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Forecasting of the Dollar Exchange rate Using Exogenous Variables with the (IV4) Method and Some Kernel Functions

Mustafa Ali Fakhri * **Firas A.Mohammed AL Mohana**

Department of Statistics, College of Administration and Economics
University of Baghdad, Iraq

***Corresponding author**

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Abstract:
This paper proposes a hybrid approach to dollar exchange rate forecasting, wherein both types of forecasting models-linear and non-linear models-have been incorporated to improve the efficiency of predictions. The work essentially combined MISO ARX model with GARCH-X models within MISO ARX framework to enhance the data. These three estimation techniques, namely IV4, RELS-HF, and RELS-SE, could optimize the modeling performance of the MISO ARX model, whereas Quasi Maximum Likelihood Estimation (QMLE) was applied for GARCH-X models.

An evaluation metric such as the Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) was used for the comparison between model performances. The findings show that RELS-SE method is superior to other estimation methods for MISO ARX models, and the results also suggest that the best forecasting accuracy was obtained for the MISO ARX (1,5,3,5,3) - GARCH (1,2)-X model. It has been effective in hybridizing the exchange rate changes model for cash volatility predictions as the hybrid model excellent catch in capturing volatility. This grows importance for a hybrid model in financial forecasting, especially in turbulent markets.

It is valuable for policy makers, financial analysts, and economic researchers. It suggests that hybrid time series models could be used to improve the accuracy of exchange rate forecasting. Future research should include investigation into the combination of machine learning techniques with hybrid econometric models to further enhance prediction improve performance.
Keywords: Hybrid Forecasting Model**,** MISO ARX Model, GARCH-X Model, Model Order Determination, QMLE, Dollar Exchange Rate Prediction.

1. **Introduction:**

Time series analysis in its various fields is one of the main research and application methods (Wang et al., 2005).The past few decades have witnessed increasing interest in theoretical and experimental developments in building time series models and their important applications in forecasting, as forecasting rules play an important role in many fields, such as business, industry, and international governmental organizations, as time series represent successive periods recorded in a specific chronological order, such as monthly, annual, or weekly values. Time series analysis aims to understand patterns, trends, and changes in data, as well as the problems they face, such as multicollinearity, heteroscedasticity, etc (Ahmed Mohammed, 2019). The major reason for using hybrid models is that time series data rarely consists of only linear or nonlinear components, as they usually include both components together, and a single model by itself cannot understand the different patterns of time series accurately. This means that using linear or non-linear time series models individually is not ideal for forecasting future values. Therefore, combining different individual models has a very effective and important effect on increasing the chance of controlling the different patterns. The basic idea of using these models is to improve the accuracy of forecasting and analysis of time data and provide a better understanding of the time behavior and factors affecting time series. Accordingly, many statistical and mathematical models have been studied, including hybrid models, which combine various statistical methods and techniques to analyze and forecast time series for the future, and these models use a combination of traditional and advanced models. One of these models is the (MISO ARX - GARCH-X) which combines two models. The first is the linear model (Multiple input single output Autoregressive with exogenous variables). The second is the non-linear model (Generalized Autoregressive Conditional Heteroscedasticity with exogenous variables), where the MISO ARX model was estimated using several estimation methods, including the (IV4) and (RELS – HF), (RELS – SE), as well as the GARCH-X model using (QMLE) method, in this study, several models were tested. The optimal model was chosen based on the model selection criteria, which are Akaike's Information Criterion (AIC) and Akaike's Final Prediction Error Criteria (FPE), Minimum Description length (MDL), Bayesian Information Criterion (BIC) (Colin et al., 2021). The structure of this paper consists of the following: in the second section, we will review the important literature related to our topic. After that, in the third section, we will focus on the research methodology and how to work the single models and hybrid models, as well as on developing the basic hypotheses. In the fourth section, we will review the results that were reached, through which the topic is understood more deeply. Finally, we will review the conclusions that emerge from this research and the directions for the future.

2. Literature Review:

In the year (2021), researchers (Kévin Colin, Laurent Bako, Xavier Bombois) studied the prediction error quantification. The collected data must be informative concerning the chosen model structure to obtain a stable estimate. In this work, the focus was on the information property of the data to identify ARX systems with multiple inputs and a single output in a closed loop. A necessary and sufficient condition was derived to verify whether the given multiple external excitations coupled with the feedback provided by the controller give data informativeness concerning the chosen model structure , the data informatics of the closed-loop direct identification of MISO ARX systems with multiple external excitations is also studied. (Colin et al., 2021).

