{RTKWD}(x) = \frac{(\theta^2+\theta+1)-(\theta^2+\theta+\theta x+1)e^{-\left(\theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\theta^2+\theta+1)-(\theta^2+2\theta+1)e^{-\left(\theta+\frac{1}{\beta^{\alpha}}\right)}} \quad (14)$$

Figures (1), and (2) illustrate the pdf, and cdf, of $RTKWD$ in some cases of Θ, α and β, respectively.

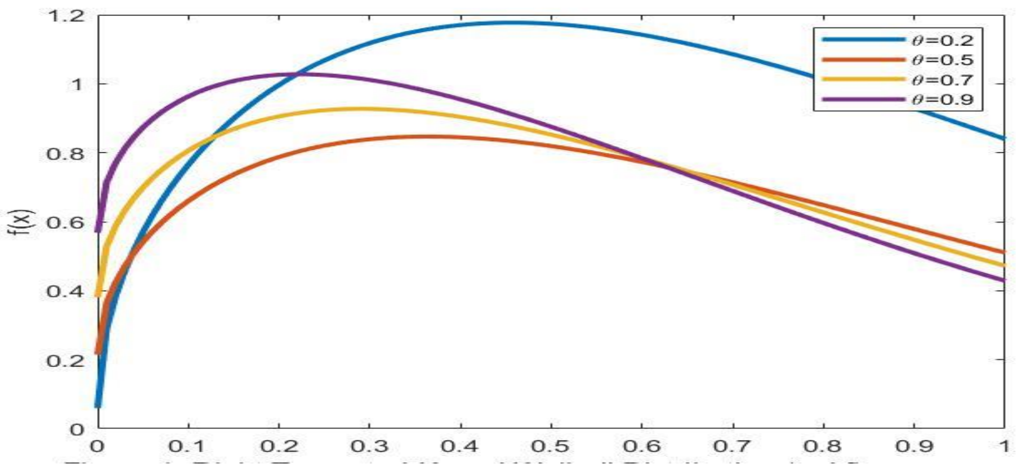

Figure 1: Probability density function of $RTKWD$

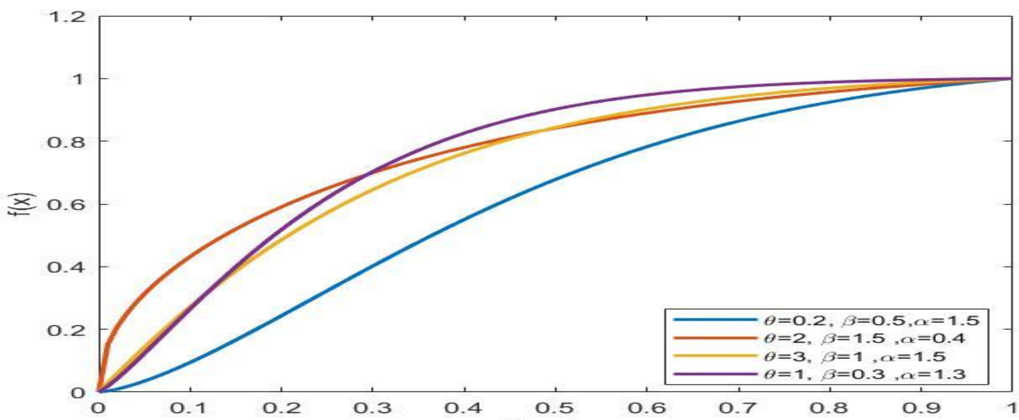

Figure 2: Cumulative distribution function for $RTKWD$

Figure 1 indicates that the RTKWD generates various shapes such as symmetrical, left-skewed, and rotated- J. In addition, Figure 2 demonstrates that the cdf of the RTKWD does not decrease with the increase of x and the distribution parameters increased.

