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Spatial Quantile Autoregressive Model: A Review

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Abstract:
This paper is specifically a detailed review of the Spatial Quantile Autoregressive (SARQR) model that refers to the incorporation of quantile regression models into spatial autoregressive models to facilitate an improved analysis of the characteristics of spatially dependent data. The relevance of SARQR is emphasized in most applications, including but not limited to the fields that might need the study of spatial variation and dependencies. In particular, it looks at literature dated from 1971 and 2024 and shows the extent to which SARQR had already been applied previously in other disciplines such as economics, real estate, environmental science, and epidemiology.
Accordingly, evidence indicates SARQR has numerous benefits compared to traditional regression models: These estimates are robust to outliers and heterogeneous spatial effects and capture fully conditional distributions with respect to mean regression models. The review supports future work toward enhancing estimation approaches and possible SARQR application extensions to other fields. The spatial modeling has applicability in the research, decision-making, and profession formulation because it encourages a broader SARQR application in economic analysis, infrastructure planning, and public health policy. Future research must aim at refining estimation methods and integrating SARQR with other models of analysis to optimize its usefulness in utilizing sophisticated spatial data.
Keywords: Quantile regression; Spatial quantile Autoregressive; Weight matrix; Spatial autoregressive model; Check function; Moran's I.

1. Introduction:

Regression analysis has been used in statistical modeling for more than 200 years to measure the relationships between variables. One of the most popular statistical techniques for capturing effects at the mean is classical regression. Conventional regressions assume that the effects of the independent variables and regression coefficients are constant across the population. Nonetheless, such average effects may not always be of interest in various domains and may, at times, exhibit considerable heterogeneity.

When spatial data is analyzed using standard statistical methods, the problem of spatially correlated observations may arise. Unreliable and skewed estimators can result from using standard assumptions about uncorrelated errors and data. Therefore, spatial effects are the primary focus of spatial econometrics, a branch of economics. Spatial econometrics represents the theory and scientific methods for analyzing spatial economic series, where spatial data for each variable are distributed based on location rather than time, as is typical in econometrics. Spatial dependence occurs when one area depends on another, while spatial heterogeneity arises when there is variability in relationships between areas, or when the same dependent variable yields unequal responses in different locations within a given area. One effect of spatial heterogeneity is that regression coefficients can vary spatially. If there is spatial heterogeneity in regression coefficients, the regression model becomes less capable of explaining the actual data phenomenon. In some cases, testing for spatial effects by including outliers in the data can cause a method to fail in addressing these effects. Therefore, it is common practice to remove outliers. However, removing outliers may not always be the correct approach, as sometimes outliers can provide valuable information that other data cannot. Excluding outliers from the model can lead to biased results that do not reflect the true underlying phenomenon.

Spatial analysis, or spatial data analysis, is a method of measuring spatial relationships between phenomena based on location measurements and adjacency to interpret spatial relationships and leverage them. Comprehending the rationale behind the dispersion of phenomena on the surface of the Earth and forecasting their future behavior. Spatial analysis relies on the assumption that each phenomenon has a spatial extent or range and a specific spread and distribution. This type of analysis aims to uncover the mutual spatial relationships between the components of the phenomenon as well as between multiple types of phenomena in the same spatial extent to construct a spatial model of the spatial phenomena that provides a clear representation of those phenomena (Shawq et al., 2023). The type of point pattern distribution random, uniform, or clustered determines the methodology of spatial analysis. Serial correlation is another name for spatial autocorrelation (Mashee Al-Ramahi et al., 2022). The use of statistical theory or geographic methods plays a significant and important role when it comes to comprehending the potential for exhibiting geographical and temporal developments, meaning events that change over time and are influenced by location (Mashee Al Ramahi & Al Bahadly, 2020).

The SARQR model combines the SAR model with QR. One of the key advantages of the QR model is that it aims to minimize asymmetric weighted absolute errors to eliminate data variability. Another significant advantage is that it produces a model that accounts for outliers, which is crucial. The combination of the spatial autoregressive model and quantile regression results in a robust model for addressing issues of dependency and variability in spatial data analysis.

