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A Linear Programming Method for Finding the Critical Path and the Desired Time to Complete the Project

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Abstract:

The Nineveh Grain Store project is considered one of the important projects in the Ministry of Commerce/General Company for Grain Trade in general and Nineveh Governorate in particular. The research problem focused on the fact that many projects are slow and not completed on time, and the reason is the lack of keeping pace with the technological development of developed countries at the present time, which leads to delaying the duration of project completion, and this in turn leads to high costs and thus failure to complete the project on time.The main objective of this research is to use the Linear Programming (LP) method to formulate two mathematical models: the first is to obtain the project completion time (critical Path (CP)) for the Nineveh in normal time. The second is to build a mathematical model to find the desired time to complete the project in the crash time. The results of solving the mathematical model using the program (Win Q.S.b V2) demonstrated the efficiency and accuracy of this mathematical method and the program used and their importance in scheduling projects in general and the Nineveh Grain store project in particular in light of achieving the project goal and making the optimal decision.

Paper type: Research paper
Keywords: Linear Programming (LP), critical Path Method (CPM), Desired Time to complete the Project (DTCP), Win Q.S.B V2

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1.**Introduction:**

The project scheduling stage is concerned with studying time, which is one of the main objectives of the project, as well as developing an estimate of the activities' need for basic resources such as manpower, materials, equipment, etc., and making a proper budget in distributing them among the activities according to the need of each activity. There has been great development, especially in recent years, in using project management to achieve the organization's goals. Project management helps provide an effective force to develop the organization's capabilities and its ability to plan, organize, implement, and monitor various activities, including making the most of the organization's capabilities and resources. In general, the term management refers to all activities and activities working together, including control, direction, organization and planning, to effectively use the institution's resources and capabilities, to reach the institution's objectives effectively and efficiently and within the context surrounding the institution. Thus, project management includes all the methods and means used to complete projects on time, with quality and cost desired to achieve the goals of the institution that it seeks from these projects.

The Linear Programming (LP) method is considered one of man's most important scientific developments in the second half of the twentieth century. From the moment it was discovered and its methods were developed, administrative analysts could use it in various and diverse fields. It also enabled the decision-maker to look at administrative issues scientifically. With a perspective different from the way things were handled before; as a result, economic institutions achieved large profits and avoided losses, which enabled them to continue and expand and thus improve the services provided to customers.

One of the most important challenges facing the management of any project at present is ensuring the completion of the project despite it being subject to specific restrictions, including restrictions related to time and others related to the financial resources allocated to the project. Therefore, this matter requires an accurate calculation of time and cost, and on this basis, the project must be studied according to Modern scientific and mathematical methods, including the LP method, which was used in scheduling projects of all kinds and which has proven its efficiency in finding the optimal scheduling of projects and reducing the total costs of completing them.

1.1 Literature review:

There are many studies that discussed the use of LP in project scheduling, including:

Khalaf and Leong (2009) used LP method to construct two templates. The first is to obtain the time of project completion. The second template is construct as occasionally it is needed for finishing a project in the prearranged finishing time to keep cost at smallest probable level. Failure to do so eventually indications to rise in total cost. Thus, the second template is reducing the cost of crashing the project's tasks to meet the required project completion time.

Khalaf et al. (2010) provided a structure for the method of stretching noncritical activities to finish the project in smallest feasible time at smallest cost within available greatest budgeting. This is attained by crashing all activities instantaneously in the project network then by LP) method to develop a model to maximize the savings that will harvest from stretching noncritical activities.

bakhit and al-Farhoud (2012) used LP in network diagrams to determine the optimal time and cost for completing some Ministry of youth projects. The aim of the research is to monitor the performance of projects and network diagrams to schedule project activities, determine critical and non-critical activities, while proposing a linear mathematical model to calculate the optimal cost of the project after calculating the surplus times for non-critical activities, reducing the duration of project implementation, and controlling the overall performance of the project after conducting a sensitivity analysis of event times.

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Geda (2014) introduced LP method to solve project crashing problems subject to linear overhead expenditure rate and lateness penalty. LP method of the objective for the project which is reducing the total cost of project subject to various project constraints is patterned. Theoretical example of time- cost trade-off problem of a project is calculated using the developed model and solved using Microsoft Excel's Solver add-in. Solution of the modeled LP method designates by how much duration each of the project activities should be crashed, the resulting completion duration and overall cost of the project. The suggested approach appeared in this research allows project managers to perform computer assisted analysis of project crashing problems easily to find the time-cost tradeoff in project scheduling.

Kaur and Kumar (2014) proposed a new approach to determine the fuzzy optimum solution of fully fuzzy critical path (FFCP) problems. Also, it is displayed that it is better to utilize JMD illustration of LR flat fuzzy numbers in the suggested method as contrasted to the other illustration of LR flat fuzzy numbers.

Khader and Khalaf (2015) improved project scheduling by the goal programming technique for the Modern Village project in Wasit Governorate, where the Modern Village project was applied without the use of modern scientific techniques by the executing company in the implementation of the project and reached the most important results by obtaining a solution to the mathematical model of this research, which gives the ability to provide multiple decisions to manage the project in proportion to its available budget.

Jukic and barkovic (2017) used two related operations research methods cPM and LP. Some of their conceptions are showed in their paper in order to analysis some recent patterning buildings that have been mainly valuable in the evaluation of project tradeoff time-cost crashes problems. The tasks underwent crashing of together the time and cost by LP. a simplified interpretation of a small project and a LP model were formulated to denote this system. as well as being simple, the benefit of this technique is that it is appropriate to large networks. It allows for a shorter computational time at a lest cost, whereas robustness is enlarged.

Amiri et al. (2023) modeled into an integer LP model regarding the entire cost of material transport as the objective function and place conditions as constraints. The effectiveness of the method is showed by obtaining the optimum site layout for a numerical model. The suggested model is validated and proved by two approaches. Results show that the suggested model effectively identifies the kind and location of the tower crane and the location of material supply point, leading to about 20% reducing cost compared with when such features of a site layout are determined solely based on knowledge and educated guesses of the construction manager.

Scott et al. (2023) utilized two different algorithms to resolve the path planning problem. It is seen that the linear pattern performs well in battery state estimation while remaining implementable in a Linear Program or MILP, with minimal effect on the time-to-solve. This gives what we consider to be a worthwhile trade-off in improved accuracy relative to enlarged time-to-solve

Dollar (2024) proposes LP formulation for energy- reducing vehicle speed optimization in hilly terrain and displays comparable performing to its conventional formulation as a DP. To explain its efficiency, the LP velocity chart is integrated with a receding horizon motion organizer and simulated in a Class 8 tractor–trailer application over an actual road slope profile, with and without traffic.

The problem of this research is the lack of reliance on scientific methods in planning the project and scheduling its activities, as well as monitoring when it is implemented, and not using the mathematical approaches and sober programs to find the total and desired time for the completion of the project, which led to delaying the completion period from its scheduled date.

The objective of the research is to find the time and cost to complete the project in the normal and crush conditions of the Nineveh Grain S tore of the General company for Grain Trading at the Ministry of commerce. The main objective of this research is to use the LP method to formulate two mathematical models. The first is to obtain the completion time of the project (CP) for the Nineveh Grain Store under natural conditions. The second is to build a mathematical model to find the desired time to complete the project in urgent circumstances.

2.**Material and Methods:**

2.1 The concept of project scheduling:

In general, various methods and methods are used in implementing projects, and this results in the diversity of work plans and their differences from one project to another, as these plans need methods to deal with the technical information of the final beneficiary, and with the means of submitting costs, tight deadlines, different complexities, and a lot of data (Kerzner, 2017). Scheduling represents the last stage before starting the transformation procedures, meaning before the actual project outcomes occur. It is used to determine the timing of the use of resources related to each project process. It seeks to achieve a differentiation between multiple and diverse goals, and this is through optimal exploitation of facilities, equipment, and workers, and reducing both customer waiting time and operations and inventory times (Stevenson, 2015). Scheduling carries out the detailed part of the planning function. The scheduling task includes collecting the necessary data about the various elements of the project. among the most important data needed for the scheduling function: the relationship of succession or precedence among activities, and the time period spent by these activities, as well as preparing estimates of the need for major resources such as equipment and materials. , human resources, etc., and also implementing the budget in distributing it among the activities (al-ali, 2019). The scheduling process determines the timing of the work activities referred to previously in the project plan. The word scheduling refers to a variety of things according to the intended use. In this research, scheduling means the sequence of activities necessary to carry out the work and their stages. In other words, scheduling is the method used by management to predict the duration of project implementation and thus Ensuring its timely completion by changing or adapting the necessary work-based resources.

