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The Bi-level Programming Approach to Improve the Inventory Control System with a Practical Application

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Abstract:

In this research, we investigated addressing the challenges associated with the seasonal allergic medical drug inventory system. The focus was on determining the optimal demand by calculating the Economic Order Quantity (EOQ) and achieving the lowest cost within the Pharmaceutical Industries and Medical Supplies company in Samarra. The primary objective was to efficiently meet the demand for seasonal allergy medications by identifying the optimal demand for medical drugs. The study encompassed two types of seasonal allergy medications, namely Samatifen drink and VALIAPAM 2 pills. The calculation of the lowest cost involved two methods: the multi-component production model without deficit, with restrictions, the solution was done using the classical method (normal) and mathematical analyses, utilizing tools such as QM and Win Qsb. Additionally, linear quadratic bilevel programming (LQBP) was employed. The LQBP model comprised an upper-level decision maker (leader) and a lower-level decision maker (follower). The transformation of the bilevel model into a single-level model was accomplished through the application of Karush-Kuhn-Tucker (KKT) conditions, and the solution was obtained using the modified simplex algorithm. The study's findings underscore the effectiveness of the LQBP method in identifying the optimal solution for the inventory problem by calculating the lowest cost. This approach significantly reduced medical drug inventory-related costs, with a value of 1,496,700,000 Iraqi Dinars (ID) and a production of 1200,700 ID. Notably, this cost was considerably lower than the total cost value obtained using the classical method, which was 1,719,166 ID/year, with a production of 2526,1773 ID. Therefore, bilevel programming (BLPP) demonstrates superior efficiency, providing more accurate and cost-effective solutions. This research emphasizes the potential of bilevel programming in optimizing medical drug inventory systems and contributes to the advancement of operational research in the healthcare sector.

Paper type: Research Paper
Keywords: The linear-quadratic bi-level programming (LQBP), Modified Simplex Algorithm, Karush-Kuhn-Tucker conditions (K.K.T), Multi-item production model (no shortage) With -Restriction ,The Classic method.

1.Introduction:

Inventory is defined as the sum of the Retained of raw materials, intermediate materials, and final goods owned by the company or company produce it. So, Inventory receives great attention from many parties, due to its many forms and uses. the decision-making process at the level of the relevant entity requires that the management of that entity maintain a stock of the raw and intermediate materials produced that deal with that Stock for medical drugs Seasonally, it is an old problem and there are limited solutions available. The tremendous progress in the field of bilevel programming (BLP) has sparked interest in finding innovative solutions to this problem. so, the stock of medical drugs one of the important problems facing manufacturers, pharmacies and medical centres is that a full stock of medical drugs leads to prohibitive costs and excessive consumption of resources. When demand decreases while the limited storage may expose patients to the risk of not having the necessary medicines available in hospitals to treat them upon request. so, it was lost Seasonal prescription drugs, specifically, presented a particularly challenging inventory problem for organizations, given changing demand. In this context, the General Company for the Manufacture of Pharmaceuticals and Medical Supplies in Samarra faces great challenges in managing its stock of medicines such as (syrup Samatifen and Valiapam tables) therefore the development in the field of electronic computers helped the researcher one operations research specialists complete all analyzes and studies required by the research very quickly. This development contributed to the emergence and development methods A new solution aimed at solving such issues that were difficult to reach solve it and find the optimal solutions for them. This research aims to shed light on optimal strategies to address the complexities in inventory control by ensuring the optimal economic demand for seasonal medical drugs, the...purpose it is to increase efficiency, reduce costs, and ensure patient safety by providing the necessary medications in the time you save.To find the optimal strategy for the storage problem by finding the lowest cost, the researcher used two solution methods. The first method included using the classical (ordinary) method of a multi-component gradual (without deficit) production model.(Production without Shortage Model for multi item)While the second method included the use of the Bi-level linear-quadratic programming method Linear Quadratic Bi-level Programming (LQBP), so It is one of the complex and difficult problems of this type (NP-Hard) It is represented by the presence of two decision makers, the decision maker at the top level of the independent variable(Leader) goal reduce costs while the decision maker is the lowest level of the dependent variable(Follower) GoalH Quantities and prices set by the importing company and provided by the company supplying the drug which in general, it is convex and non-differentiable, so it is difficult to solve directly. To draw conclusions from our Bi-level programming model, we transformation the model is transformed into a one-level model, using KKT conditions. This transformation resulted in a mathematical program with complementarity constraints (MPCC) (mathematical program with complementarity constraints). For the arithmetic solution, has been used functions the penalty proposed, where the individual problem is transformed restricted to an unconstrained problem and then solve it modified simplex algorithm method. The proposed approach achieves an effective solution, and it is worthwhile, and it will be programmed in a software environment in language R. Ready-made statistical programs were used (QM, Win QSB, R Programming) to conduct the necessary mathematical and statistical analyses. Our analysis is also based on real data regarding the production and demand of medicines from the mentioned company. Our results confirm the effectiveness of the (LQBP) in getting the perfect solution to the inventory problem. Preliminary results indicate a significant reduction in inventory-related costs while ensuring the availability of medical drugs. So, when Comparison Combined with the classic regular method, the Bi-level programming model displays superior efficiency, resulting in more accurate and cost-effective inventory control solutions determine the optimal demand for selected medical drugs.