The research is organized into sections: Section 2 literature review and hypothesis development. Section 3 spatial weight matrix, QR, SAR, and SARQR models, and spatial effects. Section 4 discussion of literature review, while Section 5 conclusion.

2. Literature Review:

The literature review is an important and essential step in order to identify and understand the key research and studies that have contributed to the development of the Spatial Autoregressive Quantile Regression (SARQR) model. This section will present the most significant research that has addressed quantile regression, spatial autoregressive regression, and the SARQR model. Some of these studies have explored this model from different angles, with a focus on how spatial dependence among variables affects the model. We will review a selection of these studies.

(Hallin et al., 2009) Using time series asymptotic assumptions, local linear spatial quantile regression is examined with an emphasis on asymptotic behavior. The method produces far more precise information than the mean regression approach that is employed in most spatial modeling methods.

(Su & Yang, 2011) was the first, as far as we know, to construct a spatial quantile autoregressive (SQAR) model by combining these two comprehensive models: the quantile regression model and the spatial autoregressive model. The Instrumental Variable Quantile Regression (IVQR) method was used to estimate the internal spatial lag in the model.

Used (Liao & Wang, 2012) quantile regression and spatial econometric modeling to study housing price variation. They found significant differences in implicit prices and found that integrating spatial econometrics and quantile regression is beneficial. They discovered a U-shaped pattern across the quantiles, which may be indicative of households moving from public to private vehicle ownership. Spatial dependence is strong in the upper and lower parts but weak in the middle range.

(Febriyanti et al., 2015), conducted research, and one of the main conclusions was that the variation occurring in spatial autoregressive modeling could be addressed through creating SARQR models, according to a case study of Java GDRP. The SARQR modeling generates multiple independent models for each segment. Additionally, it explains models for a few cities or regions with values that significantly deviate from the overall GDRP average in Java.

In research (L. Zhang, 2016). investigates the influence of residence within a 100-year floodplain on the valuation of single-family residential property sales, utilizing data from house sales within the Fargo-Moorhead Metropolitan Statistical Area spanning the years 2000 to 2013. SARQR was used. To investigate the influence of flood risks on dwellings classified as conditionally higher-priced versus lower-priced, while considering spatial autocorrelation, The findings show that the negative impact of flood risks on property values is less noticeable for more expensive homes and more noticeable for those with lower prices.

In this study (Ngwira, 2019), the Integrated Nested Laplace Approximation (INLA) method was used estimate the SARQR model. Results indicated that the different ages of mothers significantly impact birth weight rates, with clear spatial effects.

In the same year, (Hussein & Akkar, 2019a) presented a study that involved estimating the Semi-Parametric Spatial Auto Regressive Error Model (SPSEM), which suffers from the issue of spatial error correlations. The study employed the two-stage method and proposed techniques to remove the effect of spatial error correlations through the spatial variance-covariance matrix of the errors. The comparison between these methods for the SPSEM model was based on a simulation approach using the Mean Absolute Percentage Error (MAPE).

(Hussein & Akkar, 2019b), presented a study aimed at estimating two models: the first is the Semi-Parametric Spatial Auto Regressive Error Model (SPSEM), which suffers from error correlations, and the second is the Semi-Parametric Spatial Auto Regressive Model (SPSAR). Various estimation methods were employed, based on a simulation approach.

(Rahim, 2020) conducted an investigation of air pressure in the Kurdistan region using the spatial autoregressive and spatial error model techniques. Estimation method used Maximum Likelihood, and the estimation results were obtained. To determine the most favorable results, the two models were compared using various comparison criteria, considering three different matrices. It was found that the rook and queen matrices outperformed in the estimation of the spatial autoregressive model, while for the spatial error model, none of the three matrices were significant.

(T. Yu et al., 2021) carried out a study addressing concept of spatial autocorrelation and skewed distribution in modeling accident rates. The SARQ model was used to understand the modeling relationships between accidents a range of relevant factors were understood, considering effects of spatial dependence. The estimation method employed in the study was two-stage least squares. The key findings highlighted the superiority of the Spatial Autoregressive Quantile Regression model over the Quantile Regression model in terms of forecast performance and goodness-of-fit. Additionally, variables' effects at various quantiles were categorized into three types: increasing, non-changing, and U-shaped.

