JEAS, Vol. 31 No. 145 (2025) pp (163-174)
P-ISSN 2518-5764 - E-ISSN 2227-703X
DOI :https://doi.org/xxxxxxxxxxxxxxx

Journal of Economics and Administrative Sciences

Comparing Methods for Estimating Gamma Distribution Parameters with Outliers Observation

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Received:22/9/2024 Accepted:24/11/2024 Published: 1 /2/ 2025

Abstract:

The purpose of our research is to work to maintain parameters for the Gamma Distribution within a better frame of precision in the context of other methods and dealing with outliers. Outliers are common and pose a threat to modelling because one little outlier scales a statistical model and brings about the estimations of parametric index validity. The objective of this study was to find useful applications for the detection and mitigating effect of outliers accomplished through the Hampel filter, the nonlinear fit of the Gamma distribution using the Median Rank Regression (MRR) method for the calculation of the shape parameter and scale parameters.

This study then generated simulated data drawn from various parameter values of the Gamma distribution modelled with outliers and ran through the proposed Hampel-MRR method. These results were compared with those produced by the MLE and classical MRR methods based on MSE performance measures. From this research, it was observed that the proposed method gives a more accurate and robust estimate of the parameters, especially with increasing sample sizes and varying parameter values.

Implications of this research are those of broader application for the improvement of reliability research. This is just what is needed in decomposing survivability time distribution aiming to predict system performance and maintenance interventions. Superb empowerment and clubbing the years of theory with real-time applications are big draws for this technique over its counterparts in the midpoint of the era of outliers.

Keywords: Reliability Analysis, Gamma distribution, Outliers, Hampel Filter, Median Rank Regression.

1. Introduction:

Outliers are observations that deviate significantly from the mean or expected range of values in a dataset. They may result from measurement errors, experimental mishaps, or actual anomalies in the data (Ali, Sedeeq, et al., 2023). Outliers play a critical role in statistical analysis as they can distort results, increase variance, and lead to flawed conclusions. They can also severely impact reliability analysis models, such as the gamma distribution, which is commonly used to predict system lifespan or time to failure.

Robust estimation techniques to mitigate the influence of outliers were first introduced by Huber (1981), who proposed the use of M-estimators to enhance estimation reliability. An important advancement in this area was the development of influence functions by Hampel et al. (1986), which assess the sensitivity of estimators to small data changes caused by outliers. These tools allow analysts to identify and manage the effects of outliers effectively during the estimation process. Similarly, Martin and Yohai (1986) introduced trimmed likelihood methods, which exclude outliers from likelihood computations, thereby reducing their distorting effects in reliability models such as the gamma distribution.

Practical guidance on outlier detection and treatment in reliability data has been provided by Meeker et al. (2022). This review emphasizes the need to combine various robust estimation techniques and significance tests for optimal results. Bayesian approaches have also been widely applied to address outliers. For example, Tsokos and Kim (2005) proposed Bayesian maximum likelihood estimation (MLE), incorporating prior constraints to reduce the effect of outliers. Bayesian methods are particularly adaptive, allowing the integration of prior knowledge, and making them suitable for systems with expected outliers or pre-existing information about the system's behaviour.

Other robust statistical methodologies have also been developed to mitigate the impact of extreme values. For instance, robust regression techniques, such as those by Nooghabi et al. (2010), are useful in reliability studies characterized by skewed distributions like the gamma distribution. These methods reduce the influence of outliers by weighting observations based on their distance from the median.

The Hampel filter has emerged as a valuable tool in reliability data analysis. Smith and Jones (2018) applied it to adjust detection windows and threshold values, enabling improved outlier identification. Gupta and Singh (2019) explored different Hampel filter designs and their effectiveness in eliminating outliers, demonstrating their importance in estimating gamma distribution parameters through median rank regression. Chen and Wang (2020) expanded this work by studying various window widths and thresholds in Hampel filter designs for enhanced outlier detection and parameter estimation.