1.1. Literature Review:

Bilevel programming has been a subject of research interest for many years, attracting numerous researchers seeking to identify optimal inventory control strategies. This body of research has yielded several innovative solutions to scientific problems. Several studies have explored the application of bilevel programming, including:Yibing et al (2007) proposed an approach for deriving the optimal solution to a bilevel programming problem by employing the KKT optimality conditions for the lower-level problem. This approach effectively transforms the bilevel programming problem into a single-level programming problem .Iqbal and Kamal (2012) investigated the application of bilevel programming to address real-world challenges. They developed two solution algorithms that utilize KKT conditions to transform nonlinear bilevel problems into linear problems. Heba (2020) conducted a comprehensive study of bilevel programming, examining the branch-and-select algorithm, the penalty function method, the Taylor method, and the confidence zone method. Utilizing real data from a pharmaceutical and medical supplies marketing company, the study concluded that the modified branch-and-select algorithm emerged as the optimal solution, consistently generating the lowest objective (cost) function values across different sample sizes, including small, medium, and large-scale samples.
Reza et al. (2021) focused on the critical aspect of renewable energy siting. To address uncertainty, they employed robust bilevel programming and game theory (specifically, Stackelberg competition). A robust stochastic method was implemented to enhance the model's resilience to factors such as noise and perturbation, followed by the application of the (KKT) method to solve the bilevel programming problem. The findings demonstrated that incorporating uncertainty considerations can lead to improved power generation and increased profits for suppliers. Research on inventory control has also yielded valuable insights, including : Omar (2008) conducted a study on solving the fixed periodic review model for the inventory problem using dynamic programming. The study was applied to Al-Aqsa Company, which procures generators from abroad, by analyzing the associated costs. The results indicated that applying dynamic programming to the model provides various alternatives that yield the same optimal total variable cost, allowing management to select the most suitable option. Panne et al. (2009) focused on applying genetic algorithms to solve storage models. Abdullah (2017) conducted a research study aimed at determining the optimal production and demand quantities for a 330 ml can of Pepsi and its basic components in the Baghdad Soft Drinks Company. Operating in an environment characterized by extreme uncertainty and volatility due to fluctuating demand quantities and storage costs, the researcher employed the fuzzy time series method. The fuzzy inference method was utilized to address the dispersion and uncertainty surrounding the cost of holding inventory for the final product. The research findings highlighted the significance of applying fuzzy set theory to mitigate the effects of environmental fluctuations. The results demonstrated the effectiveness of using fuzzy methodologies in inventory management under uncertain and volatile conditions, leading to improved production and demand decisions in Baghdad Soft Drinks Company. Sayal (2018) studied an inventory model that considered both explicit and fuzzy systems. The primary objective was to optimize the total production cost and determine the optimal order quantity for the sub-stock system inability. Several studies have combined bilevel programming and inventory: Naser Rajabi et al. (2021) presented a bilevel programming model to coordinate pricing and inventory decisions in a competitive supply chain consisting of a dominant producer and two Cournot follower retailers with nonlinear price-dependent demand. The supply chain produces perishable products from perishable raw materials that decay at a certain rate inside the warehouses. Three levels of warehousing were considered: raw material warehouse, final product warehouse, and retail warehouses with exponential deterioration rates. A Stackelberg–Nash–Cournot game model was developed, and equilibrium wholesale and retail prices, order quantities, lead times, and replenishment cycles were obtained using an exact methodology. The effectiveness of the modeling approach was evaluated through a numerical experiment, and the sensitivity of the equilibrium was analyzed.

The results showed that an increase in deterioration rate at retail warehouses forces the retailers to offer lower prices to raise demand and prevent the product loss. As prices fall and deterioration costs increase, the chain's profit declines. Higher levels of demand can cause higher levels of production and inventory costs for the producer, leading to a decrease in the producer's profit. A growth in the price elasticities implies a reduction in pricing power of the retailer, which can consequently lead to a lower profit for the whole chain. Fatima Ezaahra Achamrah et al. (2022) addressed the challenges related to efficient routing of multiple products and vehicles in decentralized supply chains. The supply chain includes a manufacturer that sells a variety of products through a network of independent point-of-sale (POS) locations. The study utilized a two-stage mixed-integer procedure with the manufacturer acting as the leader of a Stackelberg game to optimize inventory and routing. The study introduced a novel method that combines hybrid genetic algorithm (HGA) with deep reinforcement learning. The calculated results showed that manufacturer involvement in transshipment and lost sales costs can significantly improve the service level of the network. Additionally, combining HGA with reinforcement learning was found to be effective in improving solution quality and reducing computational time. Future research should focus on evaluating the practical applicability of these methods using real data and exploring their use in different problem scenarios.

The problem of this study for this research is how to find the optimal demand by calculating the optimal economic size and finding the lowest cost with accuracy and proven validity for seasonal medical drugs, (Samatifen Syrup and VALIAPAM tables) for the general company for Pharmaceutical Industries in Samarra. This is done using the classical method and the Bi-level programming method, by comparing the solution methods in terms of the value of the total cost to reach the best solution. The research problem ends with the crystallization of important and main questions: when applying the Bi-level programming model, does this contribute to addressing the inventory problem, achieving optimization, and reaching the optimal demand that gives the lowest cost with acceptable solutions? The current research is important in academic and practical applications because it has many scientific and applied contributions, where the first objective in current research is a modest contribution to the research library in the context of inventory control, especially with the lack of studies that have addressed this topic in the field of medical drugs Seasonality, the research purpose to find the optimal strategy for the inventory system for seasonal medical drugs (allergies), by finding the optimal demand for these drugs in the general company for the manufacture of pharmaceuticals and medical supplies, Samarra, and the second objective is applied contribution, the current research contributes to improving the efficiency of the inventory system at the general Company for the Manufacture of Pharmaceuticals and Medical Supplies, Samarra, by creating the optimal demand for seasonal medical medicines, which helps the company reduce inventory costs and ensure the availability of medicines for patients.
The results showed that the bi-level programming method is the optimal strategy for inventory, as it gave the best results.

2. Materials and methods:

Monthly data for seasonal medical drugs were collected through personal interviews with knowledgeable managers and department heads in administration, marketing, stores, planning, costing and quality control. These interviews allowed us to collect accurate data and engage in meaningful discussions that enabled us to conduct a comprehensive and objective analysis. By collecting information and data directly from trusted experts for 2022, we were able to ensure the accuracy of our results and arrive at a robust, logical, and scientific analysis. Data from manufacturer records for two types of seasonal sensitive medications were also analyzed under study, one syrups, while the two medicine is pills. This data will be used to determine the optimal amount of medication that should be manufactured to meet consumers' needs. Data includes: type1: Samatifen syrup, used for secondary prevention of cardiovascular disease risks. Type 2: VALIAPAM tablets, used to treat anxiety disorders. Relieve muscle spasms. Treat some types of seizures. Medicines are stored in warehouses on morality wooden and moderate temperature. The data was collected and analyzed using two distinct methods to find the difference between them in terms of the lowest cost to find the ideal order. The first method is to use the incremental storage model for production without shortages multi-element. It was completed use the classical (ordinary) method in the solution to analyze the available data from Quantities of storage, where it was found that the total cost is (719166 /ID) While the second method involved using LQBP. Where a bi-level model of the problem is formulated stock medical drugs, where be the top-level leader linear model while be the follower the level lower Quadratic represents Quantities.

The model was transformed into a single-level nonlinear model using terms KKT, and was used penalty function proposed, where the single constrained problem is transformed into an unconstrained problem and then solved using a modified simple algorithm and a genetic algorithm (Modified Simplex Algorithm), then programmed it using the programming language (R). Total cost results (1496700000 ID) were obtained. The results showed that the method (LQBP) is the optimal method because it gave the lowest total cost of seasonal medical drugs in the drug stock for company general authority for pharmaceuticals and medical supplies manufacturing, Samarra compared to the classic (normal) method. Therefore, the following statistical programs (QM, Excel, R Programming, Win QSP) are used for data collection and analysis. In short, the research found that the method (LQBP) is the ideal method for quantification at optimum seasonal medicinal drugs to be manufactured to meet the needs of consumers. This method has been found to give the lowest overall drug cost compared to the classical (regular) method.