(Wardhani & Yanti, 2021). In this research, modeling was carried out using the Spatial Autoregressive Quantile Regression (SARQR) method with parameter estimation using Instrumental Variable Quantile Regression (IVQR) on malnutrition data for toddlers in Bandung City in 2018. The quantile that was employed was specifically 0.1, 0.25, 0.5, 0.75, and 0.9. According to the SARQR modeling results, each quantile has a different set of parameter values, and each quantile also has a separate set of variables that significantly affect.

(Semerikova & Blokhina, 2022) This paper analyzes determinants and spatial effects in housing prices using quantile regression analysis. It examines 401 German regions from 2004-2020, focusing on their geographical location and prices. The study finds that spatial allocation plays a significant role in real estate pricing, with regions close to regional centers benefiting. There's a stronger positive relationship between regions with expensive housing and a weaker relationship with those that are less appealing. The research also reveals that areas with increased costs are more responsive to changes in infrastructure or policies, whereas areas with lower costs show a more gradual response.

This study addressed (Dai et al., 2022) partially linear spatial autoregressive models using the quantile regression technique and potentially variable coefficients. The changing coefficients were approximated using the B-spline. For the coefficients, tests of rank scores were developed for the invariant and constancy of changing hypotheses.

(Castillo-Mateo et al., 2023), study is the proposed quantile autoregressive (QAR) modeling approach is characterized by sufficient flexibility to illustrate the development of changed quantiles of the highest daily amount temperature distribution In addition to determining how the Aragon area is affected by climate change. The model's only spatial covariate, elevation, using the four included spatial processes, is capable of capturing the significant variability in climate conditions across the region, enabling geographic variation in temperature mean levels, temporal trends, and serial correlations. More specifically, compared to the observed temporal increases in the median, the 0.95 quantile displays greater rises in certain valley regions, while no increase is observed in the 0.05 quantile in the northwest.

In the same year (X. Liu & Chen, 2023), presented a study that addressed the problem of variable selection in a SARQR model and the fixed effects. if the study focused on identifying spatial effects, estimating unknown parameters, and simultaneously selecting explanatory variables by applying penalties to the relevant parameters. Additionally, the researchers introduced an algorithm for variable selection and demonstrated the asymptotic properties of the penalized estimator when dealing with large samples. The results from numerical simulations and the analysis of actual data showed that the proposed method exhibited excellent performance, confirming the effectiveness of this approach in solving the variable selection problem in such models.

(Hussein & Akkar, 2023) In this research, the SPSAR model was estimated, which describes the relationship between the response variable and the explanatory variables under the Queen spatial contiguity criterion for the spatial weights matrix. By utilizing some semi parametric estimation methods, the model was estimated based on simulation techniques.

(Lee & Huang, 2024) This study assesses, in light of two different stages of the COVID-19 pandemic, the experimental application of a well-known scale of social vulnerability to local economic and public health outcomes. When predicting disproportionate economic harms resulting from the pandemic and death tolls across the United States, quantile regression provides stronger evidence in favor of social vulnerability compared to classical least squares analysis. When expanding the regionally adjusted spatial autoregressive model to include quantile regression, it becomes evident how social vulnerability varies within large areas that have experienced varying degrees of pandemic consequences.

(Hapsery et al., 2024) presented a paper titled "Perbandingan SAR Dan SARQR Pada Pemodelan Index Pembangungan Manusia di Jawa Tengah Tahun 2022." Two significant models, the Spatial Autoregressive Quantile Regression (SARQR) model and the Spatial Autoregressive (SAR) model, both involving spatial analysis, were compared in this study. The Spatial Autoregressive Quantile Regression (SARQR), which combines the SAR approach with quantile regression, is the most suitable analysis given the potential spatial effects at certain quantiles of the independent variables

3. Research Methodology:

3.1. The Spatial Weight Matrix:

Also known as the spatial autocorrelation matrix is used to represent spatial relationships and is denoted by the symbol (W). It has dimensions of (n x n). In the spatial model, n is the number of observations that were used. It's a square matrix whose elements are positive values, and the matrix doesn’t need to be symmetric. It is built based on adjacencies, meaning the adjacency relationships for each location with other locations are in one row of the matrix. The matrix's diagonal members are all equal to zero (Gumprecht, 2005).