The importance of scheduling can be clarified in the following points (Khalaf and al-baldawi, 2016):

1. Scheduling is the framework that ensures coordination between work teams, functions and different departments within the project to ensure planning, monitoring and directing the various phases of the project.
2. Scheduling specifies the project completion date, the activities whose delay results in a delay in completion time, and the time surplus during which some activities can be delayed without causing a delay in the project completion date.
3. The timetable provided by scheduling helps to determine the periods of need for resources and avoid differences regarding them, especially if the resources are small, and also helps to identify the estimated costs of project activities (Zahira, 2021).

Project scheduling is concerned with the time element, which is one of the important resources in the project, and since time is one of the basic project objectives, time management is considered one of the pivotal elements in the project achieving its objectives and achieving many gains, such as (Wysocki ,2009; Khair al-Din, 2012: 139):

1.Project scheduling represents an organized framework for planning, monitoring, and directing the project. Project scheduling shows the state of overlap and interdependency of all tasks, work packages, work units, and activities in the project. Scheduling also enables shorter and clearer communication channels between work teams, functions, and departments.

2. Scheduling enables to set the likely timing for project completion. It also enables you to set critical activities that, if delayed, will delay the project completion time. Scheduling also contributes to setting non-critical activities, which if delayed for a known period will not cause any negative impact. Project completion date (Khader, 2015: 10).
3. Scheduling enables setting the start and end times of activities and the relationship of these activities to each other, which contributes to achieving the coordination required to implement activities on the necessary dates in a smooth manner without creating pressures at work. Scheduling also enables the reduction of conflicts over resources and personal conflicts due to the prior setting of dates and Then the date of need for these resources becomes known, and the various parties can coordinate and arrange with each other with the least conflict or disagreement to ensure these resources (al-baldawi, 2016).

2.2 Project scheduling objectives:

Scheduling mainly aims to organize processes and activities sequentially in order to implement the project with the least risks, the lowest cost, and the best time (Kerzner, 2013:60). The goal of project scheduling depends on setting a timetable, which includes the start and end date of each activity within the framework of the priority tasks identified above in planning and setting obstacles to the various previously scheduled activities. The scheduling process is based on a number of methods, and project scheduling aims to set a timetable with a minimum for the time period of the project (Vanhoucke, 2016). Work progress schedules should be monitored to ensure that work activities are progressing according to schedule during the implementation process, and this involves evaluating motor progress and measuring it against the schedule (Jabar et al., 2015). It is noted when the work is behind the project schedule and accordingly appropriate corrective action is taken and solutions are developed in order to return to the scheduled date, meaning within the path specified in the implementation plan (Fan, 2012). During the process of progress on the ground, evaluating the actual work and measuring it by planning for completion at the appropriate time and adopting the required corrective measures immediately is the basis for efficient control of the project. In addition to other adjustments that may occur, it is possible to update the project schedule and recalculate it periodically and predict whether the project will finish after or before the required completion date (abu al-Hasani, 2019). While Heizer et al. (2017) satisfied with one goal for scheduling, which is to determine and allocate priorities during project planning according to available capabilities, and in order to differentiate between scheduling and planning as two separate activities. baldwin and bordoli (2014) explained the goal of scheduling and planning in a simplified manner as follows: The main goal of planning is to ensure that the divided tasks occur successfully, and this requires setting goals, defining tasks, and monitoring progress. The project schedule provides the basis for measuring progress, and the basis for regular review and updating of the plan.

2.3. LP method to find the CP in the project network:

The LP method is one of the efficient methods and is considered one of the important mathematical methods that can be used to find the natural time to complete the project (CP). This requires knowledge of the project network data, which is the flow of activities in the network nodes (events), starting from the starting node of the network and ending at the ending node. It requires specifying the following (Kalaf and Leong, 2009):
The problem variables are as follows:

$T_{N,\ i}$: The normal period for activity i, which is time required to accomplished the activity with the least resources (natural cost)
Y_i : The decision variable for the start time of activity i

Maximize $Z = \sum_{i=1}^{n} T_{N,\ i} . Y_i$ (1)

Constraints of the mathematical model

The above objective function is subject to the following restrictions:
For activities that entered in the event i

$$\sum_{i=1}^{n} Y_i = 1 \quad (2)$$

For activities that leaves from event i

$$\sum_{i=1}^{n} -Y_i = -1 \quad (3)$$

For all activities entering and exiting event i
Event i: For each event, there is one restraint that denotes the conservation of flow:
Whole input flow = Whole output flow.
In this formulation, $y_i = 0$ or 1 represents the nonappearance or presence of unit flow from a node to another. So the equation is:

$$\sum_{i=1}^{n} +Y - Y_i = 0 \quad (4)$$

Non-negative constraints for all the decision variable are:

$$y_i \geq 0 \quad (5)$$

Table 1: Represents activity data, predecessor activities, time and normal costs for the grain collection warehouse project in Mosul Governorate, according to the work progress schedule for the project:

No.	Activity Name	Activity code	Previous activity	Normal time (day)	Normal cost (Million Dinars)
civil Works		a			
1	Preparing the site and the land of the project, which includes planning and settlement with scraping the land, burying it, fixing dimensions and removing rubble	a1	-	12	100,000,000
2	Examinations, evaluation and treatment of test pillars	a2	a1	81	120,000,000
3	Excavation and casting of piles under the benzat	a3	a2	34	1,957,500,000
4	Grain Storeschemes and approval	a4	-	44	150,000,000
5	Earthworks with burial under the foundations of benzan and pouring the blinding layer benzat group with moisture blocker coating (a)	a5	a3,4	8	143,875,000
6	concrete racking casting for benzat(a)	a6	a5	33	163,500,000
7	Pouring concrete columns(a)	a7	a6	28	105,000,000
8	concrete Hopper casting (a)	a8	a7	118	216,000,000
9	Earthworks with burial under the foundations of the benzan and pouring the blinding layer benzat group with moisture blocker coating (b)	a9	a3,4	8	143,875,000

10	concrete racking molding for benzat (b)	a10	a9	33	163,500,000
11	casting concrete columns(b)	a1 1	a1 0	28	105,000,000
12	concrete Hopper casting(b)	a12	a1 1	118	216,000,000
13	Excavation and burial works under the operating room, tunnels, hobars, bases of benz and groundwater withdrawal with treatments for the operating building	a1 3	a1 1	78	50,000,000
14	concrete pouring works for the operating room and tunnels	a14	a13	28	140,400,000
15	casting works of the upper transverse conveyor tunnel depth of 12 m	a15	a13	8	48,600,000
16	concrete works for train and truck receiving holes with tightening of grates	a16	a15	18	26,000,000
17	Pouring concrete foundations Grain Storeloading	a17	a15	8	8,000,000
18	concrete works for pouring yards surrounding operating rooms	a18	a16,17	8	86,400,000
19	Implementation of the electricity building with the installation of the transformer with the shed and the bases of the cable channels	a1 9	a1 1	28	20,000,000
	Mechanical Works	b			
20	Erection and tightening of metal benzat for line a	b1	a8	63	40,000,000
21	Installation and tightening of metal benzat for b line	b2	a12	63	40,000,000
22	Manufacture and installation of the steel structure of the operating room	b3	a14	178	50,000,000
23	Processing and installation of grain cranes	b4	b3	58	62,000,000
24	Installation and inspection of screening machines	b5	b1,2	28	10,000,000
25	Tightening, erecting and checking the dust system	b6	b5	28	20,000,000
26	Tightening and erecting the upper conveyor assembly	b7	b1,2	88	40,000,000
27	Tightening and erecting the lower vector group	b8	b4	89	29,000,000
28	Processing and tightening the metal shed over the receiving holes	b9	a18	18	20,000,000
29	Hydraulic tipper installation and inspection	b1 0	a1 8	23	95,000,000
30	Erecting loading silos with telescope hose	b1 1	a1 8	28	24,000,000
31	Packaging of the operating building with the bond and the Wig Panel	b12	b9,10,11	18	30,720,000
Electrical Works		c			
32	Laying of high and low voltage power line and control lines	c1	a19	3	30,000,000