A study by Kumar and Lalitha (2012) addressed the challenges of outlier detection in gamma regression models. They showed that using Pearson residuals provides a more efficient approach to identifying outliers, particularly in cases involving skewed or heavy-tailed data. The proposed method achieved a better balance between detecting actual outliers and avoiding false positives.

More recently, Liao et al. (2023) introduced a method for identifying multiple extreme outliers in gamma-distributed data. Using kernel density estimation and Monte Carlo simulations, their approach demonstrated enhanced robustness in detecting outliers while minimizing over-representation, where non-outlier points are incorrectly classified as outliers.

In 2024, Smith and Green focused on statistical methods and algorithms for outlier detection in dependability data. Their study highlighted the importance of novel and accurate techniques for both simple and complex datasets. Liu and Chen (2024) explored machine learning methods to address outliers in reliability analysis, while Patel and Kumar (2024) investigated Bayesian statistical techniques for improving parameter estimation in gamma distributions. These studies demonstrated the efficacy of Bayesian methods in managing outliers in reliability data.

This paper utilizes the Hampel filter to process outliers across various window sizes and threshold parameters, followed by applying median rank regression (a nonlinear method) to estimate the gamma distribution parameters (shape and scale). Simulation and real datasets are used to validate the proposed approach. The proposed method's performance is compared with the classical median rank regression method based on mean square error (MSE). The remainder of this paper is structured as follows: Section 2 discusses reliability analysis, Section 3 details the proposed method, Section 4 presents practical applications through simulations and real data, and Section 5 concludes with findings and recommendations.

2.1 Gamma distribution

The exponential distribution serves as the foundation for the gamma distribution. The gamma distribution is particularly useful for instruments or equipment that require calibration (Chakrabarty et al., 2015). It has been widely applied in various fields, including reliability engineering, life testing, insurance, meteorology, climatology, and numerous other physical contexts. If a random variable t follows a Gamma distribution, its probability density function is given by (Son & Oh, 2006).

$$f(t|\alpha,\beta) = \frac{1}{\Gamma(\alpha)(\beta)^{\alpha}} t^{\alpha-1} exp\{-\frac{t}{\beta}\} \qquad \cdots (1)$$

There is just one scale parameter and one shape in the two-parameter gamma distribution. The gamma distribution of the random variable t has scale and shape parameters of $\beta > 0$ and $\alpha > 0$, respectively (Pradhan & Kundu, 2011).

2.2 Reliability Analysis:

The application of the Gamma distribution to model and analyze time-to-failure data for systems or components is a critical aspect of reliability studies. The primary objective of such an analysis is to evaluate reliability, which is defined as the probability that a system or component will perform its intended function without failure over a specified time and under predefined conditions.

Reliability represents the likelihood that a machine or system will not fail at a given time t (t>0). This concept aligns with the idea of survival analysis in biological contexts, where the focus is on the "survival" or operational continuity of a system. Mathematically, reliability is expressed using the reliability function R(t), which is related to the cumulative distribution and probability density functions of a random variable T that represents time-to-failure (Mahmood & Algamal, 2021):

$$R(t) = P(T > t), 0 < t < \infty \cdots (2)$$

$$R(t) = \int_{t}^{\infty} f(u)du$$

$$R(t) = \int_{t}^{\infty} \left[\frac{1}{\Gamma(\alpha)\beta^{\alpha}} u^{\alpha-1} exp\{-\frac{u}{\beta}\}\right] du$$

$$R(t) = \frac{1}{\Gamma(\alpha)} \int_{t}^{\infty} \frac{u^{\alpha-1}}{\beta^{\alpha}} \exp\left\{-\frac{u}{\beta}\right\} du \cdots (3)$$