Determining spatial effects heavily relies on the choice of spatial weight matrices. Results of any spatial analysis depend on the matrix used. Therefore, considerable attention should be given to selecting the appropriate spatial weight matrix. Determining spatial correlation is a fundamental requirement before initiating any standard spatial econometric analysis or exploratory analysis of spatial data. One of the key factors in determining spatial effects is the selection of the spatial weights matrix, so choosing a suitable weights matrix is crucial, as the results of any spatial analysis depend on the specification of the matrix used. Thus, much attention should be paid to selecting the appropriate spatial weight matrix (Rusche, 2010). The matrix's general formula can be expressed as follows: (Permai et al., 2018).

$$W = \begin{bmatrix} w_{11} & \dots w_{1n} \\ . & . \\ . & . \\ . & . \\ w_{n1} & \dots w_{nn} \end{bmatrix} \dots (1)$$

3.2. Quantile Regression:

Regression analysis aims to analyze the regression line and provide a conceptual and approximate relationship between each explanatory and response variable by drawing or displaying the relationship by the direction of the approximation line .

The method described is known to provide efficient and robust regression capabilities compared to those offered by ordinary least squares (OLS) regression. It does not impose any assumptions on the error distribution, making it a resilient alternative. Referred to as a robust and alternative regression method to ordinary least squares (OLS), most studies indicate that robust regression is insensitive to outliers and heteroscedasticity, making it capable of accommodating residuals that do not follow a normal distribution, which is common in many applications (Benoit et al., 2013) (Koenker & Bassett Jr, 1978). Instead of being restricted to estimating the conditional expectation (E(Y|X)) as in ordinary mean regression, quantile regression allows for a more thorough analysis of the relationship between the response variable and the explanatory variables by estimating various conditional quantiles $(Q_\tau(Y/X), 0 < \tau < 1)$ of the response variable distribution (Al-Tai & Al-Kazaz, 2022) (Baha Alwan & Abdulmohsin Ali, 2024) (Majid & Al-Bayati, 2018a). It has been applied in various fields such as econometrics, finance, medical studies, agriculture, and others. The mathematical formula for the quantile regression model is shown below:

$Q_\tau(Y/X) = X\beta_\tau \ldots (2)$

Where:

Y: A vector of observations of the dependent variable.

X: A matrix of observations of the explanatory variables, (n× (k+1)).

$\beta\tau$: Represents the parameter vector at quantile (τ), where $0 < \tau < 1$.

τ: The quantile level, where $0 < \tau < 1$.

To obtain $\beta\tau$ according to the following formula: (K. Yu & Lu, 2003)

$\min_{\beta_\tau} \sum_{i=1}^{n} \varphi_\tau(y_i - x_i\beta_\tau) \ldots (3)$

Where $\varphi_\tau(.)$ is the check function. To select the loss function, it can be expressed as follows: (Majid & Al-Bayati, 2018b) (Marasinghe, 2014)

$$\varphi_\tau(\varepsilon) = \begin{Bmatrix} \tau\varepsilon & \text{if} & \varepsilon \geq 0 \\ -(1-\tau)\varepsilon & \text{if} & \varepsilon < 0 \end{Bmatrix} \ldots (4)$$

$\varepsilon = y_i - x_i\beta_\tau$

Alternatively, it can be phrased as follows:

$\varphi_\tau(\varepsilon) = \frac{|\varepsilon| + (2\tau - 1)\varepsilon}{2} = \varepsilon(\tau - I(\varepsilon < 0)) \ldots (5)$

3.3. Spatial Quantile Autoregressive Model:

A spatial autoregressive model (SAR) is a model that uses cross-sectional data to combine simple regression with spatial lag on the dependent variable. When analyzing spatially related data, self-correlation and spatial dependence must be taken into account to avoid biased estimation. Generally, spatial models are important for researchers due to their ability to calculate location-specific effects. Also known as the Mixed Spatial Autoregressive Model or Spatial Lag Model, this model represents a special case of the General Spatial Autoregressive Model (SAC) proposed by Anselin. Mathematically, it can be expressed as follows (Lee et al., 2023) (LeSage, 1999):

$\underline{Y} = \rho W\underline{Y} + X\underline{\beta} + \underline{\varepsilon} \ \ldots (6)$

Where:

ρ: is the parameter representing spatial effects or it's also called the spatial autoregressive parameter.