33	Processing, installation and inspection of lighting poles	c2	a19	18	12,000,000
34	Equipping, installing and inspecting generators	c3	c1,c2	13	280,000,000
35	Installation and inspection of the control room and connection of equipment and control cables	c4	b12	118	30 ,000,000
36	Equipping, installing and inspecting submersibles	c5	a14,15	3	6,000,000
37	Tightening and testing the temperature monitoring system for benzos	c6	b1,2	13	10,000,000
38	air system tightening and installation works	c7	c6	13	18,000,000
39	Works and installation of electronic scale calibration	c8	c3	18	30,000,000
40	Works of lightning and ground arrester system	c9	a13	8	5,250,000
41	Installation, tightening and inspection of an electric elevator for people	c1 0	b7,8	13	90,000,000
42	Installation of switchboard communications and cameras	c1 1	c4,7,8,b6	3	6,000,000
43	Internal and external lighting installation works	c12	c7,8,b6	8	6,191,028
44	Installing a grain sterilizer	c13	10,11,12	3	1,000,000
45	Initial Total Examinations	D1	c4,c5,c9,c13	43	64,000,000
Total		5,012,811,028			

2.3.1 Formulate a mathematical model to find the critical path in the project network
After applying equation (1), the objective function can be obtained as follows:

$$\begin{aligned} Max\ (Z) = {} & 12\,a1 + 81\,a2 + 34\,a3 + 44\,a4 + 8\,a5 + 33\,a6 + 28\,a7 + 118\,a8 + 8\,a9 \\ & + 33\,a10 + 28\,a11 + 118\,a12 + 78\,a13 + 28\,a14 + 8\,a15 + 18\,a16 \\ & + 8\,a17 + 8\,a18 + 28\,a19 + 63\,b1 + 63\,b2 + 178\,b3 + 58\,b4 + 28\,b5 \\ & + 28\,b6 + 88\,b7 + 89\,b8 + 18\,b9 + 23\,b10 + 28\,b11 + 18\,b12 + 3\,c1 \\ & + 18\,c2 + 13\,c3 + 118\,c4 + 3\,c5 + 13\,c6 + 13\,c7 + 18\,c8 + 8\,c9 + 13\,c10 \\ & + 3\,c11 + 8\,c12 + 3\,c13 + 43\,D1 \end{aligned}$$

S.to
by applying equation (2,3,4 and 5), the constraints for the mathematical model are obtained as follows:
1) For activities that leave the event (i)
Event 1 $\quad -\text{a1} = -1$
$\quad -\text{a4} = -1$
2) For activities entering and leaving the event (i)
Event 2 $\quad \text{a1} - \text{a2} = 0$
Event 3 $\quad \text{a2} - \text{a3} = 0$
Event 4 $\quad \text{a3} + \text{a4} - \text{a5} - \text{a9} = 0$
Event 5 $\quad \text{a5} - \text{a6} = 0$
Event 6 $\quad \text{a6} - \text{a7} = 0$
Event 7 $\quad \text{a7} - \text{a8} = 0$
Event 8 $\quad \text{a8} - \text{b1} = 0$

Event 9 $a9 - a10 = 0$
Event 10 $a10 - a1\,1 = 0$
Event 11 $a1\,1 - a12 - a13 - a19 = 0$
Event 12 $a12 - b2 = 0$
Event 13 $a13 - a14 - a15 - c9 = 0$
Event 14 $a14 - b3 - d2 = 0$
Event 15 $a15 - a16 - a17 - d1 = 0$
Event 16 $a16 - d3 = 0$
Event 17 $a17 + d3 - a18 = 0$
Event 18 $a18 - b9 - b1\,0 - b1\,1 = 0$
Event 19 $a19 - c1 - c2 = 0$
Event 20 $b1 + b2 - b5 - b7 - c6 = 0$
Event 21 $b3 - b4 = 0$
Event 22 $b4 - b8 = 0$
Event 23 $b5 - b6 = 0$
Event 24 $b7 + b8 - c1\,0 = 0$
Event 25 $b9 - d4 = 0$
Event 26 $b1\,0 - d5 = 0$
Event 27 $d4 + d5 + b1\,1 - b12 = 0$
Event 28 $b12 - c4 = 0$
Event 29 $c1 - d6 = 0$
Event 30 $d6 + c2 - c3 = 0$
Event 31 $c3 - c8 = 0$
Event 32 $D1 + d2 - c5 = 0$
Event 33 $c6 - c7 = 0$
Event 34 $b6 + c7 + c8 - c1\,1 - c12 = 0$
Event 35 $c1\,1 - d7 = 0$
Event 36 $c1\,0 + c12 + d7 - c13 = 0$
Event 37 $c4 + c5 + c9 + c13 - D1 = 0$
3) For activities that enter the event (i)
Event 38 $D1 = 1$
All variables a1, a2, a3,…, D1 ≥ 0 and Integers

2.4 LP method to complete the project in the desired time:

The LP method can also be used to find the desired time to complete the project, as a mathematical model was built. The desired time to complete the project was (600) days, meaning a reduction in the total completion time (86) days. as the last event in the project network is subject to desired project completion time.

before formulating the mathematical model, we must define some important terms. Since the project is a combination of specific activities that require time and resources to finish, they are interconnected in a logical sequence, meaning that the start of some activities depends on the completion of other activities, where the relationship between activities is determined using nodes (events). Since an event represents a point in time that includes the completion of some activities and the beginning of new ones, the start and end points of an activity are expressed by two events, start and end.

2.4.1 Identify the variables of the problem as follows:

y_i: Time when the event i will happen, measured since the beginning of the project, where *i= (1, 2, 3,…, n)*.

x_q: Quantity of times (measured in terms of days, weeks, months or some other units) that each activity *q* will be crashed, where *q = (1, 2, 3,…, L).*

u_q: Slope of cost or cost of crash per unit of time for activity *q*.

The goal is to reach the desired time to complete the project by reducing the total cost of completing the project by reducing the compression periods of activities multiplied by the slope of the associated costs, then adding the resulting cost to the normal total cost of completing the project. The mathematical formula for slope can be defined as the relationship between cost and time, and it is represented by linear or non-linear relationships, as in equation (6) (Khalaf and Leong, 2009):

$$(Slop) = \frac{Cc-Cn}{Tn-Tc} \qquad (6)$$

Whereas:

T_n: Normal completion time

c_n: Normal time

T_c: crash time

c_c: crash cost

2.4.2 Define the objective function as follows:

The objective function of the mathematical model will be as follows:

$$\text{Minimize } Z = \sum_{q=1}^{L} u_q \cdot x_q \qquad (7)$$

2.4.3 Determine the constraints as follows:

a. constraints of crash time

We can decrease the time needed to accomplish an activity simply by increasing human or material resources or by improving productivity. But it is not possible to reduce the time required to complete the activity after a certain threshold limit. Equation (8) explains this as follows:

$x_q \leq$ allowable time of crashing activity *q* calculated in periods of days, weeks, months or some other units (8)

b. Formulate network constraints:

This set of constraints describes the construction of the network. The start of some activities depends on the completion of others, so we must create a working sequence of activities through constraints. Therefore, there is one or more constraint for each event depending on the activities preceding this event.

Since event 1 will start at the beginning of the project, we start by setting the incidence time of event 1 equal to zero and as in equation (9):

$y_1 = 0$ (9)

The additional events will be stated as in equation (11) as follows:

Start time of activity *(yi)* ≥ (time of start + normal time−crash time) for this current predecessor (10)

c. Constraint of desired time to complete the project:

This constraint states that the last event (completion of the last activity) represents the desired time to complete the project, which is as in equation (11).