2.1 Inventory Model:

Inventory models are mathematical models used to predict the optimal level of inventory a company should hold to maximize profits It appears when the accumulation of materials or goods occurs over a specific or indefinite period for the purpose of meeting demand. These models consider numerous factors such as product demand, lead time for inventory replenishment, and cost of holding inventory. Appropriate decisions are made regarding the quantities of materials required and the timing of reinforcement of the pile is a key factor in ensuring continuity of operations. Come the importance of these decisions stems from the urgent need to balance demand requirements and available inventory. It is possible to strive to store a quantity that meets the need for a specific period, or to focus on providing the quantities needed for each period without exceeding or not inability. It is possible to aim to store a quantity that meets the need for a specific period, or to focus on providing the quantities needed for each period without the increase or lack Inventory. (Hamid, 2010). points out that inventory control has long been occupied an important site among the most important topics in operations research (Ilya, 2019). The history of the mathematical theory of inventory and production can be traced back to 1888, when it was developed walrus A variation of the magazine seller's model formula for cash flow in a bank. However, the first official model was developed to help the tide determine the appropriate size and timing to replenish inventory (Harris, 1913). Quantity model command the economic (EOQ) is the first and still the most important. Inventory includes materials that are stored to be sold or used in the manufacturing process, or that have not yet been consumed or used .The definition given by the Operational Research Society of Great Britain is an Operations research is the application of science methods to temporary problems that arise in the direction and management of large systems of materials and money in industry, government, business, and defense (A. Ravi Ravindran ,2008)). Researchers addressed the objectives of inventory control:

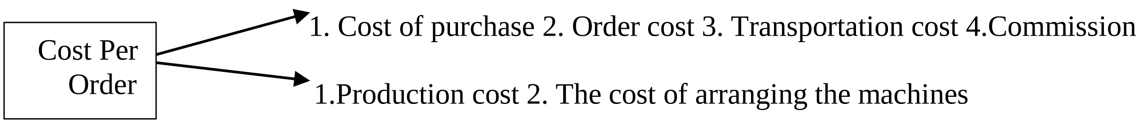
1. To ensure adequate supply of products to customers and avoid shortages as much as possible.
2. To ensure that the financial investment in stocks is minimal (i.e., to ensure that capital is blocked Working money to a minimum.
3. Efficient purchasing, storage, consumption, and accounting of materials is important.
4. Maintain a timely record of inventories of all items and maintain stock within
5. Required limits.
6. To ensure timely work of the refurbishment.
7. To provide buffer stock for variations in material delivery lead times.
8. Providing a scientific basis for both short-term and long-term material planning.

To provide a scientific basis for both short- and long-term physical planning (Anil. S, 2008). the main goal of having an inventory system is to maintain a sufficient and appropriate level of inventory. By storing inventory, a company or organization can protect itself from fluctuations short in supply and demand and making sure it gets the materials it needs when it needs it in the future. In addition to meeting the immediate needs of a company or organization, an effective inventory system can also help reduce costs by reducing .the risks and enable more efficient production processes. Inventory management plays a key role in business operations, including purchasing, storing, and using materials and supplies sufficient stock for work .Where Its importance lies in ensuring that the company maintains an appropriate level of inventory to efficiently meet customer demand (Jonathan ,2006).

2.2.Inventory Costs:

Inventory costs are associated with each quantity of inventory. Some of these costs depend on the volume of goods stored, and some of them are fixed and called fixed costs. Costs that depend on the quantity of the order are called variable costs. Therefore, inventory cost per unit time equals purchasing cost (monetary units) + ordering cost + holding cost per unit time + stock cost inability) per unit of time. Therefore, all the above costs will be briefly explained as follows (Nigel, 2007). Purchase cost or production cost refers to the total cost incurred by a company or organization in producing a product or providing a service. It includes direct costs associated with the production process, such as raw materials, labor, and energy, as well as indirect costs, such as overhead and indirect labor. (Financial Statements Tableau Exchange) order costs which are incurred Per order, which is a fixed cost that occurs when the order is placed (Donald ,2003).

This setup cost gradually decreases as the order quantity increases, starting with the issuance of the purchase order and ending with the arrival of the goods in inventory. It also includes expenses related to preparing and receiving applications, including the costs of preparing, receiving, and printing documents. It also includes the salaries of employees involved in these operations (gift, 2020). The usual unit of measurement for this cost is (dinar per course).



holding costs, also known as inventory costs, are the costs associated with storing and maintaining inventory. These costs can include the cost of the physical space required to store inventory, the cost of insurance, the cost of inspection, the cost of machinery, the cost of refrigeration, as well as the cost of insurance, taxes, on-site employee salaries and other related expenses. The cost of a shortage of inventory is the cost incurred by a company or individual because there is not enough of a particular product or resource in stock when it is needed. This can happen for several reasons, such as a sudden increase in demand for a product, unexpected delays in the supply chain, or production problems. The unit of measurement is (dinar per unit and per unit of time) (Frederick, 2021).

2.3. Derivation of the quantity model Command Economic (EOQ):

Total inventory cost per cycle = setup cost + holding cost

$$T.\frac{c}{cycle} = k + h\frac{L}{2}t_1 + h\frac{L}{2}t_2 \quad (1)$$

$$T.\frac{c}{cycle} = k + h\frac{L}{2}(t_1 + t_2) \quad (2)$$ instead

$t = t_1 + t_2$

$$T.\frac{c}{cycle} = k + h\frac{L}{2}t \quad (3)$$

whereas $L = t_1(p-d)$, and $t_1 = \frac{L}{p-d}$

by substituting t1 by taking the large triangle with base (t1) and applying the Pythagorean theorem as follows, $L = \frac{Q}{P}(P-d) = \left[1 \mapsto \frac{P}{d}\right] = b \quad L = Qb$,therefore, the total inventory cost in Eq.(4) per cycle is equal to:

$$T.C/_{perunit\ time(z)} = K + h\frac{Qb}{2}\ t \quad (4)$$

It is best to extract the total cost of storage per unit of time, as it is the most beneficia in Eq.(5):

$$\frac{T^*}{perunit}time(z) = \frac{kd}{Q} + h\frac{Qb}{2} \quad (5)$$

For the cost equation to be as low as possible to obtain the optimal quantity for quantity therefore, we must find the formula of the expression which is expressed by the first derivative with respect to and make this expression Equal to zero as follows in Eq.(6) :

$$\frac{\partial Z}{\partial Q} = -\frac{Kd}{Q^2} + \frac{hP}{2} \quad (6)$$

$$\frac{\partial Z}{\partial Q} = 0$$

$$\frac{Kd}{Q^2} = \frac{hb}{2}$$

$$Q^*(EOQ) = \sqrt{\frac{2kd}{hb}} \quad (7)$$

Optimal order quantity or economic quantity, while (b = 1-p/d) (first production cycle)
k = setup cost,
d = monthly demand rate
P = monthly production rate,
h = holding cost

2.4. Multi Item Inventory Models

It is natural that when the inventory contains more than one item, the second case will appear, which is the appearance of the constraints in the inventory process that disappeared when we had one item, as the availability of different items at the same time requires coordination and planning:

