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Some Fuzzy Least Squares Estimators for Regression Model Using Different Kernel Functions

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Abstract:

This paper presents a method for addressing the issue of outliers in fuzzy data. The method involves calculating a new distance between fuzzy numbers using various kernel functions, based on the fuzzy least squares method. The parameters of the fuzzy regression model were estimated in cases where the explanatory variables were non-fuzzy, the parameters were fuzzy, and the response variable was both fuzzy and an outlier. These estimators were then compared to the Fuzzy Least Squares method (FLS) using Mean Square Error (MSE) through simulations with different sample sizes (25, 50, 100, 150) and levels of outliers (0, 0.10, 0.20, 0.30). The results showed that this method, utilizing the new distance, achieved the best results in the presence of outliers.

Paper type: Research paper.
Keywords: Robust fuzzy regression, Fuzzy least squares method, Distance between fuzzy triangular numbers, Kernel functions, Outliers.

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1. Introduction:

There are many phenomena whose data are fuzzy, meaning that their value cannot be determined with one value, and to represent this data by estimating a model that describes it and estimating its parameters, especially when the inputs are not fuzzy and the outputs and parameters are fuzzy, with the problem of outliers in the data, and of traditional methods, the fuzzy least squares method, as it does not give effective estimates because its conditions are not applied. Many methods treat fuzzy data in the presence of outliers based on robust fuzzy methods, such as fuzzy least squares estimation methods based on kernel functions to calculate the distance between fuzzy numbers. Therefore, in this research, we will use the least squares method based on kernel functions to obtain better estimates when the response variable is a fuzzy variable, the parameters are fuzzy, and the explanatory variables are crisp. The mean square error (MSE) criterion is used to compare the effects of estimating fuzzy least squares and fuzzy least squares based on new distance.

1.1 Literature Review:

Many researchers have tried to study regression analysis specifically in uncertain data using fuzzy sets in the presence of the outliers' problem and in all areas of estimation methods to obtain the best robust fuzzy model. Zeng et al. (2018) introduced fuzzy linear regression by using fuzzy least squares estimation of fuzzy numbers in which the distance was calculated using the Euclidean distance. They concluded that the three different distances have the same estimation coefficient using the least squares method with simulation. Mohammed et al. (2019) studied the fuzzy nonparametric regression model with one explanatory variable and a fuzzy dependent variable of the symmetrical triangular type. Nonparametric estimation methods were used including the nearest neighbor method KNN the localized linear smoothing method the kernel function method and the legitimate crossing method aiming to choose the optimal value for the smoothing parameter h and the GOF and was used for comparison between modalities. It was concluded that simulation experiments are better than the local linear smoothing method. Ayden et al. (2020) attempted to find robust estimators for the parameters of the linear regression model where robust methods were used when outliers were available in the data by combining the robust M method, which contains four functions and the fuzzy setting so that the estimators were converted to fuzzy estimators using the trigonometric membership function and then using these fuzzy numbers in the function monotonicity and using the mean square error standard. It was concluded that the best M function is the Hampel function, which gave the lowest MSE. It was also concluded that the estimates of the Hampel function parameters were converted into fuzzy numbers using the trigonometric membership function. Hesamian et al. (2021) suggested an extension of classical univariate partial regression model with crisp input and triangular fuzzy outputs in which a common non-parametric estimator was combined with traditional fuzzy trigonometric arithmetic to build a univariate fuzzy regression model, then hybrid algorithm was developed to estimate bandwidth and fuzzy regression coefficient. The numerical results indicated the lower sensitivity of the proposed model to outliers and its higher precision compared to the other existing robust regression methods. Shemail et al. (2022) presented a study on fuzzy semi-parametric logistic regression the intuitive fuzzy trapezoidal number and the fuzzy semi parametric logistic regression model. They estimated the parametric part using the fuzzy least squares method and proposed weighted least squares while the nonparametric part was estimated through the Nedaria Watson estimators and the nearest neighbor estimators. It turns out that Nedaria Watson's estimators are better than those of the nearest neighbor. The results were the fuzzy ordinary least squares estimators method was better than the suggested fuzzy weighted least squares estimators, while in the non-parametric portion the Nadaraya Watson estimators were better than Nearest Neighbor estimators to estimate the model. Khammar et al. (2020) presented a new approach that fits the robust fuzzy regression model, which depends on some fuzzy quantities using the new distance between fuzzy numbers based on the kernel function and parameters estimated by the fuzzy least squares method for a robust fuzzy regression model with the presence of different types of outliers and applying the proposed methods while modeling some properties using outliers.