W: A (n×n) contiguity matrix.

ε: vector of random errors of size (n×1).

The value of the spatial autoregressive parameter falls within the range $-1 < \rho < +1$.
When the value of $\rho = 0$, it means there is no spatial autocorrelation (no spatial dependence). In this case, the regression model is the traditional model.
Neighboring values tend to be similar if $\rho > 0$, which indicates positive spatial autocorrelation. Conversely, neighboring values tend to diverge from one another if $\rho < 0$, which denotes negative spatial autocorrelation (Ali & Hadi, 2014) (Hussein & Akkar, 2019b).

Spatial Quantile Autoregressive Model this model is a combination of two models: quantile regression and spatial autoregressive regression, denoted by the symbol SARQR. It is characterized by its ability to overcome outliers and address the problem of heterogeneity in the data (Tribhuwaneswari et al., 2022). The SARQR model includes regression parameters and a spatial lag parameter, which depend on specific quantile values. The mathematical formula for this model is as follows:

$$Y = \rho_\tau WY + X\beta_\tau + \varepsilon \quad \dots (7)$$

Where:
ρ_τ: Represents the spatial autoregressive parameter at quantile (τ).

3.4. Spatial Effects:

Spatial effects consist of spatial dependence and spatial heterogeneity. To determine the presence of spatial dependence between regions, Moran's I test, one of the main measures for detecting the degree of spatial autocorrelation between elements of the studied phenomenon, was used. This test is named after the scientist who developed it, and it measures the similarity of adjacent phenomena. If the difference between adjacent parameters is smaller than the difference between all parameters, similar values will be clustered together.

It assesses the spatial distribution pattern-whether dispersed, clustered, or random. Its value ranges between -1 and +1. It is also known as a general measure and is based on the Generalized Linear Regression Model, which takes the form [$Y = X\beta + \varepsilon$]. When the values of a variable at one location are correlated with the values of the same variable at an adjacent location, it indicates spatial autocorrelation, sometimes referred to as the proximity or contiguity effect (Mohammed et al., 2022).

This idea is based on Tobler's First Law of Geography, established in 1970, which states that "everything is related to everything else, but near things are more related than distant things." This implies that things closer to each other have a stronger relationship than those further apart, meaning every phenomenon is related to another, but nearby phenomena are more strongly related than distant ones (Chen & Rodden, 2009). formula for Moran's is as follows (LeSage, 1999):

$$I = \frac{n\,(e'we)}{S_o\,(e'e)} \quad \dots (8)$$

Where:
$S_o = \sum_{i=1}^{n}\sum_{j=1}^{n} W_{ij}$: The sum of all elements in the w matrix.
W: The adjacency matrix.
n: sample size.
e: The residual vector with dimensions $n \times 1$.
To determine whether the value of Moran's I is statistically significant at a certain confidence level, the Moran test (Z) is used, as illustrated by the formula (9) below, according to the following hypotheses:

$H_0: \rho = 0$
$H_1: \rho \neq 0$

$$Z = \frac{I - E(I)}{\sqrt{V(I)}} \quad \dots (9)$$

$$E(I) = \frac{n\left(tr\left((I - X(X'X)-1)W\right)\right)}{S_o\,(n-k)}$$

$$V(I) = \frac{tr\left((I - X(X'X)-1)W(I - X(X'X)-1)W'\right) + tr\left((I - X(X'X)-1)W\right)^2 + \left(tr\left((I - X(X'X)-1)W\right)\right)^2}{(n-k)\,(n-k+1)}\left(\frac{n}{S_o}\right)^2 - \left(E(I)\right)^2$$

Where:
X' : an Idempotent Matrix that is square and symmetric.
tr: The trace of the matrix.
K: The number of explanatory variables.

The calculated value of Z is compared to critical Z value at a certain significance level. If result of the Moran test is significant, it indicates the presence of a relationship between the geographic regions that warrants attention and requires spatial analysis, i.e., the use of spatial regression models. Otherwise, a general linear model (GLM) is sufficient.