3. Reliability Function And Hazard Function:

Let X be continuous random variable with probability density function and the cumulative distribution function as in (12) and (14) respectively, then the reliability function of $RTKWD$ is:

$$R_{RTKWD} = 1 - \frac{(\Theta^2+\Theta+1)-(\Theta^2+\Theta+\Theta x+1)e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} \quad (15)$$

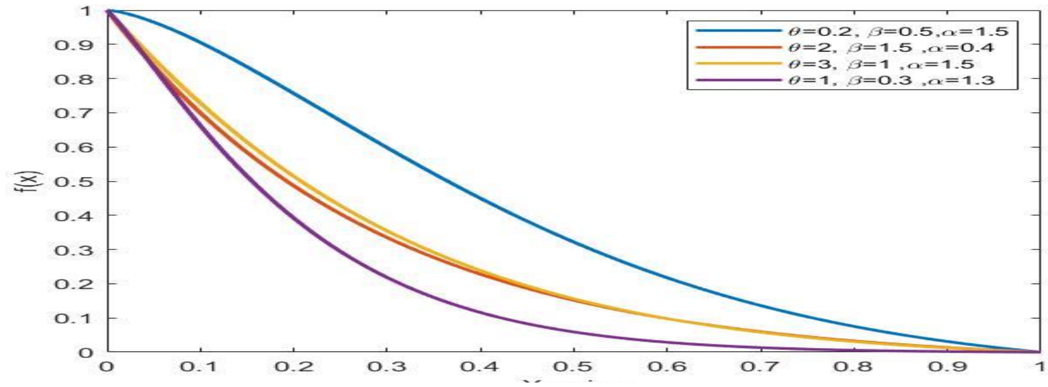

Figure 3: Reliability function for the $RTKWD$

Figure 3 shows that the reliability function of RTKWD is decreasing.

The hazard function is derived as below:

$$H_{RTKWD}(x) = \frac{u_{RTKWD}(x)}{R_{RTKWD}(x)}$$

Rewrite (15) as $R_{RTKWD} = \frac{(\Theta^2+\Theta+\Theta x+1)e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}}$

Hence

$H_{RTKWD}(x) =$

$$\frac{\left[\left(\Theta+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right)(\theta^2+\Theta+\Theta x+1)-\Theta\right]e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} \times \frac{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}}{(\Theta^2+\Theta+\Theta x+1)e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}}$$

$$H_{RTKWD}(x) = \frac{\left[\left(\Theta+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right)(\theta^2+\Theta+\Theta x+1)-\Theta\right]e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+\Theta x+1)e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} \quad (16)$$

4. Statistical Properties Of Right Truncated Komal-Weibull Distribution:

In this section, some of the statistical properties of the right truncated Komal-Weibull distribution have been derived and calculated

4.1 Moments:

In order to drive moments about origin the following steps should be followed:

$$M'_r(x) = \int_0^1 x^r\, u_{RTKWD}(x,\Theta,\alpha,\beta)dx$$

$$= \int_0^1 x^r \frac{\left[\left(\Theta+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right)(\Theta^2+\Theta+\Theta x+1)-\Theta\right]e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} dx \quad (17)$$

By putting $= \frac{1}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}}$, and using Taylor expansion

$$e^{-\left(\frac{x}{\beta}\right)^{\alpha}} = \sum_{i=0}^{\infty} \frac{(-x^{\alpha})^i}{i!\beta^{\alpha i}} \;\&\; e^{-\Theta x} = \sum_{k=0}^{\infty} \frac{(-\Theta X)^k}{k!}$$

Then equation (17) reduces into

$$M_r'(x) = v\sum_{i=0}^{\infty} \frac{(-1)^i}{i!\beta^{\alpha i}} \sum_{k=0}^{\infty} \frac{(-\Theta)^k}{k!} \left(\int_0^1 x^{\alpha i+r+k} \left[\left(\Theta + \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right)(\Theta^2 + \Theta + \Theta x + 1) - \Theta\right] dx\right) \quad (18)$$

$$= v\sum_{i=0}^{\infty} \frac{(-1)^i}{i!\beta^{\alpha i}} \sum_{k=0}^{\infty} \frac{(-\Theta)^k}{k!} \left(\int_0^1 \Theta^3 x^{\alpha i+r+k}\, dx + \int_0^1 \Theta^2 x^{\alpha i+r+k+1}\, dx + \int_0^1 \Theta^2 x^{\alpha i+r+k}\, dx + \frac{\alpha}{\beta^{\alpha}}\int_0^1 \Theta^2 x^{\alpha(i+1)+k+r-1}\, dx + \frac{\alpha}{\beta^{\alpha}}\int_0^1 \Theta x^{\alpha(i+1)+r+k}\, dx + \frac{\alpha}{\beta^{\alpha}}\int_0^1 \Theta x^{\alpha(i+1)+r+k-1}\, dx + \frac{\alpha}{\beta^{\alpha}}\int_0^1 x^{\alpha(i+1)+r+k-1}\, dx\right)$$