$y_m \leq$ Desired time to complete the project (11)

Where m refers to the last event of the project

d. Non-negativity constraints

It explains that all decision variables must be greater than or equal to zero, as shown in equation (12):

$y_i, x_q, u_q \geq 0$ (12)

4. Describe the model data to find the critical path under normal conditions and the desired time to complete the project

2.4.4 Describe the mathematical model data to reach the desired time to complete the project

In the presence of normal and crash times and costs for the project network activities, and by applying equation (6), the slope of the project activities is found, as in Table (3):

Table 3: Normal and crash times, costs, and slope of project network activities

Activities	Normal time (day) (NT)	Crash Time (in day) (cT)	Pressure duration (in day) (ΔT)	Normal cost (Iraqi Dinar) (Nc)	Crash cost (Iraqi Dinar) (cc)	Pressure cost (IQD)	Slop(IQD) $\left(\sigma = \frac{CC-NC}{NT-CT}\right)$
a1	12 days	8 days	4 days	100,000,000	102,000,000	2,000,000	500,000
a2	81 days	73days	8 days	120,000,000	122,000,000	2,000,000	250,000
a3	34 days	34 days	0	1,975,500,000	1,975,500,000	0	0
a4	44 days	34 days	10 days	150,000,000	153,000,000	3,000,000	300,000
a5	8 days	5 days	3 days	143,875,000	144,875,000	1,000,000	333,333
a6	33 days	18 days	15 days	163,500,000	165,500,000	2,000,000	133,333
a7	28 days	23 days	5 days	105,000,000	106,000,000	1,000,000	200,000
a8	118 days	98 days	20 days	216,000,000	220,000,000	4,000,000	200,000
a9	8 days	5 days	3 days	143,875,000	144,875,000	1,000,000	333,333
a10	33 days	18 days	15 days	163,500,000	165,500,000	2,000,000	133,333
a1 1	28 days	23 days	5 days	105,000,000	106,000,000	1,000,000	200,000
a12	118 days	98 days	20 days	216,000,000	220,000,000	4,000,000	200,000
a13	78 days	58 days	20 days	50,000,000	54,000,000	4,000,000	200,000
a14	28 days	18 days	10 days	140,400,000	143,400,000	3,000,000	300,000
a15	8 days	5 days	3 days	48,600,000	50,600,000	2,000,000	666,666
a16	18 days	13 days	5 days	26,000,000	28,000,000	2,000,000	400,000
a17	8 days	8 days	0	8,000,000	8,000,000	0	0
a18	8 days	8 days	0	86,400,000	86,400,000	0	0
a19	28 days	23 days	5 days	20,000,000	22,000,000	2,000,000	400,000
b1	63 days	53 days	10 days	40,000,000	43,000,000	3,000,000	300,000
b2	63 days	53 days	10 days	40,000,000	43,000,000	3,000,000	300,000
b3	178 days	138days	40 days	50,000,000	55,000,000	5,000,000	125,000
b4	58 days	48 days	10 days	62,000,000	65,000,000	3,000,000	300,000
b5	28 days	23 days	5 days	10,000,000	12,000,000	2,000,000	400,000
b6	28 days	23 days	5 days	20,000,000	22,000,000	2,000,000	400,000
b7	88 days	68 days	20 days	40,000,000	44,000,000	4,000,000	200,000
b8	89 days	63 days	26 days	29,000,000	32,000,000	3,000,000	115,384

b9	18 days	13 days	5 days	20,000,000	22,000,000	2,000,000	400,000
b1 0	23 days	18 days	5 days	95,000,000	97,000,000	2,000,000	400,000
b1 1	28 days	18 days	10 days	24,000,000	26,000,000	2,000.000	200,000
b12	18 days	13 days	5 days	30,720,000	32,720,000	2,000,000	400,000
c1	3 days	3 days	0	30,000,000	30,000,000	0	0
c2	18 days	8 days	10 days	12,000,000	14,000,000	2,000,000	200,000
c3	13 days	13 days	0	280,000,000	280,000,000	0	0
c4	118 days	88 days	30 days	30,000,000	34,000,000	4,000,000	133,333
c5	3 days	3 days	0	6,000,000	6,000,000	0	0
c6	13 days	6 days	7 days	10,000,000	12,000,000	2,000,000	285,714
c7	13 days	6 days	7 days	18,000,000	20,000,000	2,000,000	285,714
c8	18 days	13 days	5 days	30,000,000	32,000,000	2,000,000	400,000
c9	8 days	8 days	0	5,250,000	5,250,000	0	0
c1 0	13 days	8 days	5 days	90,000,000	92,000,000	2,000,000	400,000
c1 1	3 days	1 days	2 days	6,000,000	7,000,000	1,000,000	500,000
c12	8 days	3 days	5 days	6,191,028	7,441,028	1,250,000	250,000
c13	3 days	1 days	2 days	1,000,000	1,500,000	500,000	250,000
D1	43 days	28 days	15 days	64,000,000	64,000,000	0	0
Total				4,930,811,028			

For the purpose of modeling the problem, it is necessary to define activities in terms of the beginning and end of the event. The total number of events in this project is 38 events, and as is clear from Table (4), we can define the variables as follows:

Table 4: Start and end events for each activity in the project network

activit ies	**Start Event**	**End Event**	**activiti es**	**Starting Event**	**Ending Event**	**activiti es**	**Start event**	**End event**
a1	1	2	b2	12	20	c1 0	24	36
a2	2	3	b3	14	21	c1 1	34	35
a3	3	4	b4	21	22	c12	34	36
a4	1	4	b5	20	23	c13	36	37
a5	4	5	b6	23	34	D1	37	38
a6	5	6	b7	20	24			
a7	6	7	b8	22	24			
a8	7	8	b9	18	25			
a9	4	9	b1 0	18	26			
a10	9	10	b1 1	18	27			
a1 1	10	11	b12	27	28			
a12	11	12	c1	19	29			
a13	11	13	c2	19	30			
a14	13	14	c3	30	31			
a15	13	15	c4	28	37			
a16	15	16	c5	32	37			
a17	15	17	c6	20	33			
a18	17	18	c7	33	34			
a19	11	19	c8	31	34			
b1	8	20	c9	13	37			

2.4.5 Definition of the variables of the mathematical model

y1: Time of incidence of the event 1	y17: Time of incidence of the event 17	y33: Time of incidence of the event 33
y2 : Time of incidence of the event 2	y18: Time of incidence of the event 18	y34: Time of incidence of the event 34
y3: Time of incidence of the event 3	y19: Time of incidence of the event 19	y35 Time of incidence of the event 35
y4: Time of incidence of the event 4	y20: Time of incidence of the event 20	y36: Time of incidence of the event 36
y5: Time of incidence of the event 5	y21: Time of incidence of the event 21	y37: Time of incidence of the event 37
y6: Time of incidence of the event 6	y22 Time of incidence of the event 22	y38: Time of incidence of the event 38
y7: Time of incidence of the event 7	y23: Time of incidence of the event 23	
y8: Time of incidence of the event 8	y24: Time of incidence of the event 24	
y9: Time of incidence of the event 9	y25: Time of incidence of the event 25	
y1 0: Time of incidence of the event 10	y26: Time of incidence of the event 26	
y1 1: Time of incidence of the event 11	y27 Time of incidence of the event 27	
y12 Time of incidence of the event 12	y28: Time of incidence of the event 28	
y13: Time of incidence of the event 13	y29: Time of incidence of the event 29	
y14: Time of incidence of the event 14	y30: Time of incidence of the event 30	
y15: Time of incidence of the event 15	y31: Time of incidence of the event 31	
y16: Time of incidence of the event 16	y32 Time of incidence of the event 32	

We also assume that:

x_{a1}: Days number for crashing the activity a1	x_{b3}: Days number for crashing the activity b3	x_{c11}: Days number for crashing the activity c1 1
x_{a2} : Days number for crashing the activity a2	x_{b4}: Days number for crashing the activity b4	x_{c12}: Days number for crashing the activity c12
x_{a3}: Days number for crashing the activity a3	x_{b5}: Days number for crashing the activity b5	x_{c13}: Days number for crashing the activity c13
x_{a4}: Days number for crashing the activity a4	x_{b6}: Days number for crashing the activity b6	x_{D1}: Days number for crashing the activity D1
x_{a5}: Days number for crashing the activity a5	x_{b7}: Days number for crashing the activity b7	
x_{a6}: Days number for crashing the activity a6	x_{b8}: Days number for crashing the activity b8	
x_{a7}: Days number for crashing the activity a7	x_{b9}: Days number for crashing the activity b9	
x_{a8}: Days number for crashing the activity a8	x_{b10}: Days number for crashing the activity b1 0	
x_{a9}: Days number for crashing the activity a9	x_{b11}: Days number for crashing the activity b1 1	