One of the problems facing regression analysis and formulating the appropriate model is that the data related to the research under study is fuzzy and has outlier values, and the study aims to estimate more efficient parameters using the distance between fuzzy trigonometric numbers via kernel functions in the case where the response variable is fuzzy, the parameters are fuzzy, and the explanatory variables are crisp, according to fuzzy set theory and comparison of estimation methods.

2.Materials and Methods:

2.1 Fuzzy Sets

In fuzzy sets theory, the element in the sets takes a set of values confined between [0,1] with a specific membership degree; that is, the element is determined by its membership degree. Therefore, the fuzzy sets are characterized by the presence of the membership function.

$$A = \left\{\left(x_i . \mu_{\tilde{A}_{(x)}}\right)\right\} \qquad X = \{x_i . i = 1.2. \dots . n\}$$

Where $\mu\tilde{A}_{(xi)}$ is a function of membership and the degree of membership of element x_i in set A.

When the element takes a degree of membership (1), the element belongs entirely to the fuzzy sets. When the degree of membership is (0), the element does not belong to the sets, and the other degrees vary between zero and one. When the degree of membership is (0.5), this means that the element belongs (0.5) to the fuzzy set and does not belong (0.5) to the Crisp set. This element is called the equilibrium point. When the degree of membership is (0.9), that means the element belongs to the fuzzy sets by (0.9) and does not belong by (0.1), and this is closer to membership or not.

Therefore, fuzzy sets theory is an extension of the classic Crisp sets theory, and the classic sets theory is a special case of fuzzy sets theory (Mohammed, 2007; Mohammed et al., 2017).

2.2 Membership Function

It is a function that expresses the degree of membership or the degree of membership, which are real numbers within the closed interval [0,1] and it expresses the degree of membership μ (F(x)), which represents the degree of membership to the element from the variable x to the fuzzy sets (Mohammed, 2007; Farhan, 2013).

2.3 Triangular membership function

It is the most common and widespread function and contains three parameters: a, b, c, and it is expressed in the following formula (Klir, 1995)

$$M_{\mathrm{A}}(\mathrm{X}) = \begin{cases} \frac{x-\mathrm{a}}{\mathrm{b}-\mathrm{a}} & , if\ a \le x \le b \\ \frac{b-x}{c-b} & , if\ b \le x \le c \\ 0 & other\ wise \end{cases} \qquad (1)$$

a. b. c They represent the right term parameter the middle term parameter and the left term parameter respectively.

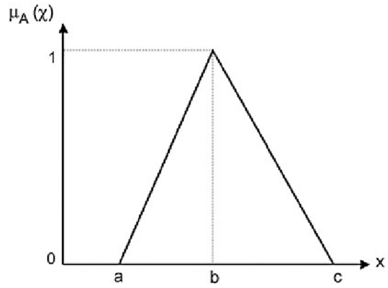

Figure 1: Triangular Function Membership.
(Mohammed, 2007)

2.4 Fuzzy Number

The fuzzy number $\tilde{A}$ is defined as a fuzzy set on the real number line R and must meet the following conditions (Barua and et al., 2013; Kareem and Mohammed, 2023)

1-There is at least one element R∈ X_0 such that $\mu_{\tilde{A}_{(x)}}$ =1

2- $\mu_{\tilde{A}_{(x)}}$is a continuous ordered pair.

3- $\tilde{A}$ It must be normal and concave (or convex).

Definition:

A fuzzy number Ñ is called a LR fuzzy number, if there are real numbers M , L and R with L , R ≥ 0. and the strictly decreasing functions L, R: $R^+ \rightarrow$ [0, 1] such that (Khammar and Arefi, 2020):

$$\tilde{N}_{(x)} = \begin{cases} L\left(\frac{m-x}{l}\right). & x \leq m. \\ R\left(\frac{x-m}{r}\right). & x > m. \end{cases} \qquad (2)$$

It is denoted by $\tilde{N}$ $= (l.m.r)_{LR}$.