Let $y = \frac{u}{\beta} \to u = \beta y \to dy = \frac{1}{\beta} du \ \to du = \beta dy$

$$R(t) = \frac{1}{\Gamma(\alpha)} \int_{g}^{\infty} \frac{(\beta y)^{\alpha-1}}{\beta^{\alpha}} \exp\left\{-\frac{\beta y}{\beta}\right\} \beta dy$$

Where $g = \frac{t}{\beta}$

$$R(t) = \frac{1}{\Gamma(\alpha)} \int_{g}^{\infty} y^{\alpha-1} \exp\{-y\}\, dy$$

$$R(t) = \frac{\Gamma(g,\alpha)}{\Gamma(\alpha)}$$

Thus, the reliability function of the Gamma distribution is:

$$R(t) = \frac{\Gamma\left(\frac{t}{\beta},\alpha\right)}{\Gamma(\alpha)} \cdots (4)$$

2.3 Maximum Likelihood Estimation:

Consider an n-size random sample drawn from a two-parameter gamma distribution with the probability density function (PDF) described in Equation (1). Applying the general form of the log-likelihood function for this model yields:

$$logL(t|\theta) = \sum_{j=1}^{n} log\left[\frac{1}{\Gamma(\alpha)\beta^{\alpha}} t_j^{\alpha-1}\, exp\{-\frac{t_j}{\beta}\}\right] \cdots (5)$$

where α and β represent the shape and scale parameters, respectively, and t_j denotes the j-th observation in the sample.

The Maximum Likelihood Estimation **(MLE)** method is employed to estimate the parameters of the gamma distribution by maximizing the log-likelihood function. This approach ensures that the estimated parameters are those most likely to have generated the observed data, as demonstrated in previous studies (Raza et al., 2018).

2.4 Median Rank Regression Estimation:

Median Rank Regression (MRR) is a widely used method in reliability analysis for estimating the parameters of a distribution (Ali et al., 2023). This approach is particularly beneficial for modelling time-to-failure data in reliability studies, especially when not all items in the sample have failed by the end of the observation period.

MRR is especially advantageous in scenarios involving small sample sizes, as its robustness often outweighs the potential loss of efficiency compared to other methods (Ali et al., 2023). By relying on the ranks of the data rather than the raw values, MRR offers a straightforward yet effective approach to parameter estimation, even in the presence of outliers or non-normal data distributions.

Steps in Median Rank Regression Estimation:

1. Organize the failure times in ascending order.
2. Assign ranks to these failure times.
3. Calculate the median rank for each failure time using the formula:

$$MRR = \frac{i-0.3}{n+0.4} \qquad \ldots \quad (6)$$

Where:

- iii is the rank of the ith failure time (sorted from smallest to largest).
- n is the total number of data points.

This method provides a systematic and robust way to estimate distribution parameters, making it a preferred choice in reliability studies, especially in datasets with challenging characteristics.

2.5 Hampel Filter:

The Hampel filter is an effective tool for handling outliers or de-noising time series data (Mustafa and Ali, 2013). It operates by analyzing local data patterns within a specified window and applying robust statistical measures to identify and mitigate anomalies. The steps involved in implementing the Hampel filter are as follows:

- **Window Size:** Select a window size k, which determines the number of neighboring data points to include on each side of the target point.
- **Median Calculation:** For each data point x_i, calculate the median M_i of the values within the window, including x_i.
- **Deviation Calculation:** Compute the Median Absolute Deviation (MAD) around the median for the data points in the window (Sedeeq et al., 2024).

$$\text{MAD}_i = \text{median}\left(\left|x_j - M_i\right|\right) \quad \ldots \quad (7)$$

Indexing: The variable j represents the index of the data points within the defined window.

Outlier Detection: A data point x_j is considered an outlier if its deviation from the median M_i exceeds a specified threshold, typically expressed as a multiple of the Median Absolute Deviation (MAD):

$$\left(\left|x_j - M_i\right|\right) > \sigma * MAD_i$$

Here, σ is a scaling parameter, commonly set to 3 for standard outlier detection.