In the same year, researchers (Wei Dai and Ka Wai Tsang) studied hybrid resampling to address (a) the long-standing inference problem of changing times and changing parameters in change-point ARX-GARCH models and (b) the problematic problem of confidence intervals. Validity, after variable selection under dispersion assumptions, for parameters in linear regression models with high-dimensional stochastic regressions and asymptotically constant noise.

For the latter problem, Consistent estimates were provided of the selected parameters and a resampling approach to overcome the difficulties inherent in post-selection confidence intervals. For the previous problem, we use a sequential Monte Carlo of latent states (representing change times and changing parameters) of a hidden Markov model. The advantages of the proposed methods are demonstrated through asymptotic efficiency theory, simulations, and experimental studies, providing detailed insights into their effectiveness. (Dai & Tsang, 2020).

In the year (2022), researcher (Philipp Ketz) studied used the results of Andrews and Cheng (2012), which are extended to allow parameters to be close to or at the boundary of the parameter space, to infer the asymptotic distributions of the two test statistics used in the two-step (test) procedure proposed by Pedersen and Rabik (2019). The latter aims to test the null hypothesis that a GARCH-X model, with exogenous variables (X), reduces to a standard GARCH model, with the “GARCH” coefficient allowed to be unidentified. Then he was presented with a characterization result for the asymptotic size of any test to test this null hypothesis before setting a numerical lower bound on the asymptotic size of the two-step procedure at the nominal 5% level. This lower bound exceeds the nominal level, revealing that the two-step procedure does not control the asymptotic size, a small simulation study was conducted, and it was found that the asymptotic theory provides good approximations to the finite sample behavior of the tests or test procedures we consider. In particular, we found that the test procedures proposed by PR can suffer from over-rejection in finite samples. (Ketz, 2022).

In the year (2023), researchers (Janczura) proposed dynamic, short-term strategies to manage the financial risks of small electricity producers and buyers who trade in wholesale electricity markets. Since electricity is mostly not storable, the economic risks resulting from electricity prices are highly volatile. It cannot be minimized using standard financing-based methods. Instead, short-term operational planning and appropriate business diversification can be used. Price risk is analyzed in terms of Markowitz's mean-variance portfolio theory. Therefore, it is crucial to forecast the variation in electricity prices correctly. To this end, the researchers jointly conducted models daily and intraday prices or arbitrage prices from Germany and Poland using ARX-GARCH type models; It has been shown that using heteroscedastic volatility significantly improves probabilistic price forecasts, mainly if a variance-fixing transformation is applied before estimating the model, and then Price forecasts to build dynamic diversification strategies that depend on risk metrics, where different objectives were taken into account in addition to the buyer and seller’s point of view (Janczura & Pu´c, 2023).

3. Methodology:

In this research section, two types of time series models will be identified (linear and non-linear models). Through these models, the hybrid model that is used in the forecasting process will be reached.

3.1 Linear Model:

The (Multiple input single output Autoregressive with exogenous variables) model is one of the most important linear models used in the hybridization process, and the (MISO ARX $(n_a, n_{b1}, n_{b2}, \ldots, n_{br}, n_k)$) model can be expressed mathematically through the following mathematical formulas (Colin et al., 2021) (Rachad et al., 2015):

3.1.1 MISO ARX $(n_a, n_{b1}, n_{b2}, \ldots, n_{br}, n_k)$ model:

$A(L)y(t) = \sum_{i=1}^{r} Bi(L)\ X_i(t - nk) + v(t)$ ... (1)

y(t) = Model outputs, i.e. the dependent variable.

$x(t) = [x_1(t), x_2(t), \ldots x_r(t)]^T$, Model input vector Explanatory variables.

r = represents the number of inputs.

v_t = represents the error term, which follows the normal distribution with a mean equal to zero and a constant variance.

na = represents the rank of the autoregression.

nbi = represents the number of input terms, $i = 1, 2, \cdots, r$

nk = represents the lag of the output from the input

Whereas the following formula gives polynomials:
$A(L) = 1 + a_1 L^{-1} + a_2 L^{-2} + \cdots + a_{na} L^{-n_a}$ ... (2)
$Bi(L) = b_{i1} L^{-1} + b_{i2} L^{-2} + \cdots + b_{inbi} L^{-n_{bi}}$ ... (3)
And (L) represents the backshift operator, where $L^{-k} y(t) = y(t-k)$.
It can be written as a regression model as follows (Zhang, 2011):
$y(t) = \mu + \varphi^T(t)\,\theta + v(t)$ ... (4)
$\varphi(t) = [-y(t-1), -y(t-2), \dots, -y(t-n_a) \quad x_1(t-n_k-1), \dots, x_1(t-n_k-n_{b1}), \dots, x_r(t-n_k-1), \dots, x_r(t-n_k-n_{br})]^T$
Model parameters.
$\theta = [a_1 \quad a_2, \dots a_{na} \quad b_{11}, \dots, b_{1nb1}, \dots, b_{r1}, \dots, b_{rnbr}]^T$
The conditional information variance (F_{t-1}) of this model is constant regardless of the rank of the model.
$Var(y_t / F_{t-1}) = \sigma_t^2$
This means that the conditional distribution of the time series y(t) follows the normal distribution as follows (Mohammed & Tawfeeq, 2021) (Hamid & Mohammed, 2018) :
$y_t / F_{t-1} \sim N(\mu t\ \sigma_t^2)$
Matrix of regression elements :$\varphi^T(t)$
A constant representing the mean of the time series :μ_t