$$M_r'(x) = v\sum_{i=0}^{\infty} \frac{(-1)^i}{i!\beta^{\alpha i}} \sum_{k=0}^{\infty} \frac{(-\Theta)^k}{k!} \left[\frac{\Theta^3}{\alpha i+r+k+1} + \frac{\Theta^2}{\alpha i+r+k+2} + \frac{\Theta^2}{\alpha i+r+k+1} + \frac{\alpha\Theta^2}{\beta^{\alpha}[\alpha(i+1)+k+r]} + \frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+r+k+1]} + \frac{\Theta}{\beta^{\alpha}[\alpha(i+1)+r+k]} + \frac{1}{\beta^{\alpha}[\alpha(i+1)+r+k]}\right] \quad (19)$$

$$M_1'(x) = E(x) = v\sum_{i=0}^{\infty} \frac{(-1)^i}{i!\beta^{\alpha i}} \sum_{k=0}^{\infty} \frac{(-\Theta)^k}{k!} \left[\frac{\Theta^3+\Theta^2}{\alpha i+k+2} + \frac{\Theta^2}{\alpha i+k+3} + \frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+1]} + \frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+2]}\right] \quad (20)$$

$$M_2'(x) = E(x^2) = v\sum_{i=0}^{\infty} \frac{(-1)^i}{i!\beta^{\alpha i}} \sum_{k=0}^{\infty} \frac{(-\Theta)^k}{k!} \left[\frac{\Theta^3+\Theta^2}{\alpha i+k+3} + \frac{\Theta^2}{\alpha i+k+4} + \frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+2]} + \frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+3]}\right] \quad (21)$$

$$M_3'(x) = E(x^3) = v\sum_{i=0}^{\infty} \frac{(-1)^i}{i!\beta^{\alpha i}} \sum_{k=0}^{\infty} \frac{(-\Theta)^k}{k!} \left[\frac{\Theta^3+\Theta^2}{\alpha i+k+4} + \frac{\Theta^2}{\alpha i+k+5} + \frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+3]} + \frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+4]}\right] \quad (22)$$

$$M_4'(x) = E(x^4) = v\sum_{i=0}^{\infty} \frac{(-1)^i}{i!\beta^{\alpha i}} \sum_{k=0}^{\infty} \frac{(-\Theta)^k}{k!} \left[\frac{\Theta^3+\Theta^2}{\alpha i+k+5} + \frac{\Theta^2}{\alpha i+k+6} + \frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+4]} + \frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+5]}\right] \quad (23)$$

4.2 Variance, Skewness and Kurtosis:

To know more about the Right truncated Komal-Weibull distribution behaviour, investigating the variance, skewness, and kurtosis is important in this subsection since the variance describes the amount of variability while skewness tells the direction of variability. By using equations (20) and (21) the variance is given as the following:

$$var(x) = M_2'(x) - \left(M_1'(x)\right)^2$$

$$var(x) = v\sum_{i=0}^{\infty} \frac{(-1)^i}{i!\beta^{\alpha i}} \sum_{k=0}^{\infty} \frac{(-\Theta)^k}{k!} \left[\frac{\Theta^3+\Theta^2}{\alpha i+k+3} + \frac{\Theta^2}{\alpha i+k+4} + \frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+2]} + \frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+3]}\right] - \left(v\sum_{i=0}^{\infty} \frac{(-1)^i}{i!\beta^{\alpha i}} \sum_{k=0}^{\infty} \frac{(-\Theta)^k}{k!} \left[\frac{\Theta^3+\Theta^2}{\alpha i+k+2} + \frac{\Theta^2}{\alpha i+k+3} + \frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+1]} + \frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+2]}\right]\right)^2$$