- **Replacement**: If x_j is detected as an outlier, replace it with the median M_i; otherwise, keep the original value.

2. Mean Squared Error:

Mean Squared Error (MSE) is a widely used metric for assessing the accuracy of estimated parameters in reliability models (Ali, 2022). It is particularly effective due to its sensitivity to large, squared differences between predicted and actual values. However, this sensitivity can also be a limitation in the presence of outliers, as they can disproportionately affect the MSE. Therefore, it is essential to consider the specific context and characteristics of the data when using MSE as an evaluation criterion (Ali et al., 2023).

2.6 Proposed Method:

The proposed method (HMRR) employs a Hampel filter to handle outliers and uses a nonlinear method based on MRR to estimate the parameters of a reliability model that has a two-parameter gamma distribution through the following:

1. Detect outliers in reliability analysis by scatter plotting the data with the estimated median rank regression for Gamma distribution parameters.

2. Employing the Hampel filter to handle outliers using several values for window size and threshold parameters (paragraph 6) to obtain the best data with the lowest mean square error.

3. Sort time data in ascending order.

4. Using Bernard's approximation approach to calculate the median rank $MRR_i = (i - 0.3)/(n + 0.4)$.

5. Finding cumulative distribution function (CDF) for Gamma distribution with estimated parameters through the nonlinear regression routine to get the objective function.

6. Using a nonlinear least squares optimization function improves the accuracy of the estimated parameters to minimize the difference between the observed mean ranks and the estimated CDF values of the gamma distribution.

7. The choice of initial parameters for MLE in the optimization function.

8. The estimated parameters for Gamma distribution (shape and scale) are extracted from the fitted parameters.

3. Simulation Study:

The data were generated from a Gamma distribution with varying shapes (10 and 20) and scales (2 and 5) parameters, as well as different sample sizes (50, 100, and 150). Two outlier values were randomly added to the generated data values and were constant for all simulations using the MATLAB program. The Gamma distribution parameters were estimated using three methods: Maximum Likelihood Estimation (MLE), Median Rank Regression (MRR), and the proposed method (HMRR). The HMRR method utilized the Hampel filter with optimal values (window size = 9 and threshold = 23.9), which provided the most effective outlier treatment, resulting in the lowest mean square error. Subsequently, nonlinear median rank regression was applied to estimate the regression parameters, using the MLE-derived values as initial estimates in the objective function (refer to the MATLAB program in the appendix). The experiment was repeated 1000 times, and the average estimated parameters from the three methods were computed, along with the mean square error (MSE) of the parameters, which are summarized in Tables 1-3:

Table 1: Results of the three methods at the shape (10) and scale (2) parameters.

Method	Sample Size	Parameter		MSE	
		Shape	Scale	Shape	Scale
MLE	50	3.4503	6.8526	43.4368	26.2980
MRR		8.2562	2.6069	7.8400	0.8088
HMRR		8.3549	2.5749	7.6655	0.7625
MLE	100	4.9723	4.3992	25.9271	6.3593
MRR		9.0950	2.2844	3.6901	0.2536
HMRR		9.1139	2.2791	3.6607	0.2496
MLE	150	5.9256	3.5920	17.2596	2.8287
MRR		9.4140	2.1792	2.3264	0.1361
HMRR		9.4219	2.1772	2.3176	0.1350

Table 2: Results of the three methods at the shape (20) and scale (2) parameters.

Method	Sample Size	Parameter		MSE	
		Shape	Scale	Shape	Scale
MLE	50	3.4456	13.3736	274.3847	135.2493
MRR		16.6164	2.5964	34.0712	0.8080
HMRR		17.0209	2.5326	32.7435	0.7146
MLE	100	5.6221	7.6976	207.3082	33.7162
MRR		18.1160	2.2899	15.3565	0.2526
HMRR		18.1987	2.2784	15.0566	0.2430
MLE	150	7.2947	5.8026	162.2324	15.0505
MRR		18.7758	2.1802	9.1556	0.1322
HMRR		18.8060	2.1763	9.0600	0.1297

Table 3: Results of the three methods at the shape (20) and scale (5) parameters.