1.Items	1,2,3 ….j ….n
	$K_1, K_1, K_2, \ldots . K_j \ldots . K_n$
2. setup cost	$d_1, d_2, d_3, \ldots . d_j \ldots . d_n$
	$P_1, P_2, P_3 \ldots . P_j . \ldots P_n$
3.Demand rate	$h_1, h_2, h_3, \ldots . h_j \ldots . h_n$
	$Z_1, Z_2, Z_3, \ldots . Z_j \ldots . Z_n$
4. product rate	

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5. Holding Cost

6. Total cost per unit time

2.4.1. Multi Items Inventory Models With no –Restriction:

$$Q_j = \sqrt{\frac{2k_j d_j}{h_j b_j}} \tag{8}$$

$$\sum_{i=1}^{n} Z_j = \sum_{i=1}^{n}\left[\frac{K_j d_j}{Q_j} + \frac{h_j Q_j b_j}{2}\right] \tag{9}$$

$$b_j = 1 - \frac{P_j}{d_j} \tag{10}$$

2.4.2. Multi Items Inventory Models With –Restriction:

There are several constraints that determine the inventory process, as inventory cannot be infinite. Therefore, to provide materials at any time when demand is extremely high, we will explain a section of them, and therefore we will discuss (Shamrti ,2010):

1. **Production without Shortage Model for Multi Items:**

The model is called the progressive replenishment model, which enhances inventory in the event of product depletion (i.e., reaching zero inventory). The real and most influential constraints on the storage process are clarified, as shown below: (Shamrti, 2010)

1. (Investment Restriction)

If b_1 represents the largest amount of money allocated to purchase and store materials (investment) to address the critical conditions for which the inventory was placed in the warehouse, therefore the mathematical relationship for it will be as shown in Eq.(11) below (Shamrti, 2010):

$$\sum_{i=1}^{n} C_j\, Q_j \le b_1 \tag{11}$$

And this relationship is intuitive because the product of the unit price and the economic quantity of the product must be less than or equal to the highest amount allocated for investment (b_1) in the storage process. However, if the investment ($\sum_{i=1}^{n} C_j\, Q_j$) amount exceeds the allocated financial amount for investment (b_1), the Lagrange Equation is applied. Through this equation, we can find a new optimal economic quantity that differs from its predecessors to meet the condition. Based on these quantities, the total cost is calculated, as illustrated in Eq.(12) below:

$$L(\lambda, Q_1, Q_2, Q_{3,\ldots\ldots\ldots\ldots} Q_n) = T, C.(Q_1, Q_2, Q_{3,\ldots\ldots\ldots\ldots} Q_n) - \lambda\left(\sum_{i=1}^{n} C_j\, Q_j - b_1\right) \tag{12}$$

Where the value of (λ) is bounded between (0,-1) and the optimal value of is extracted by determining the optimal value of from through the repeated estimation method and by repeatedly extracting the value of following the following Eq.(13):

$$Q_j = \sqrt{\frac{2\, K_j B_j}{h_j b_j - 2\lambda C_j}} \tag{13}$$

The repetition continues until reaching the result ($\sum_{i=1}^{n} C_j\, Q_j - b_1$), which equals zero or a negative value, at which point the process stops at the final value of (λ) representing the optimal value. Afterward, the total cost value is calculated based on the extracted values after estimating the (λ) according to Eq. (12)

2. (Capacity Restriction):

(b_2:Max Story area) and (f_j: is thearea required for one item j)

$$\sum_{j=1}^{n} f_j\, Q_j \;\leq b_2 \qquad (14)$$

From Eq.(14) in the case where the space required for each unit of the products to be stored is less than the space allocated to it, the constraint is not binding, and the total cost is calculated using EQ.(9).While if the volume of the storage space for each unit (that needs to be stored) ($\sum_{j=1}^{n} f_j\, Q_j$) is greater than the total storage space allocated to it (b_2) the optimal quantities of inventory and the total cost will change. To calculate them, we apply Lagrange's EQ.(16) , as shown below:

$$L(\lambda, Q_1, Q_2, Q_{3,\ldots\ldots\ldots\ldots} Q_n) = T, C.(Q_1, Q_2, Q_{3,\ldots\ldots\ldots\ldots} Q_n) - \lambda\left(\sum_{i=1}^{n} f_j\, Q_j - b_2\right) \qquad (15)$$

The value of (λ) is also bounded between (0,-1). The optimal value of (Q_j) is calculated by determining the optimal value of (λ) using the repeated estimation method for (λ). This is done by repeatedly extracting the value of (Q_j) according to the following EQ.(16):

$$Q_j = \sqrt{\frac{2\, K_j B_j}{h_j b_j - 2\lambda f_j}} \qquad (16)$$

Until the value of ($\sum_{i=1}^{n} f_j\, Q_j - b_2$) equals either zero or a negative value, at which point we stop at the last value of (λ), which is the optimal value. After that, the total cost value is extracted from the optimal quantities obtained after estimating (λ) according to Eq.(15)

3. (Number of Order Per year Restriction)

The intended meaning here is the number of repeated orders within the year, which is an actual factor hindering the complete arrival of orders to the warehouses. Additionally, these orders have associated costs, along with formal procedures imposed by the government on a certain number of orders, in addition to customs restrictions. The mathematical relationship is further elucidated below Eq.(17) by (Shamarti ,2010):

$$\sum_{j=0}^{n} \frac{D_j}{Q_j} \;\leq b_3 \qquad (17)$$

(b_3: Max . certain No.of order per year) and (D_j: is the annual demand for item (j))

In the case where the annual demand is greater than the maximum number of order repetitions per year, the constraint is not binding. The total cost and optimal quantities are calculated using Eqs. (8) and (9).In the case where the number of orders per year ($\sum_{j=}^{n} \frac{D_j}{Q_j}$) is greater than the maximum number of order repetitions per year (b_3), the optimal quantities of inventory and the total cost will change. To determine these values, Lagrange's equation is applied as follows in Eq.(18).

$$L(\lambda, Q_1, Q_2, Q_{3,\ldots\ldots\ldots\ldots} Q_n) = T, C.(Q_1, Q_2, Q_{3,\ldots\ldots\ldots\ldots} Q_n) - \lambda\left(\sum_{j=0}^{n} \frac{D_j}{Q_j} - b_3\right) \qquad (18)$$

the value of (λ) is different from the previous values and is bounded between (0, -1). The optimal value of (Q_j) is extracted by determining the optimal value of (λ) and then extracting the repeated values of (Q_j) according to the following equation.

$$Q_i = \sqrt{\frac{2(\lambda D_{j-K_jB_j})}{h_j b_j}} \qquad (19)$$

Until the result of $(\sum_{j=}^{n} \frac{D_j}{Q_j})$ equals zero or a negative value, we stop at the extraction of the final value of(λ), which represents the optimal value. Afterward, we calculate the total cost value based on the optimal quantities calculated after estimating (λ) according to Eq. (20) by (Shamarti ,2010).