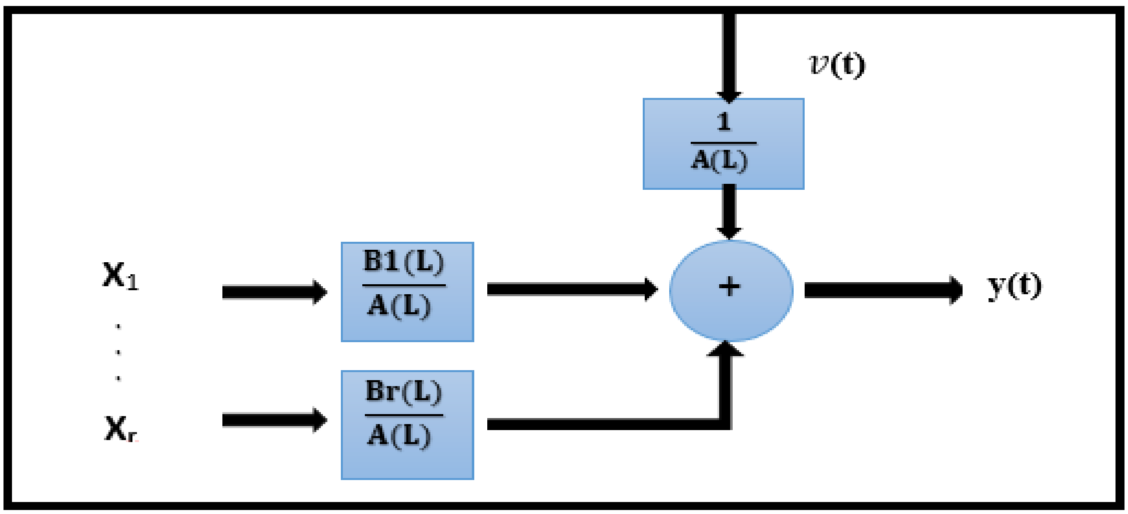

Diagram (1): The structure of the MISO-ARX model

Source: Prepared by the researchers.

3.2 Non-Linear Model:

The process of modelling the residuals of the linear model (MISO ARX($n_a, n_{b1}, n_{b2}, \dots, n_{br}, n_k$)) in equation (1) is done by applying a non-linear model (GARCH (q,p)-X($b_1, b_2, \dots, b_r$)) and the following sections explain these models:

3.2.1 GARCH (q, p)-X($b_1, b_2, \dots, b_r$) Model:

Generalized autoregressive conditional heteroscedasticity (GARCH) models have gained increasing popularity since their introduction due to their ability to predict volatility. The GARCH model is usually considered to be able to capture this volatility based on its past volatility and prior squared residuals, but it does not take into account the effect of exogenous variables on this process, where exogenous variables play a critical role, (Kazim & Hassan, 2017). Therefore, this study seeks to present and apply the GARCH-X model.

A major problem appears for us, which is how to include the external variables in the model, as for how to include external variables in the model, it is as follows:

The following formula gives the average equation (Ketz, 2022) (Md Yeasin, K.N. Singh, 2020) (Hu, 2019) (Mohammed &Yadkar, 2015).

$y(t) \;=\; \mu(t) \;+\; vt \;\ldots\; (5)$

The following formula gives the random error equation:

$\mathrm{vt} \;=\; \eta\mathrm{t}\sqrt{h_t} \;\;, \eta_t/\mathcal{F}_{t-1} \cong$ iid N $(0, 1)$ … (6)

The following formula gives the conditional variance equation:

$h_t \;= \propto_0 + \sum_{i=1}^{p} \propto_i v_{t-i}^2 + \sum_{j=1}^{q} \theta_j \, h_{t-j} + \sum_{i=1}^{r} \sum_{k=1}^{bi} \lambda_{ik} \, x_{i(t-k)}^2 \;\ldots\; (7)$

Where:

$\propto_0$: The fixed term.