Method	Sample Size	Parameter		MSE	
		Shape	Scale	Shape	Scale
MLE	50	2.8317	40.1896	294.900	1277.20
MRR		16.6163	6.4911	34.0705	5.0504
HMRR		17.0250	6.3302	32.7366	4.4635
MLE	100	4.7583	22.5822	232.5975	317.1748
MRR		18.1160	5.7247	15.3568	1.5787
HMRR		18.2001	5.6956	15.0517	1.5184
MLE	150	6.2959	16.7239	188.2427	141.1941
MRR		18.7759	5.4505	9.1556	0.8264
HMRR		18.8065	5.4405	9.0595	0.8105

Analysis of Simulation Results: Based on the data presented in Tables 1-3, the following observations can be made:

- The parameters (shape and scale) of the Gamma distribution estimated by the proposed method (HMRR) demonstrated greater accuracy compared to the classical method (MRR) across all simulation scenarios.
- The MLE method was unable to accurately estimate the Gamma distribution parameters in the presence of outliers, whereas both the proposed and classical methods exhibited robustness to outliers.
- The proposed method effectively addresses the issue of outliers.
- The accuracy of the estimated parameters improves as the sample size increases.

The accuracy of the estimated parameters (shape and scale) diminishes as their values increase.

3.1 Real Data

The real data are taken from (Murali, 2016) which represents (24) units that were tested for reliability and have a Gamma distribution and are shown in Table 4:

Table 4: Life Test Data

Life Test Data							
61	66	56	43	49	58	58	56
53	50	44	58	65	53	43	48
62	61	67	48	55	53	62	40

The data in Table (4) do not include outliers as shown in Figure 1 (all the Figures in the appendix). The parameters of the Gamma distribution were estimated through the three methods with the Kolmogorov-Smirnov test, and the results are summarized in Table 5.

Table 5: Results of the three methods for real data

Method	**Parameter**		**Kolmogorov-Smirnov Test**	
	Shape	**Scale**	**Statistics**	**p-value**
MLE	50.4908	1.0802	0.10486	0.92976
MRR	41.0037	1.3440	0.08941	0.98151
HMRR	41.0037	1.3440	0.08941	0.98151

Since the Kolmogorov-Smirnov test statistic is below the critical value (0.32286) for all three methods, the data is confirmed to follow a Gamma distribution, as supported by the p-value, which exceeds 0.01. The three estimators performed effectively in the absence of outliers. However, the classical method (MRR) demonstrated comparable efficiency to the proposed method (HMRR) and yielded more accurate Gamma distribution parameters than the MLE method. Figure 2 illustrates the MRR method, showing the nonlinear estimation of the objective function for the cumulative distribution function (CDF) of the Gamma distribution, based on the initial parameter values from the MLE. Four randomly generated outliers were introduced in place of the true values, and Figure 3 presents the corresponding box plot of the data. The estimated Gamma distribution parameters from all three methods are summarized in Table 6:

Table 6: Results of the three methods for real data with outliers

Method	Parameter		K. S. Test	
	Shape	Scale	Statistics	p-value
MLE	2.4629	35.6113	0.40718	0.0004135
MRR	32.8612	1.7919	0.16667	0.46784
HMRR	46.8403	1.2433	0.15139	0.58878

Since the Kolmogorov-Smirnov test statistic is below the critical value (0.32286) for both the MRR and HMRR methods, the data is confirmed to follow a Gamma distribution, as indicated by the p-value, which exceeds 0.01. Both the MRR and HMRR estimators performed well in the presence of outliers, while the MLE method failed to accurately estimate the Gamma distribution parameters when outliers were present (the Kolmogorov-Smirnov test statistic exceeded the critical value, and the p-value was less than 0.01). The proposed method (HMRR) provided more accurate results than the classical method (MRR). Figure 4 illustrates the MRR method, showing the nonlinear estimation of the objective function for the cumulative distribution function (CDF) of the Gamma distribution, based on the initial parameter values from MLE in the presence of outliers. Figure 5 demonstrates how the proposed method (HMRR) effectively addresses the issue of outliers.