4. Discussion of Literature Review:

The presented is a study (Fisher, 1971) on estimating spatial econometrics, addressing the standard economic problems involving spatial links among variable observations. He focused on spatial dependence in estimating cross-sectional models, which they handled through various methods. He also proposed a balanced spatial covariance matrix for disturbances and then coupled the results with more traditional estimation procedures.

(Paelinck et al., 1979) published a little volume named Spatial Econometrics. This is recognized as the original research on the topic of the study of spatial econometrics and its unique methodology. In this volume, they first discovered geographical dependency among hedonic regression residuals.

Researchers presented (Mardia & Marshall, 1984) a study where they utilized the method of maximum likelihood to estimate spatial autoregressive models when residuals are correlated. They assumed that observations were Gaussian and explored conditions ensuring the natural convergence and consistency of estimates. Spatial data analysis in this context was of primary interest. Some simulation experiments were conducted to evaluate the behavior of estimates for a small sample. Subsequently, the application of spectral approximation to maximum likelihood operations on lattices was discussed.

(K. Yu & Moyeed, 2001) presented a study in which they assumed random error terms in a Bayesian quantile regression model follow an asymmetric Laplace distribution. Regardless of the actual distribution of the data, it was found that using an asymmetric Laplace distribution for the Bayesian quantile regression model makes the method efficient and robust. Additionally, the algorithm for computing the posterior distribution of the model parameters was fast to execute and provided efficient estimators, as demonstrated through a simulation example and two real data examples.

In (Gumprecht, 2005) presented a study providing a concise and general overview of spatial data characteristics and the importance of spatial regression. The study discussed how spatial dependence in data is detected and introduced specific models and estimation techniques that can be used. Additionally, it addressed the processes of spatial auto regression and spatial moving averages, which are used to display spatial effects. Models were estimated using the methods of M.L. and the two-stage least squares method.

Developed the Markov Chain Monte Carlo method (MCMC) (LeSage & Parent, 2007), to compare Bayesian models to OLS models, incorporating SAR and spatial error models. This method addresses large-scale scenarios where the number of possible models depends on multiple explanatory variables, making posterior likelihood function computation difficult or impractical. Bayesian model averaging is used to average estimates and inferences over models using posterior probabilities as weights.

Qadam (Zietz et al., 2008), is a study in which they mentioned that in housing research, ordinary least squares (OLS) regression is often used to determine the relationship between a specific home characteristic and the sale price. Study results vary, not only in terms of statistical significance and the sizes of OLS coefficients but sometimes even in the direction of the effect. According to this study, some of the observed variation in estimated housing characteristic values can be attributed to the fact that different attributes are valued differently within a given distribution of home prices. To study this issue, quantile regression was used, either with or without accounting for spatial autocorrelation, to calculate the coefficients for a variety of variables across multiple quantiles.

(Kostov, 2009), argues that ignoring spatial dependence is akin to leaving out a significant explanatory variable, which typically leads to estimates that are inconsistent and biased. He applied spatial lag quantile regression to a hedonic land price model. Hedonic models of land prices that ignore spatial autocorrelation may lead to biased results. This leads to differences in the characteristics of pleasure effects as well as in the degrees of spatial self-relevance, which is considered the most important.

A sample of agricultural sales in Northern Ireland was subjected to this method. The computed confidence intervals compare well with those in the parametric spatial lag model, given the parametric rate of convergence for the QR estimator.

In this study (Li & Liny, 2010), a Bayesian approach using Gibbs sampling was proposed to solve the flexible network model. Although the marginal posterior of regression coefficients equals those provided by the non-Bayesian flexible network, Bayesian flexible networks have two main advantages. Firstly, due to the Bayesian procedure, the distributional results of estimates are straightforward, facilitating statistical inference. Secondly, the flexible network method selects both penalty parameters simultaneously, thus preventing the "double shrinkage problem." Simulation experiments and real-data examples demonstrate that Bayesian flexible networks perform better in variable selection while behaving similarly in terms of prediction accuracy.