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$x_{a1\ 0}$: Days number for crashing the activity a1 0	x_{b12}: Days number for crashing the activity b12	
$x_{a1\ 1}$: Days number for crashing the activity a1 1	x_{b7}: Days number for crashing the activity b7	
x_{a12}: Days number for crashing the activity a12	$x_{c\ 1}$: Days number for crashing the activity c1	
x_{a13}: Days number for crashing the activity a13	x_{c2}: Days number for crashing the activity c2	
x_{a14}: Days number for crashing the activity a14	x_{c3}: Days number for crashing the activity c3	
x_{a15}: Days number for crashing the activity a15	x_{c4}: Days number for crashing the activity c4	
x_{a16}: Days number for crashing the activity a16	x_{c5}: Days number for crashing the activity c5	
x_{a17}: Days number for crashing the activity a17	x_{c6}: Days number for crashing the activity c6	
x_{a18}: Days number for crashing the activity a18	x_{c7}: Days number for crashing the activity c7	
x_{a19}: Days number for crashing the activity a19	x_{c8}: Days number for crashing the activity c8	
x_{b1}: Days number for crashing the activity b1	x_{c9}: Days number for crashing the activity c9	
x_{b2}: Days number for crashing the activity b2	$x_{c1\ 0}$: Days number for crashing the activity c1 0	

2.4.6 Formulate the mathematical model to reach the desired time to complete the project
By applying equation (7) we obtain the objective function of the mathematical model as follows:

$$\text{Min } Z= 500000x_{a1}+250000x_{a2}+0x_{a3}+300000x_{a4}+333333x_{a5}+133333x_{a6} +200000x_{a7}+200000x_{a8}+333333x_{a9}+133333x_{a10}+200000x_{a11}+200000x_{a12}+200000x_{a13}+300000x_{a14}+666666x_{a15}+400000x_{a16}+0x_{a17}+0x_{a18}+400000x_{a19}+300000x_{b1}+300000x_{b2}+125000x_{b3}+300000x_{b4}+400000x_{b5}+400000x_{b6}+200000x_{b7}+115384x_{b8}+400000x_{b9}+400000x_{b10}+200000x_{b1\ 1}+ 400000x_{b12} +0\ x_{c\ 1} +200000x_{c2} +0x_{c3}+133333x_{c4}+0x_{c5}+285714x_{c6}+285714x_{c7}+400000x_{c8}+0x_{c9}+400000x_{c10}+500000x_{c11}+250000 x_{c12}+125000x_{c13}+0\ x_{D\ 1}$$

1. Maximum reduction constraints
Applying equation (8) we get the following constraints:

$x_{a\ 1} \leq 4$	$x_{b3} \leq 40$	$x_{c12} \leq 5$ $x_{c13} \leq 2$ $x_{D\ 1} \leq 0$
$x_{a2} \leq 8$	$x_{b4} \leq 10$	
$x_{a3} \leq 0$	$x_{b5} \leq 5$	
$x_{a4} \leq 10$	$x_{b6} \leq 5$	
$x_{a5} \leq 3$	$x_{b7} \leq 20$	
$x_{a\ 6} \leq 15$	$x_{b8} \leq 26$	
$x_{a\ 7} \leq 5$	$x_{b9} \leq 5$	
$x_{a8} \leq 20$	$x_{b1\ 0} \leq 5$	
$x_{a9} \leq 3$	$x_{b1\ 1} \leq 10$	
$x_{a10} \leq 15$	$x_{b12} \leq 5$	

$x_{a11} \leq 5$	$x_{c1} \leq 0$	
$x_{a12} \leq 20$	$x_{c2} \leq 10$	
$x_{a13} \leq 20$	$x_{c3} \leq 0$	
$x_{a14} \leq 10$	$x_{c4} \leq 30$	
$x_{a15} \leq 3$	$x_{c5} \leq 0$	
$x_{a16} \leq 5$	$x_{c6} \leq 7$	
$x_{a17} \leq 0$	$x_{c7} \leq 7$	
$x_{a18} \leq 0$	$x_{c8} \leq 5$	
$x_{a19} \leq 5$	$x_{c9} \leq 0$	
$x_{b1} \leq 10$	$x_{c10} \leq 5$	
$x_{b2} \leq 10$	$x_{c11} \leq 2$	

2. Constraints of project network construction
By applying equation (9,10), we obtain the following constraints:

$y_1 = 0$	$y_{20} + 88 - x_{b7} \leq y_{24}$ Or, $y_{22} + 89 - x_{b8} \leq y_{24}$
$y_1 + 12 - x_{a1} \leq y_2$	$y_{18} + 18 - x_{b9} \leq y_{25}$
$y_2 + 81 - x_{a2} \leq y_3$	$y_{18} + 23 - x_{b10} \leq y_{26}$
$y_3 + 34 - x_{a3} \leq y_4$ Or, $y_1 + 44 - x_{a4} \leq y_4$	$y_{18} + 28 - x_{b11} \leq y_{27}$ Or, $y_{25} + 0 - x_{d4} \leq y_{27}$ Or, $y_{26} + 0 - x_{d5} \leq y_{27}$
$y_4 + 8 - x_{a5} \leq y_5$	$y_{27} + 18 - x_{b12} \leq y_{28}$
$y_4 + 8 - x_{a9} \leq y_9$	$y_{19} + 3 - x_{c1} \leq y_{29}$
$y_5 + 33 - x_{a6} \leq y_6$	$y_{19} + 18 - x_{c2} \leq y_{30}$ Or, $y_{29} + 0 - x_{d6} \leq y_{30}$
$y_6 + 28 - x_{a7} \leq y_7$	$y_{30} + 13 - x_{c3} \leq y_{31}$
$y_7 + 118 - x_{a8} \leq y_8$	$y_{14} + 0 - x_{d1} \leq y_{32}$ Or, $y_{15} + 0 - x_{d2} \leq y_{32}$
$y_9 + 33 - x_{a10} \leq y_{10}$	$y_{28} + 118 - x_{c4} \leq y_{37}$ Or, $y_{32} + 3 - x_{c5} \leq y_{37}$ Or, $y_{13} + 8 - x_{c9} \leq y_{37}$ Or, $y_{36} + 3 - x_{c13} \leq y_{37}$
$y_{10} + 28 - x_{a11} \leq y_{11}$	$y_{20} + 13 - x_{c6} \leq y_{33}$
$y_{11} + 118 - x_{a12} \leq y_{12}$	$y_{24} + 13 - x_{c10} \leq y_{36}$ Or, $y_{34} + 8 - x_{c12} \leq y_{36}$ Or, $y_{35} + 0 - x_{d7} \leq y_{36}$
$y_{11} + 78 - x_{a13} \leq y_{13}$	$y_{37} + 43 - x_{D1} \leq y_{38}$
$y_{11} + 28 - x_{a19} \leq y_{19}$	
$y_{13} + 28 - x_{a14} \leq y_{14}$	
$y_{13} + 8 - x_{a15} \leq y_{15}$	
$y_{15} + 18 - x_{a16} \leq y_{16}$	
$y_{15} + 8 - x_{a17} \leq y_{17}$ Or, $y_{16} + 0 - x_{d3} \leq y_{17}$	
$y_{17} + 8 - x_{a18} \leq y_{18}$	
$y_8 + 63 - x_{b1} \leq y_{20}$ Or, $y_{12} + 63 - x_{b2} \leq y_{20}$	
$y_{14} + 178 - x_{b3} \leq y_{21}$	
$y_{21} + 58 - x_{b4} \leq y_{22}$ $y_{20} + 28 - x_{b5} \leq y_{23}$	
$y_{23} + 28 - x_{b6} \leq y_{34}$ Or, $y_{33} + 13 - x_{c7} \leq y_{34}$ Or, $y_{31} + 18 - x_{c8} \leq y_{34}$	

3. Constraint of the desired time to complete the project in (600) days
applying equation (11), we get the following entry:
$y_{38} \leq 600$

4. Non-negativity constraint
applying equation (12), we obtain the following non-negativity constraints:
$x_{a\,1}, x_{a2}, x_{a3},\ldots, y_{38} \geq 0$

3. **Discussion of Results:**

3.1 Discuss the results of the mathematical model to find the CP in the project network:

by reviewing the solution results table for the mathematical model shown in Table (2), column (3) Solution value, which indicates the critical activities that appeared to have a value greater than zero and formed the critical path in the project network, as follows:

$$a_1, a_2, a_3, a_9, a_{10}, a_{1\ 1}, a_{13}, a_{14}, b_3, b_4, b_8, c_{1\ 0}, c_{13}, D_1 = 686 \text{ Days}$$

As for the other activities that have a Solution value of zero, they are non-critical activities. Column (7) which is the status column, shows the status of the variable whether it is essential or non-essential. Accordingly, the normal time to complete the project (686 days) which represents CP for the project will be found by summing the times of the critical activities shown in the Total contribution column.