2.4.3 .Analysis nature Demand

To determine the nature of demand, each of the following must be calculated (Hamdi ,2017) :

1. Average for each month. $x^{-} = \frac{\sum xi}{n}$ (20)
2. Standard deviation $S = \sqrt{\frac{\sum(x_i - x^{-})^2}{n}}$ (21)

3.Coefficient of variation (V) = $\text{Standard deviation}/\text{Mean}$ (22)

After calculating Eqs. (20),(21) and (22) the nature of the request is determined as shown in the table.(1) below.

Nature of the request	Coefficient of variation	Demand
Deterministic and constant	Less than 20%	Static with time
Deterministic and variable	Less than 20%	Dynamic with time
Probabilistic and stationary	Greater than 20%	Static with time
Probabilistic and nonstationary	Greater than 20%	Dynamic with time

Table 1 : Criteria for determining the nature of the demand.

Source: Suleiman (2020)

2.5. The concept of bilevel programming problem (BLBB):

There are many concepts about BLB, and according to different Bi-level views, optimization is a branch of optimization Bi-level, which is BLPP problem is a mathematical program that contains an optimization problem in constraints,These are non-convex optimization problems (non-(convex.-Convex) which takes the hierarchical form and is not differentiable to help the decision maker reach the optimal solution, as BLPP is a nested mathematical optimization technique used to solve decision-making problems with multiple objectives and constraints, so you can generally know that it is an optimization problem constrained by another optimization problem and it is a tool Important for decision-making they are used by decision makers who need to find optimal solutions to complex and uncertain problems have been formulated Stack Elberg In his study of the market economy in 1934 (Zhijun Xiong ,2022). Adopting BLPP makes BLPP more stable and simpler, followers have only one choice i.e., value set by the leader. If it does not the choice is made by follower in, it will be that solving BLPP is more difficult due to the unique solution having a unique follower solution. In this case, these BLPP are divided into two categories based on follower selection, weak BLBB and strong two-level planning programming (Yulan 2008).It is a scientific and practical tool to solve decision-making problems (Ahmed ,2020). In addition to using, it with financial models, it is also widely used in many fields such as economics, transportation, and finance. (Marwan ,2009).

Hierarchical optimization techniques or multilevel programming (MLP) are extensions of Stackelberg games used to solve decentralized planning problems for multiple decision makers (DMs) in hierarchical organizations (Suropati, 2009). Multilevel organization or hierarchical organization, (Benoît ,2007) study for two-level optimization, (Bijay ,2003) In BLPP, the DMs (top and bottom) are in two different hierarchies: each layer independently controls a set of decision variables with different and conflicting goals.

2.5.1. Linear – quadratic bi-level programming (LQBP):

Quadratic programming is a quadratic optimization problem that has been widely used since its development in the 1950s, particularly by Wolff and Frank. It is a simple type of nonlinear programming that can accurately model many convex real-world systems(Eghbal, 2014).

2.5.2. The concepts of the problems:

we are looking at special classes of bilevel programming is Linear Quadratic Bilevel Programming (LQBP). The general formulation of this problem is as follows: we are looking at special classes of bilevel programming: Linear Quadratic Bilevel Programming (LQBP). The general formulation of this problem is as follows (Masatoshi,2012):

$$\underset{x}{Max}\ F\ (x,y) = a^T x + b^T y \qquad (23)$$

S.t

$$\underset{y}{Max}\ f\ (x,y) = (x^T, y^T) Q (x^T, y^T)^T + c^T x + d^T y \qquad (24)$$

$$S.t\ \ Ax + By \leq r$$

$$x, y\ \geq 0$$

wher a, c $\in R^{n_1}$,,,,,, and $b, d\ \in\ R^{n_2}\ A\ \in\ R^{m\times n_1}\ B\ \in\ R^{m\times n_2}\ r\ \in\ R^m\ x \in\ R^{n_1} y \in\ R^{n_2}\ f(x,y)$
$F\ (x,y)$
are the objective function of the leader and follower, respectively. Also is symmetric positive simi-definite matrix.$Q\ \in\ R^{(n_1+n_2)\times(n_1+n_2)}$ Suppose that.

$Q = \begin{bmatrix} Q_2 & Q_1^T \\ Q_1 & Q_0 \end{bmatrix}$, Which $Q_0 \in R^{n_2\times n_2} Q_1 \in R^{n_1\times n_1} Q_2 \in R^{n_0\times n_0}$ Then the follower problem of the LQBP is:

$$\underset{y}{Max}\ f\ (x,y) = d^T y + 2Q_1 xy + y^T Q_0 y \qquad (25)$$

$$S.t.By \leq r - Ax \qquad (26)$$

$$y\ \geq 0$$

2.6. Conditions Karush Kuhn-Tucker (KKT)

Reformulation is also introduced for the primal KKT reformulation and used to recover the M -stationarity conditions (Stephan ,2012). The terms are as follows, summarized in the table. (2) below:

Table. 2: Criteria for determining the nature of the demand.

Required conditions		
Solution space	Objective function	Type of optimization
Convex set	Convex	Maximization
Convex set	Concave	Minimization

Source: Heba (2020)

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Therefore, we conclude that the necessary conditions in the case of minimum are as follows:

- $\frac{\partial f}{\partial x} - \sum_i \lambda_i\, h\,(x) + \sum_i \mu\, gi\,(x)$
- $g_i(x_i) \leq 0$ for $i = 1,2, \ldots m$
- $\mu\left(g_i(x)\right) = 0$ (Complete slackness)
- $\mu \leq 0$
- $x_i \geq 0$

In Problem Eq.(13) the presence of numerous non-linear equality constraints underscores the non-convex nature of the programming involved. This characteristic suggests the existence of local optimal solutions that do not equate to global solutions, thus complicating the resolution of Problem Eq.(13). To navigate this complexity, the following method is proposed: the application of a penalty function to transform Problem Eq.(13) into an unconstrained problem. By applying all constraints to the upper-level objective function, each accompanied by a respective penalty, the subsequent penalized problem is:

$$\max_{x} f(x, y) = a^T x + b^T y - M(2Q_1 x + 2Q_0 y - Bu + v + d) - N(uw + vy)$$

$$\text{st } Ax + By + W = r \quad (27)$$

$$x, y, u, v, w \geq 0$$

Which N is a large positive number and M is a matrix of large positive numbers and $M \in R^{n_1+n_2}$, Where we solve the Eq. (27)

2.7. A real-life example:

2.7.1. State Company For Pharmaceutical and Medical Appliance Industries Samarra:

The company is considered the best and largest local source for providing medical medicines with international specifications and seeks to continue the growth process until it becomes one of the most prominent pharmaceutical companies in the region. The company's headquarters is in the city of Samarra, one kilometer south of the center of Salah al-Din Governorate, one kilometer north of Baghdad and next to the famous Malviya Lighthouse in Samarra. An area of buildings and roofs is about 70,000 square meters, and the factory area is about 520,000 square meters. The company expects that the demand for its products will increase at a growth rate of no less than 10% annually and for the next three years for the reason referred to, in addition to the good reputation that our products enjoy as they depend on Modern pharmacopoeia standards (American). USA and British BP). In addition, the company obtained the international quality certificate ISO (ISO:9001-2008) at the end of 2013. It was completed. (ISO:9001-2015) Work is also underway to obtain the ISO (Environment) (ISO:14001-2015) and Health and Safety (ISO:45001-2018) certificates in 2021 1965 according to the offer concluded with the Soviet Union.