Now to evaluate the skewness of the right truncated Komal-Weibull distribution since: $sk = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}} = \frac{E(x^3) - 3\mu E(x^2) + 2\mu^3}{(\sigma^2)^{\frac{3}{2}}}$, So

$$sk=\frac{\left\{\begin{array}{l} v\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+4}+\frac{\Theta^2}{\alpha i+k+5}+\frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+3]}+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+4]}\right]\\ -3v\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+2}+\frac{\Theta^2}{\alpha i+k+3}+\frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+1]}+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+2]}\right]\\ v\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+3}+\frac{\Theta^2}{\alpha i+k+4}+\frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+2]}+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+3]}\right]\\ +2\left(v\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+2}+\frac{\Theta^2}{\alpha i+k+3}+\frac{\alpha\Theta^2+\Theta}{\beta^{\alpha}[\alpha(i+1)+k+1]}+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+2]}\right]\right)^3 \end{array}\right\}}{\left\{\begin{array}{l} v\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+3}+\frac{\Theta^2}{\alpha i+k+4}+\frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+2]}+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+3]}\right]\\ -\left(v\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+2}+\frac{\Theta^2}{\alpha i+k+3}+\frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+1]}+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+2]}\right]\right)^2 \end{array}\right\}^{\frac{3}{2}}} \quad (25)$$

and the kurtosis will be equal to:

$$kr=\frac{\mu_3}{(\mu_2)^2}-3=\frac{E(x^4)-4\mu E(x^3)+6\mu^2E(x^2)-3\mu^4}{(\sigma^2)^2}-3$$

$$kr=\frac{\left\{\begin{array}{l} v\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+5}+\frac{\Theta^2}{\alpha i+k+6}+\left[\frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}\alpha(i+1)+k+4}\right]+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+5]}\right]\\ -4\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+2}+\frac{\Theta^2}{\alpha i+k+3}+\frac{\alpha\Theta^2+\Theta}{\beta^{\alpha}[\alpha(i+1)+k+1]}+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+2]}\right]\\ \sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+4}+\frac{\Theta^2}{\alpha i+k+5}+\frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+3]}+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+4]}\right]\\ +6\left(v\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+2}+\frac{\Theta^2}{\alpha i+k+3}+\frac{\alpha\Theta^2+\Theta}{\beta^{\alpha}[\alpha(i+1)+k+1]}+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+2]}\right]\right)^2\\ v\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+3}+\frac{\Theta^2}{\alpha i+k+4}+\frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+2]}+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+3]}\right]\\ -3\left(v\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+2}+\frac{\Theta^2}{\alpha i+k+3}+\frac{\alpha\Theta^2+\Theta}{\beta^{\alpha}[\alpha(i+1)+k+1]}+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+2]}\right]\right)^4 \end{array}\right\}}{\left\{\begin{array}{l} v\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+3}+\frac{\Theta^2}{\alpha i+k+4}+\frac{\alpha\Theta^2+\Theta+1}{\beta^{\alpha}[\alpha(i+1)+k+2]}+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+3]}\right]\\ -\left(v\sum_{i=0}^{\infty}\frac{(-1)^i}{i!\beta^{\alpha i}}\sum_{k=0}^{\infty}\frac{(-\Theta)^k}{k!}\left[\frac{\Theta^3+\Theta^2}{\alpha i+k+2}+\frac{\Theta^2}{\alpha i+k+3}+\frac{\alpha\Theta^2+\Theta}{\beta^{\alpha}[\alpha(i+1)+k+1]}+\frac{\alpha\Theta}{\beta^{\alpha}[\alpha(i+1)+k+2]}\right]\right)^2 \end{array}\right\}^2}-3 \quad (26)$$

4.3 Moment Generating Function:

The moment generating function of RTKWD can be derived as follows:

$$\mathcal{M}_x(t)=E(e^{tx})=\int_0^1 e^{tx}\,u_{RTKWD}(x)\,dx$$

$$=\int_0^1 e^{tx}\,\frac{\left[\left(\Theta+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right)(\Theta^2+\Theta+\Theta x+1)-\theta\right]e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}}\,dx$$

$$=\frac{1}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}}\int_0^1 e^{tx}\left[\Theta^3+\Theta^2+x\Theta^2+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\Theta^2+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\Theta+\frac{\alpha}{\beta^{\alpha}}x^{\alpha}\Theta+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right]e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}dx \quad (27)$$

by using the following Taylor expansion to simplify equation (27):