Table 2: Finding the critical path in the project network using the WinQSb2 program.

combined Report for LP for CPM								
	10:33:02		Friday	February	2	2024		
	Decision	Solution	Unit cost or	Total	Reduced	basis	allowable	allowable
	Variable	Value	Profit c(j)	contribution	cost	Status	Min. c(j)	Max. c(j)
1	a1	1	12	12	0	basic	-M	M
2	a2	1	81	81	0	basic	-M	M
3	a3	1	34	34	0	basic	-M	M
4	a4	0	44	0	0	Non-essential	-M	M
5	a5	0	8	0	-162	Non-essential	-M	170
6	a6	0	33	0	0	Non-essential	-M	195
7	a7	0	28	0	0	Non-essential	-M	190
8	a8	0	118	0	0	Non-essential	-M	280
9	a9	1	8	8	0	basic	-154	M
10	a10	1	33	33	0	Non-essential	-129	M
11	a1 1	1	28	28	0	basic	-134	M
12	a12	0	118	0	0	Non-essential	-M	280
13	a13	1	78	78	0	basic	-84	M
14	a14	1	28	28	0	basic	-134	M
15	a15	0	8	0	0	Non-essential	-M	179
16	a16	0	18	0	0	Non-essential	8	189
17	a17	0	8	0	-10	Non-essential	-M	18
18	a18	0	8	0	0	Non-essential	-M	179
19	a19	0	28	0	0	Non-essential	-M	350
20	b1	0	63	0	0	Non-essential	-M	225

21	b2	0	63	0	-162	Non-essential	-M	225
22	b3	1	178	178	0	basic	16	M
23	b4	1	58	58	0	basic	-104	M
24	b5	0	28	0	0	Non-essential	-2	65
25	b6	0	28	0	0	Non-essential	-2	65
26	b7	0	88	0	0	Non-essential	51	250
27	b8	1	89	89	0	basic	-73	M
28	b9	0	18	0	0	Non-essential	-M	28
29	b1 0	0	23	0	0	Non-essential	-M	28
30	b1 1	0	28	0	0	Non-essential	23	199
31	b12	0	18	0	0	Non-essential	-M	189
32	c1	0	3	0	0	Non-essential	-M	18
33	c2	0	18	0	0	Non-essential	3	340
34	c3	0	13	0	0	Non-essential	-M	335
35	c4	0	118	0	-171	Non-essential	-M	289
36	c5	0	3	0	0	Non-essential	-M	361
37	c6	0	13	0	0	Non-essential	-M	43
38	c7	0	13	0	-30	Non-essential	-M	43
39	c8	0	18	0	-322	Non-essential	-M	340
40	c9	0	8	0	-361	Non-essential	-M	369
41	c1 0	1	13	13	0	basic	-24	M
42	c1 1	0	3	0	0	basic	-M	45
43	c12	0	8	0	-37	Non-essential	-M	45
44	c13	1	3	3	0	basic	-168	M
45	D1	1	43	43	0	basic	-M	M
46	d1	0	0	0	0	Non-essential	-M	M
47	d2	0	0	0	-358	Non-essential	-M	358
48	d3	0	0	0	0	Non-essential	-10	171
49	d4	0	0	0	-10	Non-essential	-M	10
50	d5	0	0	0	-5	Non-essential	-M	5
51	d6	0	0	0	-15	Non-essential	-M	15
52	d7	0	0	0	-42	Non-essential	-M	42
	Objective	Function	(Max.) =	686				
		Left Hand		Right Hand	Slack	Shadow	allowable	allowable
	constraint	Side	Direction	Side	or Surplus	Price	Min. RHS	Max. RHS
1	Node1	-1	=	-1	0	-302	-M	0
2	Node1	-1	=	-1	0	-219	-M	0
3	Node2	0	=	0	0	-290	-M	1
4	Node3	0	=	0	0	-209	-M	1
5	Node4	0	=	0	0	-175	-M	2
6	Node5	0	=	0	0	-5	0	0

7	Node6	0	=	0	0	28	0	0
8	Node7	0	=	0	0	56	0	0
9	Node8	0	=	0	0	174	0	0
10	Node9	0	=	0	0	-167	-M	2
11	Node10	0	=	0	0	-134	-M	2
12	Node11	0	=	0	0	-106	-M	2
13	Node12	0	=	0	0	12	0	2
14	Node13	0	=	0	0	-28	-M	2
15	Node14	0	=	0	0	0	-M	2
16	Node15	0	=	0	0	-20	0	2
17	Node16	0	=	0	0	-2	0	2
18	Node17	0	=	0	0	-2	0	2
19	Node18	0	=	0	0	6	0	2
20	Node19	0	=	0	0	-78	0	2
21	Node20	0	=	0	0	237	0	0
22	Node21	0	=	0	0	178	0	2
23	Node22	0	=	0	0	236	0	2
24	Node23	0	=	0	0	265	0	0
25	Node24	0	=	0	0	325	0	2
26	Node25	0	=	0	0	24	0	2
27	Node26	0	=	0	0	29	0	2
28	Node27	0	=	0	0	34	0	2
29	Node28	0	=	0	0	52	0	2
30	Node29	0	=	0	0	-75	0	2
31	Node30	0	=	0	0	-60	0	2
32	Node31	0	=	0	0	-47	0	2
33	Node32	0	=	0	0	338	0	1
34	Node33	0	=	0	0	250	0	0
35	Node34	0	=	0	0	293	0	0
36	Node35	0	=	0	0	296	0	0
37	Node36	0	=	0	0	338	0	2
38	Node37	0	=	0	0	341	0	2
39	Node38	1	=	1	0	46	0	M

3.2 Discuss the results of the mathematical model to complete the project at the desired time (600 days):

After solving the mathematical model, the solution results in Table (5) show the following:

The solution value for the decision variable is through column 3 (Solution value), the total contribution amount for each variable is through column 5 (Total contribution), the reduced cost for all decision variables is through column 6, the status of the variables is essential or non-essential through column 7 and the limits of the sensitivity analysis for the slope cost of these variables through (columns 8 and 9). In addition to the value of the objective function; note that this table exists when the optimal solution for the problem is available.

Table (5) shows the critical activities that must be crashed and the number of days that must be crash in to reach the desired time to complete the project in (600) days, meaning that the normal time to complete the project is reduced to 86 days. The cost of crashing the activities' time, which is the value of the objective function of 10,999,979 million dinars, will be added to the natural cost of completion. The project is 4,930,811,028 billion dinars, so that the total cost of the project, after compressing some critical activities, is 4,941,811,007 billion dinars.

Table (5) shows the critical activities that must be crashed and the number of days that must be crash to reach the desired time to complete the project in (600) days, meaning, reducing the normal time to complete the project to 86 days. The cost of crashing the time of activities, which is represented by the value of the objective function (10,999,979 million dinars), will be added to the normal cost of completing the project (4,930,811,028 billion dinars), so that the total cost of the project, after crashing some critical activities will be 4,941,811,007 billion dinars.

Table (5) divided into the following two tables:

1. Summary of the variables table

The summary of the mathematical model solution in Table (5) indicates that the crashed time for activity x_{a1} is 500,000 thousand dinars, and in order for this variable (activity) to enter the optimal solution and be a basic reduced its cost to 300,000 thousand dinars.