(Lum & Gelfandy, 2012) presented a study with spatially dependent errors using the MCMC algorithm and demonstrated the suitability of this model within a Bayesian structure. Its application was showcased using a dataset of home sales prices. Subsequently, they extended the asymmetric Laplace process to accommodate large datasets through the asymmetric Laplace predictive process and demonstrated use asymmetric Laplace predictive process on a dataset of birth weights, considering common variables of mothers. Particularly in the first example, it is shown that the conditional quantile predictive advantage can be obtained by incorporating spatial structure through the asymmetric Laplace process or asymmetric Laplace predictive process.

Also in the same year (Trzpiot, 2012)conducted a study using spatial quantile regression instead of traditional spatial regression. The goal was to integrate quantile regression with spatial econometric modeling. The findings show substantial variance between quantiles, proving that simple regression is insufficient on its own. In certain regions of the distribution, quantile estimates for a spatial lag model show a significant degree of spatial dependency.

(S. Liu & Hite, 2013) conducted a study on green spaces and housing prices by integrating spatial econometrics techniques. This approach is essential because the value of housing may be influenced by the characteristics of nearby properties. Put differently, by integrating spatial econometric techniques and merging them with quantile regression, this work enabled spatial heterogeneity in the estimation to comprehend the influence of location on various home price levels. The study data is from Delaware County, Ohio. By incorporating spatial lag, we can compare the results, which revealed significant variation with or without spatial effects across quantiles. This suggests that buyers of luxury homes might place a different value on green areas than buyers of mid-range or affordable homes.

Conducted (Helbich et al., 2014), a study in which they highlighted that in real estate economics, spatial heterogeneity modeling remains a contentious issue. The results indicated the necessity of always considering heterogeneity, even though dummy variables are less suitable for explaining partial geographical relationships. Spatial diversity in transactions is of paramount importance for locally operating decision-makers, necessitating a specific understanding of the local or regional housing market. This aids in better understanding local housing price anomalies and improving policy measures.

(Yang & He, 2015) They used observed daily precipitation data from seven stations in Illinois, along with simulated precipitation, humidity, and the highest temperature recorded by the ERA-40 reanalysis model, as predictors. The authors employed a linear QR model to estimate the conditional quantile functions at 0.95 and 0.99 quantiles. They demonstrated how their approach could leverage spatial correlation to improve efficiency over standard quantile regression estimates.

(H. Zhang & Wang, 2016), chose relevant macroeconomic variables and tried to address the query, "How effective are these policies?" in order to assess the diverse spatial effectiveness of urban housing macro-control policies on housing prices using spatial quantile regression. Home purchase limitations are successful in curbing speculative demand for homes, according to estimation results using a recently released dataset covering sixty-eight big cities of various sizes, ranging from small to medium, from the beginning of 2011 to the end of 2013.

Effectively lowering home prices is difficult, though, particularly in places where prices are high. Other policies can effectively reduce housing prices, but their efficacy varies depending on the city and the level of housing prices.

In 2017, researchers (Benoit & van den Poel, 2017), presented the R package Bays for Bayesian estimation of QR, encompassing both binary and continuous dependent variable approaches. The efficient Gibbs sampling approach was employed in the MCMC estimation. In addition to the main algorithms of the package, some support functions are provided. In other words, functions for MCMC chain analysis, summarizing results, quantile regression estimation, and plotting quantile processes. There is also a function available to compute expected probabilities for binary dependent variables. The Bayes QR program is a useful tool for estimating quantile regressions using a Bayesian framework, as demonstrated with data examples.

(Permai et al., 2018) conducted a study aiming to understand the impact of spatial factors on one of the Sustainable Development Goals, namely the ratio of employment to the national population in each city of Indonesia. Several methods were employed to construct the spatial weight matrix to illustrate the impact of spatial factors on observations.

These methods included weight matrices such as the adjacency matrix, the distance matrix, and a matrix based on correlations. Based on the results obtained from the study and using criteria to compare the best outcomes with different matrices i.e., determining the most effective weight matrix it was found that the best estimation was achieved when using the correlation matrix, followed by the adjacency matrix. The distance matrix, however, provided the least accurate estimation.