$e^{-\left(\frac{x}{\beta}\right)^{\alpha}} = \sum_{i=0}^{\infty} \frac{(-x^{\alpha})^i}{i!\beta^{\alpha i}}$, $e^{xt}.e^{-\Theta x} = e^{(t-\Theta)x} = \sum_{k=0}^{\infty} \frac{[(t-\Theta)x]^k}{k!}$ and

we obtain:

$$\mathcal{M}_x(t) = \frac{1}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} \sum_{i=0}^{\infty} \frac{(-1)^i}{i!\beta^{\alpha i}} \sum_{k=0}^{\infty} \frac{(t-\Theta)^k}{k!} \left(\int_0^1 x^{\alpha i+k} \left[\Theta^3+\Theta^2+x\Theta^2 + \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\Theta^2 + \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\Theta + \frac{\alpha}{\beta^{\alpha}}x^{\alpha}\Theta + \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right] dx \right)$$

$$\mathcal{M}_x(t) = \frac{1}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} \sum_{i=0}^{\infty} \frac{(-1)^i}{i!\beta^{\alpha i}} \sum_{k=0}^{\infty} \frac{(t-\Theta)^k}{k!} \left(\frac{\Theta^3+\Theta^2}{\alpha i+k+1} + \frac{\Theta^2}{\alpha i+k+2} + \frac{\alpha\Theta^2+\alpha\Theta+\alpha}{\beta^{\alpha}\alpha(i+1)+k+} + \frac{\alpha\Theta}{\beta^{\alpha}\alpha(i+1)+k+1} \right) \quad (28)$$

4.5 The distribution of order statistics:

Let $X_1, X_2, \dots, X_n$ be a random sample of size n of the right truncated Komal-Weibull distribution and suppose $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ is the corresponding order statistics then probability distribution function and cumulative of the i[th] order statistics say $f_{RTkwd:i}(x)$ could be derived as follows:

$$u_{RTkwd:i}(x) = \frac{n!}{(i-1)!(n-i)!} u_{RTkwd}(x,\Theta,\alpha,\beta)[U_{RTkwd}(x,\Theta,\alpha,\beta)]^{(i-1)}[1-U_{RTkwd}(x,\Theta,\alpha,\beta)]^{(n-i)} \quad (29)$$

By substituting (5) and (7) in (20) we get

$$u_{RTKWD:i}(x) = \frac{n!}{(i-1)!(n-i)!} \left[\frac{\left[\left(\Theta+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right)(\Theta^2+\Theta+\Theta x+1)-\Theta\right] e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} \right] \left[\frac{(\Theta^2+\Theta+1)-(\Theta^2+\Theta+\Theta x+1)e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} \right]^{(i-1)} \left[1 - \frac{(\Theta^2+\Theta+1)-(\Theta^2+\Theta+\Theta x+1)e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} \right]^{(n-i)} \quad (30)$$

Put $i = 1$ to get the probability distribution function of the first order statistics $X_{(1)} = min(X_1, X_2, \dots, X_n)$ as follows:

$$u_{RTKWD:1}(x) = n \left[\frac{\left[\left(\Theta+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right)(\Theta^2+\Theta+\Theta x+1)-\Theta\right] e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} \right] \left[1 - \frac{(\Theta^2+\Theta+1)-(\Theta^2+\Theta+\Theta x+1)e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} \right]^{(n-1)} \quad (31)$$

Also, when $= n$, the probability distribution function of the n[th] order statistics is: $X_{(n)} = max(X_1, X_2, \dots, X_n)$ as follows:

$$u_{RTKWD:n}(x) = n \left[\frac{\left[\left(\Theta+\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\right)(\Theta^2+\Theta+\Theta x+1)-\Theta\right] e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} \right] \left[\frac{(\Theta^2+\Theta+1)-(\Theta^2+\Theta+\Theta x+1)e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}} \right]^{(n-1)} \quad (32)$$