In a special case, if L(x) = R(x) for all x ∈ [0, 1], then $\tilde{N}$ is called the LL fuzzy number. On the other hand, if $L(x) = R(x) = 1 - x$ for all $x \in [0.1]$. then $\tilde{N}$ is called triangular fuzzy numbers and is denoted by $(m.l.r)_T$. Also for $l = r$. $\tilde{N}$ is a symmetric triangular fuzzy numbers as $\tilde{N}$ $= (m.l)_T$.

2.5 Fuzzy Linear Regression Model

The linear fuzzy regression model estimates the significant relationship between the response variable and the independent variables in a fuzzy with a linear function. Thus, it is called Fuzzy Linear Regression (FLR). This research will adopt a fuzzy model, where the inputs are crisp, and the outputs and parameters are fuzzy; the fuzzy regression model follows:

$$\tilde{Y} = \tilde{\beta}_0 + \tilde{\beta}_1 x_1 + \tilde{\beta}_2 x_2 + \cdots + \tilde{\beta}_n x_n + \tilde{\varepsilon} \qquad (3)$$

i =1, 2, …, n

Where:

$\hat{\tilde{Y}}$: - Fuzzy response variable.

x_1 . x_2 ... x_n: - Crisp independent variables.

$\tilde{\beta}$: - Fuzzy parameters.

$\tilde{\varepsilon}$: - Fuzzy random error.

2.6 Outliers

Outliers have been defined in more than one way. They are those observations that appear to deviate significantly from the rest of the sample observations, and are inconsistent and illogical when compared to the rest of the data set. The focus of this research is the fuzzy regression model when the outliers are in the dependent variable of the model (Mohammed, 2007).

2.7 Distance based on kernel function between Fuzzy numbers (LR)

The fuzzy regression model, which consists of crisp explanatory variables, was studied. The dependent variable is fuzzy, and the parameters are fuzzy, that is, fuzzy numbers. The measurement of these variables is not represented by a point but by an interval. This variable type is very common in practice, so using least squares probability theory or one of the other alternative methods would not be the appropriate way to analyse and process this type of data. It was therefore necessary to use new methods based on fuzzy set theory to process this type of data. These methods depend on the distance between the two fuzzy numbers and calculate this distance using the kernel function, and we will determine the new distance between the two fuzzy numbers.

Definition: Suppose that $\tilde{A}$ and $\tilde{B}$ are two LR fuzzy numbers. Distance between $\tilde{A}$ and $\tilde{B}$ based on kernel function (kernel distance) is defined as (Arefi and Akbari, 2020):

$\tilde{A} = (m_a . l_a . r_a)_{LR}$, $\tilde{B} = (m_b . l_b . r_b)_{LR}$

$$D_K(\tilde{A}.\tilde{B}) = \left[\frac{1}{3}[\|\emptyset(m_a) - \emptyset(m_b)\|^2 + \|\emptyset(m_a - \lambda l_b)\|^2 + \|\emptyset(m_a + \rho r_a) - \emptyset(m_b + \rho r_b)\|^2]\right]^{1/2}$$

$$= \left[\frac{1}{3}[K(m_a.m_a) - 2K(m_a.m_b) + K(m_b.m_b) + K(m_a - \lambda.m_a - \lambda l_a) - 2K(m_a - \lambda l_{a.}m_b - \lambda l_b) + K(m_b - \lambda l_{b.}m_b - \lambda l_b) + K(m_a - \rho l_{a.}m_a - \rho l_a) - 2K(m_a - \rho l_{a.}m_b - \rho l_b) + K(m_b - \rho l_{b.}m_b - \rho l_b)]\right]^{1/2} \quad (4)$$

Where $Z = \{z_1 . z_2 \dots . z_n\}$.

$\emptyset : Z \rightarrow F$ be a nonlinear mapping from the input space Z to a high dimensional feature space F, $\lambda = \int_0^1 L^{-1}(w)dw$ and $\rho = \int_0^1 R^{-1}(w)dw$.