Presented research In their study (Tuofu et al., 2021), they used SARQR model to assess the impacts of proximity to different types of blue spaces such as (rivers, lakes, and wetland parks) on property prices in Changsha, China, across various conditional distributions. The study revealed that spatial dependence in property values is significantly important between quantiles of price, suggesting that adding spatial lag variables in studies on hedonic pricing is essential to prevent skewed outcomes. Additionally, it was discovered that the beneficial impact of spatial correlation grows as the price quantiles increase; expensive homes tend to cluster together, suggesting that wealthy homebuyers are more concerned with rare public amenities and environmental resources. The geographical clustering of expensive residences may be due in part to this feature of the property market in large Chinese cities.

Worked the study of Dengue fever (Rizki & Ammar, 2022), The quantity of dengue fever sufferers in West Java Province in 2019 reached 25,282 cases, higher than in 2018. is an important concern for the government in suppressing the transmission of dengue fever by knowing the factors that influence the incidence in West Java. This research uses the spatial autoregressive quantile regression method. With five quantile levels are available, namely 0.1, 0.25, 0.5, 0.75, 0.90, parameter estimation was done using instrumental variable quantile regression, and parameter significance testing conducted employing the Z test. The findings indicated that there is positive spatial autocorrelation, so modeling needs to include spatial effects. Apart from that, the modeling in this research also requires differences in risk levels in the model. So, to accommodate spatial effects as well as differences in risk levels, modeling was carried out using the Spatial Autoregressive Quantile Model.

In the same year (Chen et al., 2022), a study was conducted that focused on modeling spatial variance with adjustments for inaccurate classification. Infectious disease datasets are generally captured with elevated temporal precision. Inaccurate classification, such as underreporting or over reporting, can lead to biased estimates and poor decision-making. When decisions relate to global pandemic issues, it is essential to use all available tools to provide the best information to policymakers. This research uses a modern Bayesian model based on reducing reporting bias and the effects of location and expands it to include exaggeration in reporting and the incorporation of prior information from experts, in addition to programming the model using alternative software that achieves faster results.

The researchers (Gaosheng & Yang, 2023), conducted a study and stated in it that although spatial autoregressive models have gained popularity recently, functional independent variables are not taken into account. A functional quantile spatial autoregressive model has been proposed as a useful approach. First, the nonparametric slope function is estimated using a principal component analysis. Then, the parameters are estimated using the instrumental variable method. They may be of interest in the dependent variable's quantile to researchers in many real-world applications. According to simulation results, the estimates obtained have the ability to lessen estimate bias. To achieve good performance with the method, real data were used in this research, specifically economic growth data.

(Siregar, 2024), conducted a study and analyzed drug use distribution in Karo Regency using the SARQR model. The study found approximately 50 people die daily due to drug use, making North Sumatra the second-highest region for drug abuse. The factors or variables contributing to the increased risk include the availability and ease of access to drugs, as well as individual, environmental, familial, and social factors. The study's results, after estimating the SARQR model parameters, revealed that the most influential factors on drug abuse were age, gender, and occupation.

5. Conclusion:

The present paper comprehensive analysis and straightforward of Spatial Autoregressive Quantile Regression (SARQR) model, contributing to understanding the challenges and issues in data analysis, particularly spatial data. Previous studies indicate that SARQR represents a significant advancement in spatial modeling by combining the advantages of spatial autoregressive models and quantile regression models. This hybrid approach allows researchers to effectively address spatial variability and dependence, which are common obstacles in geographical data analysis. Among the identified future challenges is the need to develop new methods for more effective parameter estimation, especially in contexts involving highly complex or heterogeneous spatial data. Expanding the study of SARQR in different contexts may also enhance our understanding of its more effective application. Furthermore, additional research is recommended to explore how SARQR can be integrated with other analytical techniques, maybe resulting in the creation of models that are more robust and flexible.

Finally, this review underscores the importance and effectiveness of further research in this field, as SARQR is a valuable tool for spatial data analysis, particularly when error distributions are non-normal or unknown, and its application across various fields.

Authors Declaration:
Conflicts of Interest: None
-We Hereby Confirm That All The Figures and Tables In The Manuscript Are Mine and Ours. Besides, The Figures and Images, which are Not Mine, Have Been Permitted Republication and Attached to The Manuscript.
- Ethical Clearance: The Research Was Approved by The Local Ethical Committee in The University.

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