Hence the cumulative distribution function n^th order statistics is:

$$U_{RTKWD:i}(x) = \sum_{k=i}^{n}\binom{n}{k}[U_{RTkwd}(x)]^k[1-U_{RTkwd}(x)]^{(n-k)}$$

Then

$$U_{RTKWD:i}(x) =$$

$$\sum_{k=i}^{n}\binom{n}{k}\left[\frac{(\Theta^2+\Theta+1)-(\Theta^2+\Theta+\Theta x+1)e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}}\right]^k\left[1-\frac{(\Theta^2+\Theta+1)-(\Theta^2+\Theta+\Theta x+1)e^{-\left(\Theta x+\left(\frac{x}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}}\right]^{(n-k)} \quad (33)$$

5. The Maximum Likelihood Estimation:

Although (FISHER, 1934b) presents the method of maximum likelihood for the first time, but this method is considered the most widely used method in estimating problems(Atiya Kalaf et al., 2021)). Many authors choose the maximum likelihood estimation to estimate different parameters of different statistical functions see(A. Mohammed, 2019; Raheem et al., 2021; Salman et al., 2018,) et al 2017, Raheem et al, 2021, Haddad and Batah, 2021):

Let $(x_1, x_2, \dots, x_n)$ be a random sample of size n drawn independently from the right truncated Komal- Weibull distribution the likelihood function L of this sample is given by:

$$L(\Theta\backslash\beta,\alpha,x) = \prod_{i=1}^{n} u_{RTKWD}(x_i,\Theta,\beta,\alpha)$$

$$L(\Theta\backslash\beta,\alpha,x) = \prod_{i=1}^{n}\frac{\left[\left(\Theta+\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}\right)(\Theta^2+\Theta+\Theta x_i+1)-\Theta\right]e^{-\left(\Theta x_i+\left(\frac{x_i}{\beta}\right)^{\alpha}\right)}}{(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}}$$

$$L(\Theta\backslash\beta,\alpha,x) = \frac{1}{\left[(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}\right]^n}\left[\left(\Theta+\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}\right)(\Theta^2+\Theta+\Theta x_i+1)-\Theta\right]e^{-\left(\Theta x_i+\left(\frac{x_i}{\beta}\right)^{\alpha}\right)} \quad (34)$$

Taking the natural logarithm of (33) to maximize the likelihood (Haddad & Batah, 2021; Taha & Salman, 2022) gives us the following equation:

$$lnL(\Theta\backslash\beta,\alpha,x) = ln\frac{1}{\left[(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}\right]^n}+\sum_{i=1}^{n} ln\left[\left(\Theta+\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}\right)(\Theta^2+\Theta+\Theta x_i+1)-\Theta\right]-\sum_{i=1}^{n}\left(\Theta x_i+\left(\frac{x_i}{\beta}\right)^{\alpha}\right)$$

$$= -nln\left[(\Theta^2+\Theta+1)-(\Theta^2+2\Theta+1)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}\right]+\sum_{i=1}^{n} ln\left[\Theta^3+\Theta^2x_i+\Theta^2\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}+\Theta^2+\Theta\alpha\left(\frac{x_i}{\beta}\right)^{\alpha}+\Theta\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}+\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}\right]-\sum_{i=1}^{n}\left(\theta x_i+\left(\frac{x_i}{\beta}\right)^{\alpha}\right) \quad (35)$$

assume that the parameters β *and* α are known then to estimate the unknown parameter Θ The partial derivative of (35) will be taken with respect to Θ is

$$\frac{dlnL}{d\Theta} = \frac{-n\left[(2\Theta+1)+\left((\Theta^2+2\Theta+1)-(2\Theta+1)\right)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}\right]}{(\Theta^2+2\Theta+1)-\left((\Theta^2+2\Theta+1)\right)e^{-\left(\Theta+\frac{1}{\beta^{\alpha}}\right)}}+\frac{3\Theta^2+2\Theta x_i+2\Theta\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}+2\Theta+\alpha\left(\frac{x_i}{\beta}\right)^{\alpha}+\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}}{\sum_{i=1}^{n}\left[\Theta^3+\Theta^2x_i+\Theta^2\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}+\Theta^2+\Theta\alpha\left(\frac{x_i}{\beta}\right)^{\alpha}+\Theta\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}+\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}\right]}-\sum_{i=1}^{n}x_i \quad (36)$$