The distance of the kernel function according to equation (4) is the average error applied to the middle, upper and lower terms of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$. In order to implement the kernel distance, we have to define a function k (.,.). There are different kernel functions will be studied as follows: -

1) Gaussian kernel: $k(x_i . x_k) = \exp\{-\frac{(x_i - x_k)^2}{2h^2}\}$,
2) Triweight kernel: $k(x_i . x_k) = \frac{35}{32}(1 - |x_i - x_k|^2)^3$ for $|x_i - x_k| \leq 1$
3) Epanecknikiv kernel: $k(x_i . x_k) = \frac{3}{4}(1 - (x_i - x_k)^2)$ for $|x_i - x_k| \leq 1$
4) Uniform kernel: $k(x_i . x_k) = \frac{1}{2}$ for $|x_i - x_k| \leq 1$
5) Biweight kernel: $k(x_i . x_k) = \frac{15}{16}(1 - |x_i - x_k|^2)^2$ for $|x_i - x_k| \leq 1$

Where h is bandwidth parameter.

2.8 Estimating of parameters of model

2.8.1 Fuzzy Least Squares Estimation (FLS)

The parameters of the fuzzy regression model are estimated based on the usual least squares formula with the fuzzy model, which is called the fuzzy least squares method, as in the following formula (Kalnins,2018; Kareem and Mohammed, 2023):

$$\hat{\tilde{\beta}} = Min \sum_{i=1}^{n} (\tilde{y}_i - \tilde{\beta}_0 - \sum_{j=1}^{p} \tilde{\beta}_j x_{ij})^2 \qquad , \tilde{\beta} = (\tilde{\beta}_0 . \tilde{\beta}_1 . \dots . \tilde{\beta}_n) \qquad (5)$$

And by using a Lagrange multiplier by deriving the above equation with respect to $\tilde{\beta}$ it results

$$\hat{\tilde{\beta}}^{FLSE} = \arg\min \sum_{i=1}^{n} d^2 (\tilde{y}_i - \tilde{y}_i{}^*) \qquad (6)$$

$$d^2 = (\tilde{y}_i . \tilde{y}_i{}^*) = \left[\left(y_i{}^C - y_i{}^{*C} \right)^2 + [(y_i{}^C - x_i{}^L) - (y_i{}^{*C} - y_i{}^{*L})]^2 + (y_i{}^C - y_i{}^U) - (y_i{}^{*C} - y_i{}^{*U}) \right]^2 \quad (7)$$

2.8.2 Fuzzy least squares estimation based on the kernel distance

The fuzzy regression model estimated by the fuzzy least squares method is based on the kernel distance in (4). The distance is calculated from the sum of squares of $\hat{\tilde{Y}}_i . \tilde{Y}_i$ as follows (Khammar et al., 2020):

$$S = \sum_{i=1}^{n} D_K^2 \left(\tilde{Y}_i . \hat{\tilde{Y}}_i \right) = 2n - \frac{2}{3} \sum_{i=1}^{n} \Big(K\big(y_i . \sum_{j=0}^{n} \beta_j^m x_{ij}\big) + K\Big(y_i - \lambda S_{li} . \sum_{j=0}^{p} \beta_j^m x_{ij} - \lambda \sum_{j=0}^{p} \beta_j^l x_{ij}\Big) + K\Big(y_i + \rho S_{ri} . \sum_{j=0}^{p} \beta_j^m x_{ij} + \rho \sum_{j=0}^{p} \beta_j^u x_{ij}\Big)\Big) \qquad (8)$$

2.9 Mean Square Error (MSE)

For the purpose of comparing the fuzzy least squares method and the fuzzy least squares method based on different kernel functions in calculating the distance between fuzzy numbers and to obtain the best estimator, the mean square error (MSE) criterion was used as follows (Mansson and Shukur, 2012):

$$MSE = \frac{1}{n} \sum_{i=1}^{n} D_i^2 (\tilde{y}_i . \hat{\tilde{y}}_i) \qquad i = 1.2.3. \dots . n \qquad (9)$$

$$D^2 \left(\tilde{y}_i . \hat{\tilde{y}}_i \right) = \left| y_i^M - y_i^{*M} \right| + \left| y_i^l - y_i^{*l} \right| + \left| y_i^R - y_i^{*R} \right|$$

Where $\boldsymbol{D_i^2}$ represents the distance between two fuzzy numbers between the real fuzzy number and the estimated fuzzy number. The smaller value is the best estimator of the model.