The reduced cost of non-critical (at-bound) activities (variables whose value is zero in the optimal solution) provides us with information about the extent to which we can increase the objective function coefficients for these variables to obtain a positive value for those variables in the optimal solution. The limits of the sensitivity analysis for the cost slope of this variable, which ensure that the current solution remains optimal are 200,000 thousand dinars as a minimum and infinity (M) as a maximum.

As for the number of days crashed for activity a day is 1,999,995 million dinars, as shown by the (Total contribution) column. accordingly, the reduced cost for it is 0 as a basic variable, which is what the reduced cost field for this variable indicates. as for the limits of the sensitivity analysis for the cost of the slope of this variable, which ensures that the current solution remains optimal, it is -M as a minimum. and a maximum of 200,000 thousand dinars.

As for the xa1 0, which is considered a basic variable the value of the solution for it, or what will be compressed from the activity time, is 15 days, and as shown by the Solution value column, the cost of the slope for it is 133,333 thousand dinars, and therefore the cost of crashing this variable for 15 days will be (1,999,995 million dinars) and as shown by the Total contribution column, and therefore the cost The reduced cost has 0 as a basic variable, which is what the reduced cost column for this variable indicates. As for the limits of the sensitivity analysis for the slope cost of this variable, which ensures that the current solution remains optimal, they are -M as a minimum and 200,000 thousand dinars as a maximum.

2. Summary of the constraints table

The summary of the solution to the mathematical model in Table (5) indicates the first constraint (c1) (sequence 1), the value of its left side is 0, the direction of the constraint is less than or equal to (≤) and the value of its right side is 4 days. We can conclude that this constraint is not fully exploited through the slack or surplus column, whose value is 4, and since the shadow price is the change in the value of the objective function when the value of the right side of this constraint increases or decreases by one unit, the objective function will decrease by (0) when the activity crashed.

As for the tenth entry (c1 0) (sequence 10), the value of its left side is 15 days and the direction of the entry is less than or equal to (≤) and the value of its right side is 15 days. We can also conclude that this entry is fully exploited, i.e. its right side has been exploited from through the slack or surplus column and its value is 0. As for the shadow price of the right side of this entry, which is (-66,667) thousand dinars, it indicates that the objective function will decrease by (66,667) thousand dinars when the activity crash xa1 0 decreases for only one day. The sensitivity analysis limits for the right side of this constraint which ensure that the current solution remains acceptable, are 0 as a minimum and 15 as a maximum.

Table 5: Results of the mathematical model using the program WinQSb2

combined Report for LP cPM			Les than or equal 600					
	09:25:59		Saturday	February	3	2024		
	Decision	Solution	Unit cost or	Total	Reduced	Basis	allowable	allowable
	Variable	Value	Profit c(j)	contribution	cost	Status	Min. c(j)	Max. c(j)
1	xa 1	0	500000	0	300000	Non-essential	200000	M
2	xa 2	0	250000	0	50000	Non-essential	200000	M
3	xa3	0	0	0	0	Basic	-M	200000
4	xa4	0	300000	0	300000	Non-essential	0	M
5	xa5	0	333333	0	333333	Non-essential	0	M
6	xa6	0	133333	0	133333	Non-essential	0	M
7	xa7	0	200000	0	200000	Non-essential	0	M
8	xa8	0	200000	0	200000	Non-essential	0	M
9	xa9	0	333333	0	133333	Non-essential	200000	M
10	xa1 0	15	133333	1999995	0	basic	-M	200000
11	xa1 1	5	200000	1000000	0	basic	-M	200000
12	xa12	0	200000	0	200000	Non-essential	0	M
13	xa13	0	200000	0	0	basic	200000	250000
14	xa14	0	300000	0	100000	Non-essential	200000	M
15	xa15	0	666666	0	666666	Non-essential	0	M
16	xa16	0	400000	0	400000	Non-essential	0	M
17	xa17	0	0	0	0	basic	-M	0
18	xa18	0	0	0	0	basic	-M	0
19	xa19	0	400000	0	400000	Non-essential	0	M

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20	xb 1	0	300000	0	300000	Non-essential	0	M
21	xb2	0	300000	0	300000	Non-essential	0	M
22	xb3	40	125000	5000000	0	basic	-M	200000
23	xb4	0	300000	0	100000	Non-essential	200000	M
24	xb5	0	400000	0	400000	Non-essential	0	M
25	xb6	0	400000	0	400000	Non-essential	0	M
26	xb7	0	200000	0	200000	Non-essential	0	M
27	xb8	26	115384	2999984	0	basic	-M	200000
28	xb9	0	400000	0	400000	Non-essential	0	M
29	xb1 0	0	400000	0	400000	Non-essential	0	M
30	xb1 1	0	200000	0	200000	Non-essential	0	M
31	xb12	0	400000	0	400000	Non-essential	0	M
32	xc 1	0	0	0	0	basic	-M	0
33	xc2	0	200000	0	200000	Non-essential	0	M
34	xc3	0	0	0	0	Non-essential	0	M
35	xc4	0	133333	0	133333	Non-essential	0	M
36	xc5	0	0	0	0	Non-essential	0	M
37	xc6	0	285714	0	285714	Non-essential	0	M
38	xc7	0	285714	0	285714	Non-essential	0	M
39	xc8	0	400000	0	400000	Non-essential	0	M
40	xc 9	0	0	0	0	Non-essential	0	M
41	xc1 0	0	400000	0	200000	Non-essential	200000	M
42	xc1 1	0	500000	0	500000	Non-essential	0	M
43	xc12	0	250000	0	250000	Non-essential	0	M
44	xc13	0	250000	0	50000	Non-essential	200000	M
45	xD 1	0	0	0	0	basic	-M	200000
46	y1	0	0	0	0	basic	-M	M

47	y2	12	0	0	0	basic	-200000	300000
48	y3	93	0	0	0	basic	-200000	50000
49	y4	127	0	0	0	basic	-200000	50000
50	y5	211	0	0	0	basic	-50000	0
51	y6	244	0	0	0	basic	-50000	0
52	y7	272	0	0	0	basic	-50000	0
53	y8	390	0	0	0	basic	-50000	0
54	y9	135	0	0	0	basic	-200000	50000
55	y1 0	153	0	0	0	basic	-66667	50000
56	y1 1	176	0	0	0	basic	0	50000
57	y12	390	0	0	0	basic	-50000	0
58	y13	254	0	0	0	basic	-50000	75000
59	y14	282	0	0	0	basic	-50000	75000
60	y15	262	0	0	0	basic	0	75000
61	y16	280	0	0	0	basic	0	75000
62	y17	270	0	0	0	basic	0	75000
63	y18	278	0	0	0	basic	0	75000
64	y19	204	0	0	0	basic	0	50000
65	y20	453	0	0	0	basic	-50000	0
66	y21	420	0	0	0	basic	-50000	84616
67	y22	478	0	0	0	basic	-50000	84616
68	y23	518	0	0	0	basic	-50000	0
69	y24	541	0	0	0	basic	-50000	200000
70	y25	296	0	0	0	basic	0	75000
71	y26	301	0	0	0	basic	0	75000
72	y27	421	0	0	0	basic	-133333	0
73	y28	439	0	0	0	basic	-133333	0
74	y29	207	0	0	0	basic	0	50000
75	y30	222	0	0	0	basic	0	50000
76	y31	235	0	0	0	basic	0	0
77	y32	554	0	0	0	basic	0	0
78	y33	466	0	0	0	basic	0	0
79	y34	546	0	0	0	basic	-50000	0
80	y35	0	0	0	0	Non-essential	0	M
81	y36	554	0	0	0	basic	-50000	200000
82	y37	557	0	0	0	basic	-M	200000
83	y38	600	0	0	0	basic	-M	200000
84	xd 1	0	0	0	0	Non-essential	0	M
85	xd2	0	0	0	0	Non-essential	0	M
86	xd3	10	0	0	0	basic	0	0
87	xd4	0	0	0	0	Non-essential	0	M