the nonlinear equation (36) does not give the value of the estimated parameter $\hat{\Theta}$ by sitting $\frac{dlnL}{d\Theta}$ to zero, Hence Newton Raphson iteration will be suggested to use for obtaining the analytical solution. The iteration equation to $\hat{\Theta}$ is given by:

$$\hat{\Theta}_{n+1} = \hat{\Theta}_n - \frac{f(\hat{\Theta}_n)}{\acute{f}(\hat{\Theta}_n)}$$

where $f(\hat{\Theta}_n)$ could be obtained by replacing each θ in (34) by $\hat{\Theta}_n$ then

$$\hat{\Theta}_{n+1} = \hat{\Theta}_n - \left\{ \frac{ \frac{-n\left[(2\hat{\Theta}_n+1)+\left((\hat{\Theta}_n^2+2\hat{\Theta}_n+1)-(2\hat{\Theta}_n+1)\right)e^{-\left(\hat{\Theta}_n+\frac{1}{\beta\alpha}\right)}\right]}{(\hat{\Theta}_n^2+\hat{\Theta}_n+1)-(\hat{\Theta}_n^2+2\hat{\Theta}_n+1)e^{-\left(\hat{\Theta}_n+\frac{1}{\beta\alpha}\right)}} + \frac{3\hat{\Theta}_n^2+2\hat{\Theta}_n x_i+2\hat{\Theta}_n\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}+2\hat{\Theta}_n+\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha}+\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}}{\sum_{i=1}^{n}\left[\hat{\Theta}_n^3+\hat{\Theta}_n^2 x_i+\hat{\Theta}_n^2\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}+\hat{\Theta}_n^2+\hat{\Theta}_n\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha}+\hat{\Theta}_n\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}+\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}\right]} - \sum_{i=1}^{n} x_i }{ \frac{-2n(\Theta_n^2+\hat{\Theta}_n+1)+n(2\hat{\Theta}_n+1)(2\hat{\Theta}_n+1)}{(\hat{\Theta}_n^2+\hat{\Theta}_n+1)^2} + \frac{\left(\begin{array}{c}\sum_{i=1}^{n}\left[\hat{\Theta}_n^3+\hat{\Theta}_n^2 x_i+\hat{\Theta}_n^2\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}+\hat{\Theta}_n^2+\hat{\Theta}_n\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha}+\hat{\Theta}_n\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}+\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}\right]\left(6\hat{\Theta}_n+2x_i+2\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}+2\right) \\ -\left(3\hat{\Theta}_n^2+2\hat{\Theta}_n x_i+2\hat{\Theta}_n\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}+2\hat{\Theta}_n+\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha}+\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}\right)^2\end{array}\right)}{\left(\sum_{i=1}^{n}\left[\hat{\Theta}_n^3+\hat{\Theta}_n^2 x_i+\hat{\Theta}_n^2\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}+\hat{\Theta}_n^2+\hat{\Theta}_n\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha}+\hat{\Theta}_n\frac{\alpha}{\beta}\left(\frac{x_i}{\gamma}\right)^{\alpha-1}+\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1}\right]\right)^2} } \right\} \quad (37)$$

6. Conclusion:

In this paper, the right truncated of a new lifetime mixture with three parameters distribution called right truncated Komal-Weibull distribution has been proposed and studied. The shape of pdf, cdf, and the hazard rate function have been discussed. In addition, some mathematical properties such as reliability function, moments about origin, variance, coefficients of skewness, and the mode and moment generating function have been derived to know more about the distribution function behavior. And to percent additional information and knowledge about the right truncated Komal-Weibull distribution the distribution of order statistics and the maximum likelihood estimation have been investigated.

Authors Declaration:

Conflicts of Interest: None

-We Hereby Confirm That All The Figures and Tables In The Manuscript Are Mine and Ours. Besides, The Figures and Images, which are Not Mine, Have Been Permitted Republication and Attached to The Manuscript.

- Ethical Clearance: The Research Was Approved by The Local Ethical Committee in The University.

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