2.10 The formulations of the resulting methods based on kernel functions with fuzzy least squares estimators are as follows:

- Fuzzy Least Squares method with Gaussian kernel (**FLSG**)
- Fuzzy Least Squares method with Triweight kernel (**FLST**)
- Fuzzy Least Squares method with Epanecknikiv kernel (**FLSE**)
- Fuzzy Least Squares method with Uniform kernel (**FLSU**)
- Fuzzy Least Squares method with Biweight kernel (**FLSB**)

3- Discussion of Results

3.1 Simulation

The R 4.2.1 statistical programming was used to write the simulation program. The written program includes four basic stages for estimating the fuzzy regression model as follows:

Step 1: Define default values

At this stage, parameter values are chosen as in Table 1

Table 1: Fuzzy parameter values

	β_0	β_1	β_2	β_3	β_4	β_5
L	-12.4	0.34	-0.36	-0.028	1.97	0.35
M	-10.4	0.38	-0.030	-0.015	2.03	0.48
R	-6.0	0.42	-0.009	-0.008	2.10	0.63

Step 2: Generating data

1- Five independent variables were generated from normal distribution.
2- Different default values for the standard deviation of random errors (σ = 0.1,2) were chosen.
3- Outliers in the data were chosen as (α = 0, 0.10, 0.20, 0.30).
4- Different sample sizes were chosen (n=25, 50, 100, 150).
5- Each experiment was repeated 1,000 times.
6- Fuzzy random error follows the normal distribution.

$$Y_L = -12.4 + 0.34x_1 - 0.36x_2 - 0.028x_3 + 1.97x_4 + 0.35x_5 + \varepsilon_L$$
$$Y_M = -10.4 + 0.38x_1 - 0.030x_2 - 0.015x_3 + 2.03x_4 + 0.48x_5 + \varepsilon_M$$
$$Y_R = -6.0 + 0.42x_1 - 0.009x_2 - 0.008x_3 + 2.10x_4 + 0.63x_5 + \varepsilon_R$$

Step 3: Estimation

At this stage, the estimation process for the regression parameters is performed using the estimation methods of interest, as follows:

- Fuzzy Least Squares Estimator (FLSE) method.
- Fuzzy least squares estimation based on the kernel distance.

Step 4: Comparison step between methods

In the stage of comparison between the methods to compare the different estimation methods for the models and find the best estimators, the mean square error (MSE) criterion was used in equation (9).

The following results have been obtained for Table 2, Table 3, Table 4, and Table 5.

Table 2: MSE for different kernel methods when (α= 0)

σ	Sample Size	FLS	FLSG	FLST	FLSE	FLSU	FLSB
$\sigma = 0.1$	**n=25**	0.38714	0.38856	0.38805	0.38831	0.402578	0.401901
	n=50	0.26012	0.26496	0.26677	0.26508	0.292367	0.29202
	n=100	0.17818	0.22416	0.22121	0.22366	0.172162	0.172462
	n=150	0.14933	0.13432	0.13496	0.13410	0.17028	0.17019
$\sigma = 2$	**n=25**	8.15732	7.90977	7.99684	7.99963	7.716986	7.654253
	n=50	10.9766	9.78180	9.90793	9.84582	10.52324	10.52509
	n=100	3.95531	3.54158	3.46988	3.44314	3.837656	3.863256
	n=150	3.69981	3.47192	3.46800	3.51779	2.57852	2.623859

Table 3: MSE for different kernel methods when (α=0.10)

σ	Sample Size	FLS	FLSG	FLST	FLSE	FLSU	FLSB
$\sigma = 0.1$	n=25	11.77242	10.72412	10.86997	10.81184	11.4688	11.50578
	n=50	8.814746	7.63063	7.66998	7.62651	8.757381	8.798775
	n=100	6.79336	6.36009	6.34040	6.31537	5.66153	5.685438
	n=150	5.303626	4.22901	4.25571	4.25565	5.120462	5.142251
$\sigma = 2$	n=25	15.08173	14.69672	14.89826	14.93616	12.93513	13.01085
	n=50	10.82364	9.78180	9.90793	10.52324	9.84582	10.52509
	n=100	8.73535	8.15505	8.32410	8.31371	7.204448	7.266898
	n=150	7.71823	7.01297	6.89843	6.87943	6.398112	6.479754