2.7.2. Classic method (ordinary) :

2.7.2.1.Data Collection Method:

To collect data for our studies we used several methods :

1.Presonal interviews with knowledgeable managers and heads of administration marketing ,stores and planning departments allowed us to collect data for the year 2022.

2. Analysis of data by manufacturer records of two types of sensitive seasonal medicines drink and pills.

The economic size(E.Q) and total cost of a multi-item production model without shortage Model for Multi Items are calculated for two types of medications. Unit of measurement of bottle filling=100 mg ,Production rate(P) > Demand rate (d) ,andCoefficient of variation less than 20% for all pharmaceuticals, demand deterministic and variable. Based on the available data, the optimal economic volume and the lowest total cost are calculated, as illustrated below:

A budget of (7000000) thousand Iraqi dinars for the year 2022 to manufacture two types of anti-allergy medicines. The storage capacity allocated for these medicines is (145000) square meters, and the advertising and marketing cost allocated to each unit is (400,000) thousand Iraqi dinars. The annual demand for these medicines is (1000,000) (one million and eight hundred) vials/strips of ten tablets, and the production quantity allocated to the three medicines is (2,322,900) two million and three hundred and twenty-two thousand dinars /Vial/strip 10 tablets.in table. (3)

Table 3: Annual medical drug data year (2022).

	Samatifen(year)	VALIAPAM 2(year)
1	Production rate (P_1)=500000 Unit	Production rate (P_2)=386400 Unit
2	Demand rate(d_1)=100000 Unit	Demand rate (d_2)=80000 Unit
3	First production cycle (b_1)= 0.80	First production cycle (b_2)=0.792
4	Setup cost(K_1)= 11000 ID	Setup cost(K_2)=9400 ID
5	Holding cost(h_1)= 431 ID	Holding cost(h_2)= 604 ID
6	Price per unit C1=900 ID	Price per unit C2=660ID
7	Area (f) =8 cm	Area (f)= 5 cm
8	E.O.Q (Q_1)=2526	E.O.Q (Q_2)=1773

Source: prepared by the researcher based on the outputs (Win QSB and QM) (Microsoft Excel 2022)

The optimal quantities of inventory for two items are calculated using Eq (8) in the normal form.

$$Q_1 = \sqrt{\frac{2k_1d_1}{h_1b_1}} = 2526 \qquad Q_2 = \sqrt{\frac{2k_2d_2}{h_2b_2}} = 1773$$

1. (Investment constraint)

By applying the following Eq.(11) we obtain:

$\sum_{i=1}^{n} C_j\, Q_j$ =900*2526+660*1773 = 3443580 < 7000000

The constraint is not binding, as the product of the optimal quantities, their prices, and their size is less than the maximum investment amount, which is (7000000). To find the total cost using equation Eq.(9)we get:

$$\sum_{i=1}^{n} Z_j = \sum_{i=1}^{n}\left[\frac{K_jd_j}{Q_j} + \frac{h_jQ_jb_j}{2}\right] = 870953 + 848213 = 1719166 \text{ ID/ Year}$$

2. (Capacity Restriction)

By applying equation Eq.(14), we test the equation and obtain the following:

$\sum_{j=1}^{n} f_j\, Q_j$ = 2526*8+1773*5 =29073 < 145000 The constraint holds.

$\sum_{j=1}^{n} f_j\, Q_j < 145000$

The optimal quantity and total cost are calculated by Eqs.(1) and (2).

3. (Number of Order Per year Restriction)

by applying Eq. (17)

$$\sum_{j=0}^{n} \frac{D_j}{Q_j} = 85 < 1000000$$

We conclude that the annual demand is greater than the maximum number of order repetitions per year. Therefore, the constraint is not binding. The optimal quantity and total cost are calculated using Eqs.(8)and (9), which were presented earlier.

$T^* /_{perunit\ time(z)} = \sum_{i=1}^{n} Z_j$=1719166 ID /year

The total cost In the other part, the total cost will be calculated using (LQBB) method and then solve using the developed simple algorithm.

2.7.3. LQBP:(Model)

The mathematical model that describes (LQBP) based on the data available to us for two types of allergies medications in table (3).

$\text{Min } F(S_1, S_2): 825S_1^2 + 630\, S_2^2$ (upper level)

$\text{Min } f\,(S_1, S_2): 800S_1^2 + 600\, S_2^2$ (Lower level)

st:

$850\, S_1^2 + 660S_2^2$	≤ 7000000	(Investment constraint)
$100000S_1^2 + 80000S_2^2$	≤ 1000000	(Space constraint)
$8S_1^2 + 5S_2^2$	≤ 145000	(Demand Constraint)

$S_1, S_2, \geq 0$

By applying the Modified Simplex Algorithm, we obtain:

$L(S_1, S_2, \mu_i) = F(S_1, S_2) + \sum_{i=1}^{m} \mu_i g_i(S_1, S_2)$ (set of feasible solutions for the upper level)

$L(S_1, S_2, \mu_i) = f(S_1, S_2) + \sum_{i=1}^{m} \mu_i g_i(S_1, S_2)$ (set of feasible solutions for the lower level)

Since the values of are known, the Lagrange multipliers (μ_i) are found as follows. $(i = 1,2)$

$$\nabla F(S_1, S_2) = \mu_i \nabla\, g_i(S_1, S_2)$$

$$\nabla F(S_1, S_2)_{(S_1)} = \frac{\partial F(S_1, S_2)}{\partial S_1} = 1650\, S_1$$

$$\mu_1 \nabla\, g_i(S_1, S_2)_{(S_1)} = \frac{\mu_1 \partial\, g_i(S_1, S_2)}{\partial S_1} = 1700\mu_1 S_1$$

$$1650\, S_1 = 1700\mu_i S_1 \longrightarrow \mu_1 = 0.97$$

$$\nabla f(S_1, S_2)_{(S_2)} = \frac{\partial F(S_1, S_2)}{\partial S_2} = 1200S_2$$

$$\mu_2 \nabla\, g_2(S_1, S_2)_{(S_2)} = \frac{\mu_2 \partial\, g_2(S_1, S_2)}{\partial S_2} = 1320\mu_2 S_2$$

$$1200S_2 = 1320\mu_2 S_2 \longrightarrow \mu_2 = 0.90$$

Using KKT conditions following problem is obtained and from the lower level:

$$\mathcal{L}(S_1, S_2, \lambda_i, \mu_i) = f(S_1, S_2) + \sum_i \lambda_i h_i(S_1, S_2) + \sum_j \mu_i g_i(S_1, S_2)$$