4. Conclusions:

1. The parameters (shape and scale) of the Gamma distribution estimated by the proposed method (HMRR) were more accurate than those estimated by the classical method (MRR) across all simulation cases and real data, even in the presence of outliers.
2. The proposed method effectively addresses the issue of outliers.
3. The accuracy of the estimated parameters improves as the sample size increases.
4. The accuracy of the estimated parameters (shape and scale) decreases as their values increase.
5. For real data, all three estimators performed well in the absence of outliers.
6. The classical method (MRR) demonstrated comparable efficiency to the proposed method (HMRR) and yielded more accurate Gamma distribution parameters than the MLE method for real data without outliers.

5.Recommendations:

1. The proposed method should be applied to estimate the Gamma distribution parameters in the presence of outliers.
2. Future research will focus on the linear robust estimation of median rank regression for the Gamma distribution.
3. Future studies will also explore the robust estimation of median rank regression for other lifetime distributions (e.g., Weibull, Exponential, etc.).

Authors Declaration:

Conflicts of Interest: None

We Hereby Confirm That All the Figures and Tables in The Manuscript Are Mine and Ours. Besides, The Figures and Images, which are Not Mine, Have Been Permitted Republication and Attached to The Manuscript

Ethical Clearance: The Research Was Approved by the Local Ethical Committee at The University.

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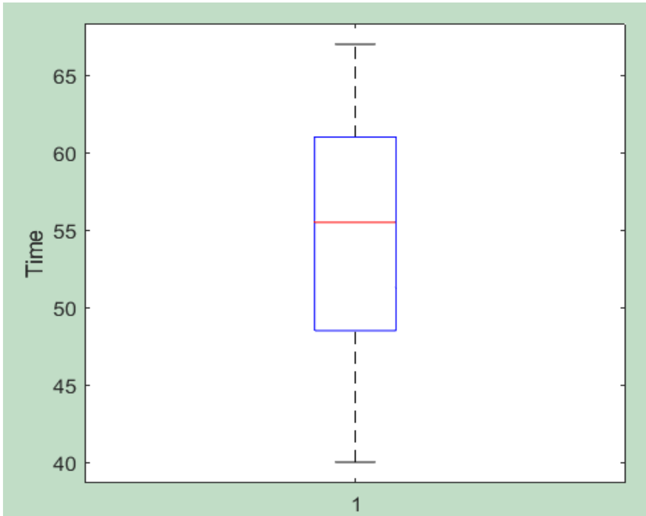