Table 4: MSE for different kernel methods when (α=0.20)

σ	Sample Size	FLS	FLSG	FLST	FLSE	FLSU	FLSB
$\sigma = 0.1$	n=25	20.92863	15.25513	15.49348	15.44455	18.22387	18.19134
	n=50	12.21243	11.40573	11.42064	11.39767	10.76698	10.80429
	n=100	8.62948	7.07238	7.02521	7.01354	8.361582	8.36174
	n=150	8.37348	6.50582	6.42359	6.43062	8.218509	8.219832
$\sigma = 2$	n=25	22.24873	18.43374	18.58039	18.56289	18.5436	18.43728
	n=50	13.83144	12.80867	12.85548	12.80228	12.91759	12.95024
	n=100	9.65391	9.55407	9.49532	9.45657	8.225185	8.980115
	n=150	9.00556	8.965848	8.819413	8.795122	8.179773	8.298138

Table 5: MSE for different kernel methods when (α=0.30)

σ	Sample Size	FLS	FLSG	FLST	FLSE	FLSU	FLSB
$\sigma = 0.1$	n=25	22.92355	21.94186	21.40251	21.61535	21.54326	21.63817
	n=50	15.94135	13.51254	13.21953	13.24130	14.90031	14.89318
	n=100	14.04288	11.18523	10.83809	10.85877	13.22415	13.2244
	n=150	12.30512	11.02033	10.69609	10.69343	10.77121	10.83296
$\sigma = 2$	n=25	30.38427	28.80604	28.09751	28.10713	22.14557	22.16284
	n=50	15.07804	14.46584	14.00513	13.95232	14.84458	14.88321
	n=100	13.85176	11.55813	11.24946	11.25030	12.82928	12.84501
	n=150	11.63028	11.45265	11.16694	11.12651	10.74069	10.77846

Discuss the results based on the information presented in the table.

- In all cases, the fuzzy least squares method based on the kernel distance outperforms the fuzzy least squares method (FLS).
- Table 2 shows the remarkable convergence of the results of the MSE values between the methods used in the absence of outliers for all sample sizes and with different variance values.
- The results of Tables (3, 4, and 5) showed the least squares method based on kernel functions is better than the FLS method in the case of differing ratios of outliers, sample sizes, and variance values.
- The MSE value increases by increasing the standard deviation.

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4. Conclusions

Through the results presented previously, several conclusions were achieved as below:

- The fuzzy least squares method based on different kernel functions, gave better results when compared than the FLS method.
- When there are no outliers and a difference in the variance value and sample size, the results converge for the two methods used.
- The results showed that the methods used in this research gave better estimation results when using the new distance between fuzzy numbers in the presence of outliers.

Authors Declaration:

Conflicts of Interest: None

-We Hereby Confirm That All The Figures and Tables In The Manuscript Are Mine and Ours. Besides, The Figures and Images, Which are Not Mine, Have Been Permitted Republication and Attached to The Manuscript.

- Ethical Clearance: The Research Was Approved By The Local Ethical Committee in The University.

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بعض مقدرات المربعات الصغرى الضبابية لأنموذج الانحدار الضبابي باستخدام دوال كيرنل المختلفة

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مستخلص البحث:
في هذا البحث تم تقديم طريقة لحساب مسافة جديدة بين الأعداد الضبابية باستخدام دوال مختلفة للنواة بالاعتماد على طريقة المربعات الصغرى الضبابية. تم تقدير معلمات أنموذج الانحدار الضبابي في حالة كانت المتغيرات التوضيحية غير غامضة والمعلمات ضبابية ومتغير الاستجابة ضبابي ويحتوي على القيم الشاذة. ومقارنتها بطريقة المربعات الصغرى الضبابية (FLS) وتمت المقارنة بين هذه المقدرات باستخدام معيار متوسط مربع الخطأ (MSE) عن طريق المحاكاة وعبر أحجام عينات مختلفة (25، 50، 100، 150) وقيم شاذة مختلفة (0، 0.10، 0.20، 0.30). وقد تبين أن الطريقة المقترحة باستخدام المسافة الجديدة حققت أفضل النتائج في ظل وجود القيم المتطرفة.

نوع البحث: ورقة بحثية.
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