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$\mathcal{L}(S_1, S_2, \mu_i) = f(S_1, S_2) + \mu_1 g_1(X) + \mu_2 g_2(X)$

$$L(S_1, S_2, \mu_i) = 800S_1^2 + 600S_2^2 + \mu_1(850\,S_1^2 + 660S_2^2 \,-7000000) + \mu_2(100000S_1^2 + 80000S_2^2 - 1000000) + \mu_3(8S_1^2 + 5S_2^2 - 145000)$$

Condition 1: $\frac{\partial L\,(X,\mu)}{\partial S} = 0$

$\frac{\partial L}{\partial S_1} = 2*\ 800S_1 + \mu_1(2*850S_1) + \mu_2(2*100000S_1) + \mu_3(2*8S_1)$

$=16000S_1 + 1700\,\mu_1 S_1 + 200000\mu_2 S_1 + 16\mu_3 S_1 \quad =0$

$\frac{\partial L}{\partial S_2} = 2*\ 600S_2 + \mu_1(2*660S_2) + \mu_2(2*80000S_2) + \mu_3(2*5S_2)$

$= 1200S_2 + 1320\mu_1 S_2 + 160000\mu_2 S_2 + 10\mu_3 S_2 = 0$

Condition 2: $g_i(S_1, S_2) \leq 0$

$850\,S_1^2 + 660S_2^2 \qquad \leq 7000000$

$100000S_1^2 + 80000S_2^2 \qquad \leq 1000000$

$8S_1^2 + 5S_2^2 \qquad \leq 145000$

Condition 3: $\mu_i g_i(x) = 0$

$\mu_1 g_1 = 0 \longrightarrow \mu_1(850\,S_1^2 + 660S_2^2 - 7000000) = 0$

$\mu_2 g_2 = 0 \qquad \mu_2(100000S_1^2 + 80000S_2^2 - 1000000) = 0$

$\mu_3 g_3 = 0 \longrightarrow \mu_3(8S_1^2 + 5S_2^2 - 145000) \ = 0$

Condition 4 : $\mu_1, \mu_2, S_1, S_2, \geq 0$

Final version of the model:

Min $f(S_1, S_2)$: $800S_1^2 + 600S_2^2$

S.to

$16000S_1 + 1700\,\mu_1 S_1 + 200000\mu_2 S_1 + 16\mu_3 S_1 \quad =0$ (1)

$1200S_2 + 1320\mu_1 S_2 + 160000\mu_2 S_2 + 10\mu_3 S_2 \quad = 0$ (2)

$850\,S_1^2 + 660S_2^2 \qquad \leq 7000000$ (3)

$100000S_1^2 + 80000S_2^2 \qquad \leq 1000000$ (4)

$8S_1^2 + 5S_2^2 \qquad \leq 145000$ (5)

$\mu_1(850\,S_1^2 + 660S_2^2 - 7000000) = 0$ (6)

$\mu_2(100000S_1^2 + 80000S_2^2 - 1000000) = 0$ (7)

$\mu_3(8S_1^2 + 5S_2^2 - 145000) \ = 0$ (8)

$\mu_1, \mu_2, S_1, S_2, \geq 0$

The optimal quantity and total cost for each drug were obtained using the modified simplex algorithm, the steps for algorithm :

Let the main iteration number ,k=0 and the objective function value at the optimal solution at the k-th iteration

$Z_0^* = -\infty$

Step 1: If Ax+By+W=r in the problem (27) is infeasible go to Step 5.Otherwise find an arbitrary basic feasible solution of Ax+By+W=r . Let (X_B, Y_B, W_B) be the associated basic inverses H^{-1}.The variables are divided into two separates classes basic and non-basic variables can be written according to the non-basic variables as follows:

$\begin{bmatrix} X_B \\ Y_B \\ W_B \end{bmatrix} = H^{-1}(b - P_N X_N - Q_N Y_N - I_N W_N)$ where P_N, Q_N, I_N are matrixes which correspond with the columns of the non-basic variables X_N, Y_N, W_N respectively .

Step 2: By replacing equation $\begin{bmatrix} X_B \\ Y_B \\ W_B \end{bmatrix} = H^{-1}(b - P_N X_N - Q_N Y_N - I_N W_N)$ in to of the objective function of the problem (27),the objective function can be written as follows:

$$z = z_0 + a^T x + b^T y - M(2Q_1 x + 2Q_0 y - Bu + v + d) - N(uw + vy)$$

which z_0 is the current value of the objective function .For all non-basic variables we calculate $Z_j - C_j$.

According to the usual rule in the simplex method if all $Z_j - C_j$ are positive, the simplex method will be finished, and we go to step 4.Otherwise go to step3.

Step 3:According to the usual rule in the simplex method enter the non-basic primal variable with the smallest.

$Z_j - C_j$ or the non-basic dual variable with the largest $Z_j - C_j$into the basis. Also, the leaving variable is determined using the usual minimum ratio rule: Then go to Step 1

$$\frac{r_k}{y_{kj}} = \min\left\{\frac{r_i}{y_{ij}} | y_{ij} \geq 0, j \in N\right\}$$

Step 4: If Z_K^* involves M or N ,go to Step 5.Otherwise let k=k+1 , $Z_0 = Z_K^*$ and go to step1.

Step5: If k=0 then the problem (27) is infeasible. Otherwise, the obtained solution at the last iteration is the optimal solution.

which was programmed using R _programming as shown in table .(4) below :

```
#code
rm(list = ls())#Clear objects.
cat("\f")#Clear commands.
options(warn=-1)
library(nloptr)
library(pracma)
C=c(825,630)
nC=length(C)
A.eq=matrix(c(16000,1200,1700,1320,200000,160000,16,10),nrow=nC)
a.eq=matrix(c(1600,1200))
A.in=matrix(c(850,100000,8,660,80000,5),ncol=nC)
b.in=matrix(c(700000,100000,145000))
ss=c(2526,1773); uu=c(0.97,0.90,0.95); z0=145000
x0=c(ss,uu)
# Define the objective function to minimize
objective_function=function(S,µ){
  FF=0;
  for(i in 1:nC){
    FF=FF+C[i]*S[i]^2  }
  return(FF+sum(µ)*0)  }
# Define the equality and inequality constraints
equality_constraints=function(S,µ){ res=NULL
  for(i in 1:nC){ res=c(res,S[i]*(a.eq[i]-sum(A.eq[i,]*µ))) }
  return(res)}
inequality_constraints=function(S,µ){
 nµ=length(µ)
 res=NULL
 for(i in 1:nµ){
res=c(res,sum(A.in[i,]*S^2)-b.in[i]) }
 return(res)}
equality_constraintsµ=function(S,µ){
```

```
 nμ=length(μ)
 res=NULL
 for(i in 1:nμ){
 res=c(res,μ[i]*(sum(A.in[i,]*S^2)-b.in[i]))}
 return(res)}
ConOptim=function(x0,OBJ,InEq,Eq,JacobEq){
  local_opts=list("algorithm"="NLOPT_LN_BOBYQA","xtol_rel"=1e-6)
  lower_bounds=rep(0,length(x0))
  upper_bounds=c(rep(Inf,nC),rep(1,length(x0)-nC))
  res=nloptr(x0=x0,eval_f=OBJ,
        eval_grad_f=GradObj,eval_g_ineq=InEq,
        eval_g_eq=Eq,eval_jac_g_ineq=JacobInEq,
        eval_jac_g_eq=JacobEq,
        opts=list("algorithm"="NLOPT_LD_AUGLAG",
               "local_opts"=local_opts,
               "lower"=lower_bounds,
               "upper"=upper_bounds))
  xx=(res$solution)
  xx[1:nC]=round(xx[1:nC]*nC)
  zz=res$objective/z0
  return(list(xx,zz))}
####
OBJ=function(x){
  nx=length(x)
  S=x[1:nC]
  μ=x[(nC+1):nx]
  objective_function(S,μ)}…
result=ConOptim(x0,OBJ,InEq,Eq,JacobEq)
ss=result[[1]]
zz=result[[2]]
print(ss)
print(zz)
```