Figure 1. Box plot for real data

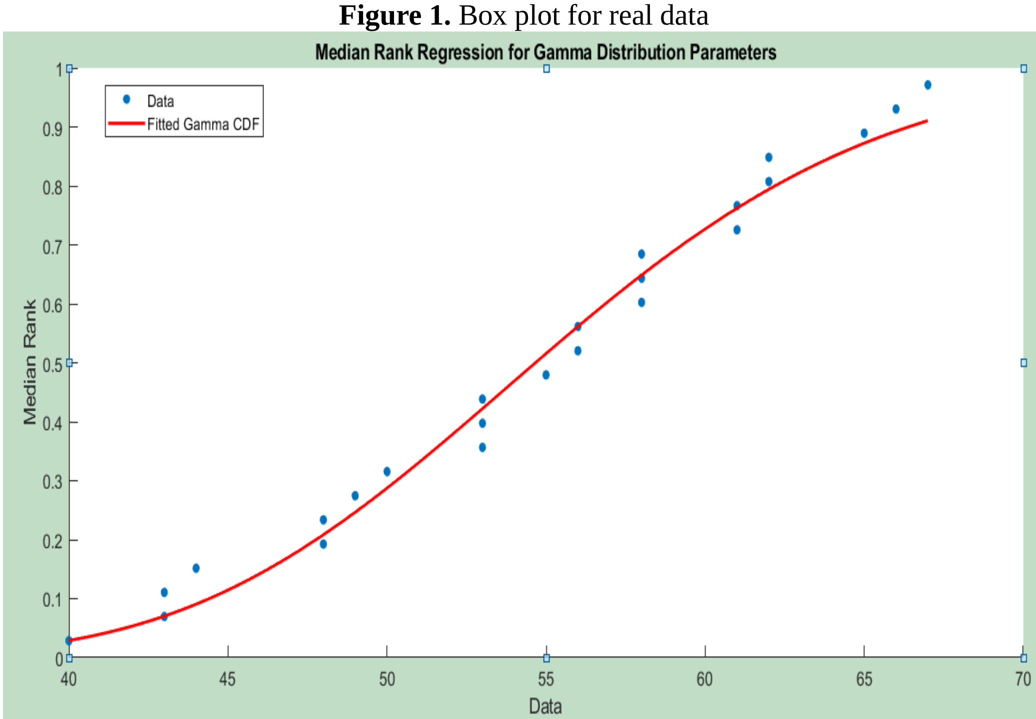

Figure 2. Classical Method (Median Rank Regression) for real data

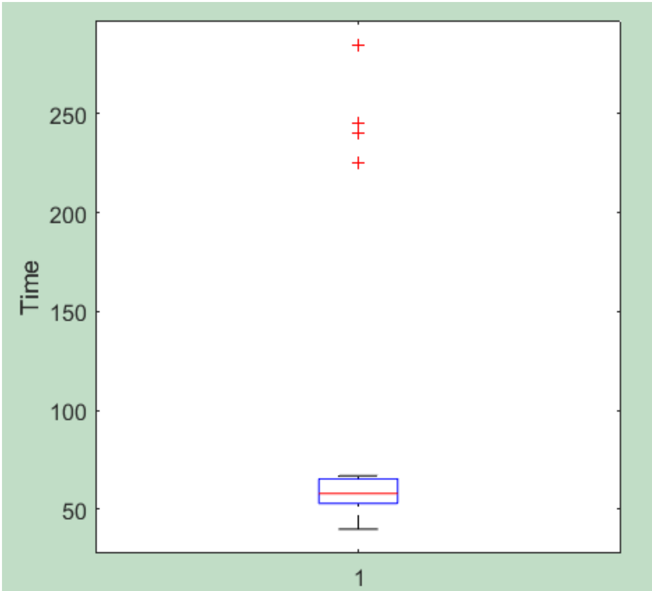

Figure 3. Box plot for real data with outliers