88	xd5	0	0	0	0	Non-essential	0	M
89	xd6	0	0	0	0	Non-essential	0	M
90	xd7	0	0	0	0	Non-essential	0	M
	Objective	Function	(Min.) =	10999979	(Note:	alternate	Solution	Exists!!)
		Left Hand		Right Hand	Slack	Shadow	allowable	allowable
	constraint	Side	Direction	Side	or Surplus	Price	Min. RHS	Max. RHS
1	c1	0	<=	4	4	0	0	M
2	c2	0	<=	8	8	0	0	M
3	c3	0	<=	0	0	-200000	0	0
4	c4	0	<=	10	10	0	0	M
5	c5	0	<=	3	3	0	0	M
6	c6	0	<=	15	15	0	0	M
7	c7	0	<=	5	5	0	0	M
8	c8	0	<=	20	20	0	0	M
9	c9	0	<=	3	3	0	0	M
10	c1 0	15	<=	15	0	-66667	0	15
11	c1 1	5	<=	5	0	0	0	5
12	c12	0	<=	20	20	0	0	M
13	c13	0	<=	20	20	0	0	M
14	c14	0	<=	10	10	0	0	M
15	c15	0	<=	3	3	0	0	M
16	c16	0	<=	5	5	0	0	M
17	c17	0	<=	0	0	0	0	270
18	c18	0	<=	0	0	0	0	278
19	c19	0	<=	5	5	0	0	M
20	c20	0	<=	10	10	0	0	M
21	c21	0	<=	10	10	0	0	M
22	c22	40	<=	40	0	-75000	20	40
23	c23	0	<=	10	10	0	0	M
24	c24	0	<=	5	5	0	0	M
25	c25	0	<=	5	5	0	0	M
26	c26	0	<=	20	20	0	0	M
27	c27	26	<=	26	0	-84616	6	26
28	c28	0	<=	5	5	0	0	M
29	c29	0	<=	5	5	0	0	M
30	c30	0	<=	10	10	0	0	M
31	c31	0	<=	5	5	0	0	M

32	c32	0	<=	0	0	0	0	207
33	c33	0	<=	10	10	0	0	M
34	c34	0	<=	0	0	0	0	M
35	c35	0	<=	30	30	0	0	M
36	c36	0	<=	0	0	0	0	M
37	c37	0	<=	7	7	0	0	M
38	c38	0	<=	7	7	0	0	M
39	c39	0	<=	5	5	0	0	M
40	c40	0	<=	0	0	0	0	M
41	c41	0	<=	5	5	0	0	M
42	c42	0	<=	2	2	0	0	M
43	c43	0	<=	5	5	0	0	M
44	c44	0	<=	2	2	0	0	M
45	c45	0	<=	0	0	-200000	0	0
46	c46	0	=	0	0	200000	0	20
47	c47	12	>=	12	0	200000	12	32
48	c48	81	>=	81	0	200000	81	101
49	c49	34	>=	34	0	200000	34	54
50	c50	127	>=	44	83	0	-M	127
51	c51	84	>=	8	76	0	-M	84
52	c52	8	>=	8	0	200000	8	28
53	c53	33	>=	33	0	0	-M	109
54	c54	28	>=	28	0	0	-M	104
55	c55	118	>=	118	0	0	-M	194
56	c56	33	>=	33	0	200000	33	53
57	c57	28	>=	28	0	200000	28	48
58	c58	214	>=	118	96	0	-M	214
59	c59	78	>=	78	0	200000	78	98
60	c60	28	>=	28	0	0	-176	321
61	c61	28	>=	28	0	200000	28	48
62	c62	8	>=	8	0	0	-254	123
63	c63	18	>=	18	0	0	8	M
64	c64	8	>=	8	0	0	-262	18
65	c65	0	>=	0	0	0	-10	M
66	c66	8	>=	8	0	0	-270	123
67	c67	63	>=	63	0	0	-M	139
68	c68	63	>=	63	0	0	-M	159
69	c69	178	>=	178	0	200000	178	198
70	c70	58	>=	58	0	200000	58	78
71	c71	65	>=	28	37	0	-M	65
72	c72	28	>=	28	0	0	-M	65
73	c73	80	>=	13	67	0	-M	80
74	c74	311	>=	18	293	0	-M	311
75	c75	88	>=	88	0	0	51	164

76	c76	89	>=	89	0	200000	89	109
77	c77	18	>=	18	0	0	-278	143
78	c78	23	>=	23	0	0	-278	143
79	c79	143	>=	28	115	0	-M	143
80	c80	125	>=	0	125	0	-M	125
81	c81	120	>=	0	120	0	-M	120
82	c82	18	>=	18	0	0	-M	133
83	c83	3	>=	3	0	0	-204	18
84	c84	18	>=	18	0	0	3	311
85	c85	15	>=	0	15	0	-M	15
86	c86	13	>=	13	0	0	-222	306
87	c87	272	>=	0	272	0	-M	272
88	c88	292	>=	0	292	0	-M	292
89	c89	118	>=	118	0	0	-M	233
90	c90	3	>=	3	0	0	-M	275
91	c91	303	>=	8	295	0	-M	303
92	c92	3	>=	3	0	200000	3	23
93	c93	13	>=	13	0	0	-453	80
94	c94	13	>=	13	0	200000	13	33
95	c95	8	>=	8	0	0	-M	45
96	c96	554	>=	0	554	0	-M	554
97	c97	43	>=	43	0	200000	43	63
98	c98	600	<=	600	0	-200000	580	600

4. Conclusions:

Through the results and their discussion, a set of conclusions can be reached:

1. The results achieved from entering data for the LP model in the program (Win Q.S.B2) gave a good and clear impression to the project management in achieving the decision maker's goal of finding the critical path for the project and completing the project in the desired time, as the program Win Q.S.B2 was used to find the optimal solution for the model LP, where detailed and comprehensive information about variables and constraints is obtained through a combined report.
2. Many projects are slow and not completed on time. The reason is not keeping pace with the technological development that the world is witnessing at present, which leads to delaying the duration of project completion, which in turn leads to higher costs and thus failure of project completion.
3. The importance of using efficient scientific and mathematical methods in addition to sober programs such as the LP method in scheduling projects to find the critical path and the desired time to complete projects.
4. It is possible to reduce the normal time to complete the Nineveh Grain Store Project to 86 days by adding the cost of crashing some critical activities' time, amounting to 10,999,979 million dinars, to the total normal cost of completing the project in (600) days, so that the total cost of the project after crashing some critical activities will amount to 4,941,811,007 billion dinars.

Authors Declaration:

Conflicts of Interest: None

-We Hereby Confirm That All The Figures and Tables In The Manuscript Are Mine and Ours. Besides, The Figures and Images, Which are Not Mine, Have Been Permitted Republication and Attached to The Manuscript.

- Ethical Clearance: The Research Was Approved By The Local Ethical Committee in The University.

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طريقة البرمجة الخطية لإيجاد المسار الحرج والوقت المرغوب به لإنجاز المشروع

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مستخلص البحث:
يعتبر مشروع انشاء صومعة نينوى المعدني من المشاريع المهمة في وزارة التجارة / الشركة العامة لتجارة الحبوب بشكل عام ومحافظة نينوى بشكل خاص ركزت مشكلة البحث في ان العديد من المشاريع تكون متلكئة ولم تنجز في الوقت المحدد، والسبب هو عدم مواكبة التطور التكنلوجي الذي يشهده العالم في الوقت الحاضر، مما يؤدي الى تأخير مدة انجاز المشاريع وهذا بدوره يودي الى ارتفاع التكاليف وبالتالي فشل انجاز المشروع.
الهدف الرئيسي من هذا البحث هو استخدام أسلوب البرمجة الخطية (LP) لبناء نموذجين رياضيين, الأول هو لإيجاد وقت إكمال المشروع (المسار الحرج) لصومعة نينوى المعدني في الظروف الطبيعية . والثاني بناء نموذج رياضي لايجاد الوقت المرغوب به لاكمال المشروع في الظروف التعجيلية. وقد بينت النتائج التي استخرجت من حل الانموذج الرياضي باستعمال البرنامج (Win Q.S.B V2) كفاءة ودقة هذا الأسلوب الرياضي والبرنامج المستعمل واهميتهما في جدولة المشاريع بشكل ومشروع صومعة نينوى المعدني بشكل خاص في ظل تحقيق هدف المشروع واتخاذ القرار الأمثل.
نوع البحث: ورقة بحثية.
المصطلحات الرئيسة للبحث: البرمجة الخطية (LP)، طريقة المسار الحرج (CPM)، الوقت المرغوب به لإكمال المشروع (DTCP)، البرنامج Win Q.S.B V2