Table 4 : The total cost and the minimum quantity produced for each medication(Samatifen and VALIAPAM) for year (2022)

Best solution by LQBP(ID)		
(S_1^*, S_2^*)	(μ_1, μ_2)	Objective function(Z^*)
(1200,700)	(0.97 ,0.90)	1496700000

Source: prepared by the researcher based on the outputs (R Programming) (Microsoft Excel 2022)

The results extracted above indicate that the optimal quantities to produce the required drugs using the LQBP method, which is necessary to be determined by the manufacturer accurately and at the lowest costs for two types of seasonal allergy drugs (VALIAPAM 2 & Samatifen) are as follows:

1. The optimal amount of Samatifen 100 mg is (1200) vials .
2. The optimal amount of (VALIAPAM) 2 mg is (700) cartons of strip 10 tablets .
3. The lowest total cost achieved for all medicines is (1496700000)ID.

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3. Discussion of Results :

In this thesis, an analytical study was addressed in the application of both the classical (normal) method on a multi-element (deficit-free) gradual production model, and the results showed that the lowest amount of the drug (Samatifen) (Q_1) is (2526) while the lowest economic quantity of the drug (VALIAPAM 2) (Q_2) is (1773) as well as the lowest total cost achieved is (Z^*) (1719166) ID /year, which gave the lowest results unlike the second method of linear quadratic bi-level programming (LQBP) which gave the lowest total cost achieved for the stock of medical drugs in the State Company for the Manufacture of Medicines and Medical Appliances Samarra and for each drug (Samatifen) (S_1) is (1200) vials while the drug (VALIAPAM 2) (S_2) is (700) cartons of strip 10 tablets as well as the lowest total cost achieved for all medicines is (Z^*) (1496700000) ID/year, where It was found that the two-level programming method puts a lot of flexibility to decision-makers in determining the optimal quantities for drug orders, which are determined by the company (the lowest costs for medicines) in finding the optimal strategy for the storage problem, unlike the results of the classical (normal) method, where it gave high cost solutions in reaching the optimization in the analysis of data on medical drugs.

4. Conclusions:

The results showed from table.(4) after using the method programming (LQBP) then solve by using the modified simplex algorithm to and using the classic (normal) method. The researcher found that method LQBP a lot of flexibility on decision makers in determining the lowest cost and optimal quantities of drug orders that are determined by the company ,because it gave the lowest cost for medicines (1496700000ID) in finding the optimal strategy for the storage problem, in contrast to the results of the classical (ordinary) method, which gave irrational solutions (prohibitive cost) (1719166ID)in reaching the demand optimization in analyzing data on medical drugs.

Authors Declaration:

Conflicts of Interest: None

-We Hereby Confirm That All The Figures and Tables In The Manuscript Are Mine and Ours. Besides, The Figures and Images, Which are Not Mine, Have Been Permitted Republication and Attached to The Manuscript.

- Ethical Clearance: The Research Was Approved By The Local Ethical Committee in The University.

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أسلوب البرمجة ثنائية المستوى لتحسين نظام السيطرة على الخزين مع تطبيق عملي

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هذا العمل مرخص تحت اتفاقية المشاع الابداعي نسب المُصنَّف - غير تجاري - الترخيص العمومي الدولي 4.0
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مستخلص البحث:

في هذا البحث يتم معالجة المشاكل المرتبطة بنظام مخزون الأدوية الطبية الموسمية (الحساسية) من خلال تحديد الطلب الأمثل عن طريق حساب كمية الطلب الاقتصادي (E.O.Q) وتحقيق أقل تكلفة في الشركة العامة للصناعات الدوائية والمستلزمات الطبية في سامراء. كان الهدف هو تلبية الطلب على الادوية الحساسية الموسمية بكفاءة من خلال تحديد الطلب الأمثل على الأدوية الطبية. شملت . تضمنت الدراسة نوعين من أدوية الحساسية الموسمية (شراب Samatifen وحبوب VALIAPAM 2). حيث تم حساب كمية الطلب الاقتصادي وأقل تكلفة باستخدام طريقتين : نموذج الإنتاج متعدد العناصر بدون عجز مع القيود تم حل هذه الطريقة باستخدام الطريقة الكلاسيكية (العادية) وتم استخدام التحليلات الرياضية باستخدام برامج مثل QM و.Win Qsb بينما الطريقة الثانية البرمجة المزدوجة الخطية التربيعية (LQBP). وهو يتألف من: صانع القرار من المستوى الأعلى (القائد) وصانع القرار من المستوى الأدنى (التابع). تم تحويل نموذج النموذج ثنائي المستوى إلى نموذج أحادي المستوى باستخدام شروط كاروش-كوهن-توكر Karush-Kuhn-Tucker (KKT)، وتم الحصول على الحل باستخدام خوارزمية السمبلكس المعدلة.تؤكد نتائج هذه الدراسة فعالية طريقة LQBP في إيجاد الحل الأمثل لمشكلة المخزون ، مما يؤدي إلى انخفاض كبير في تكاليف المخزون المرتبطة بالأدوية الطبية. القيمة المتحصل عليها (1496700000) دينار عراقي انتاج 1200 700,د.ع /السنة أقل بكثير من إجمالي قيمة التكلفة باستخدام الطريقة الكلاسيكية (1719166). لذلك ، فإن البرمجة ثنائية المستوى (BLPP) تُظهر كفاءة فائقة ، مما يؤدي إلى حلول أكثر دقة وفعالية من حيث التكلفة. يركز هذا البحث على إمكانية استخدام البرمجة ثنائية المستوى في تحسين أنظمة مخزون الادوية الطبية ويساهم في تطوير مجال البحوث في قطاع الرعاية الصحية.

نوع البحث: ورقة بحثية .
المصطلحات الرئيسة للبحث: البرمجة ثنائية المستوى التربيعية الخطية (LQBP)،شروط K.K.T, طريقة سمبلكس المطورة،نموذج انتاج بدون عجز متعدد العناصر مع وجود قيود ,الطريقة الكلاسيكية (العادية)