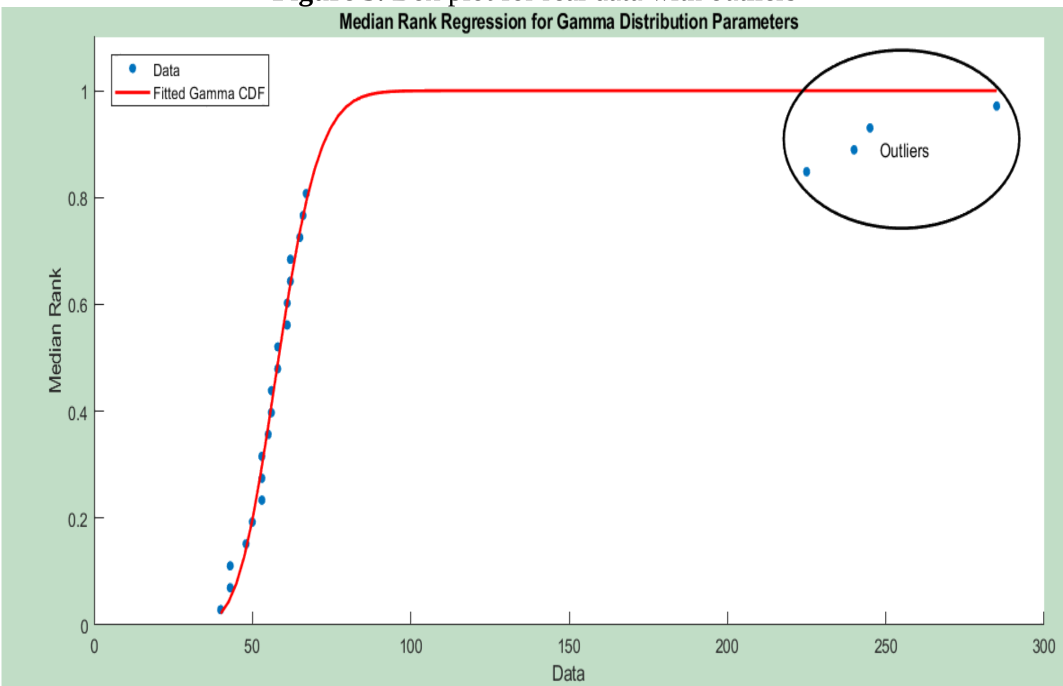

Figure 4. Classical Method (MRR) for real data with outliers

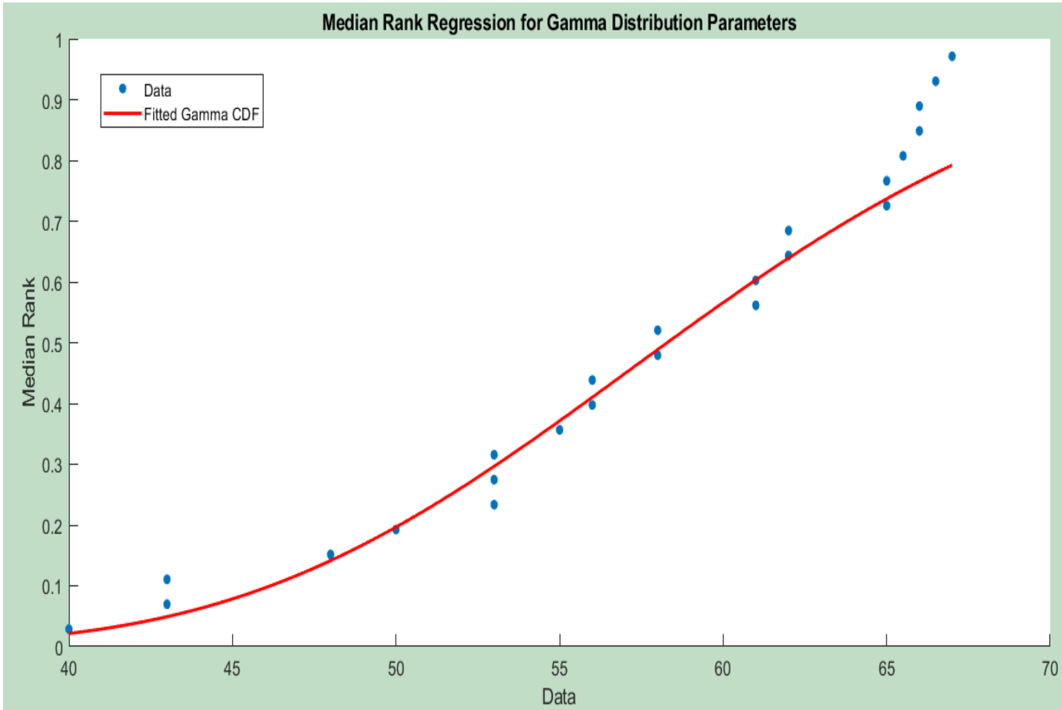

Figure 5. Proposed Method (Median Rank Regression) for real data with outliers