$\theta_j, \propto_i$: The parameters of the GARCH (q, p) model.

λ_{ik} : The parameter of the exogenous variables.

$x_{i(t-k)}^2$: The square of the exogenous variable (i) at time t-k.

According to the restrictions or conditions imposed on the parameters, which ensures a positive conditional variance, where:

$\propto_0 \geq 0$

$\propto_i, \theta_j, \lambda_k \geq 0$

i = 1, 2,3, ……, p-1, $\propto_p > 0$

j = 1, 2,3, ……, q-1, $\theta_q > 0$

k = 1, 2,3, ……, b-1, $\lambda_b > 0$

3.3 Stages of Building Hybrid Model

3.3.1 The first stage:

The first stage in building hybrid models depends on choosing the best model from the models (MISO ARX$(n_a, n_{b1}, n_{b2}, \ldots, n_{br}, n_k)$, and this is based on choosing the best orders $(n_a, n_{b1}, n_{b2}, \ldots, n_{br}, n_k)$ for the model (MISO ARX).

Depending on Akaike's Information Criterion (AIC), Akaike's Final Prediction Error Criteria (FPE), and Minimum Description length (MDL) These criteria are mathematically defined as follows (Anderson, 2004):

$$\mathrm{AIC} = \ln\left(\frac{1}{n}\sum_{t=1}^{n} \upsilon^2(t)\right) + \frac{2F}{n} \cdots (8)$$

$$\mathrm{FPE} = \left[\frac{1+\left(\frac{F}{n}\right)}{1-\left(\frac{F}{n}\right)}\right] * \frac{1}{n}\sum_{t=1}^{n} \upsilon^2(t) \ldots (9)$$

$$MDL = \ln\left(\frac{1}{n}\sum_{t=1}^{n} \upsilon^2(t)\right) + \frac{F\ln(n)}{n} \cdots (10)$$

Where:

n: Sample size.

v_t : represents the error term

F: represents the number of parameters.

3.3.2 The second stage:

The parameters of the hybrid model are estimated through a two-step procedure. Three estimation methods (IV4, RELS-HF, and RELS-SE) are used in the first step to estimate the model parameters (MISO ARX). The model's parameters (GARCH-X) are estimated in the second step using the (QMLE) method. The following statistical methods are used to estimate the parameters of the model (MISO ARX):

3.3.2.1 Four-Stage Instrument Variable Method (IV4):

The (IV4) method is an extension of the instrumental variable (IV) method, and the (IV) method can be considered an alternative approach to modifying the least squares (LS) method since the error v(t) in optimal cases is independent of the previous data, i.e. z^{t-1} as:

$z^{(t-1)} = [x_1(1) \dots x_r(1), \dots, x_1(t-1) \dots x_r(t-1) y(1) \dots y(t-1)]$

If there is a relationship between them, it suggests that the information contained in z^{t-1} about y(t) is greater than what is extracted by $\hat{y}$(t) to address this issue, we can select the instrumental variables vector $\zeta(t)$ with finite dimensions that is derived from z^{t-1}, So it achieves the following:

$$\frac{1}{n}\sum_{t=1}^{n} \zeta(t)\, e(t) = 0$$

Thus, the estimator of the instrumental variables can be obtained in

$$\hat{\theta}_{IV} = \left[\mathrm{SOL}\frac{1}{n}\sum_{i=1}^{n} \zeta(t)[\mathrm{y}(t) - \varphi^T(t)\theta] = 0\right]$$

$$\hat{\theta}_{IV} = \left[\frac{1}{n}\sum_{t=1}^{n} \zeta(t)\varphi^T(t)\right]^{-1} \left[\frac{1}{n}\sum_{t=1}^{n} \zeta(t)\mathrm{y}(t)\right]$$

For large sample sizes, for $\hat{\theta}_{IV}$ to tend to θ. $\frac{1}{n}\sum_{t=1}^{n} \zeta(t)\, e(t)$ Must tend to zero in general, the desirable characteristics of the instrumental variables $\zeta(t)$ are:

$\bar{E}\, \zeta(t)\, \varphi^T(t)$ (Nonsingular)

$\bar{E}\, \zeta(t)\, e(t) = 0$

This means that the instrumental variables must be related to the elements of the regression vector and not to the error term, so the aids can be generated as follows:

$\zeta(t) = [-\chi(t-1) \dots -\chi(t-n_a) \quad x_1(t-1) \dots x_1(t-n_b) \dots x_r(t-1) \dots x_r(t-n_b)]^T$

χ(t) is generated from the input through a linear system.

The basic and specific steps of this method can be obtained by applying a four-step algorithm, which is as follows (Ljung, 1999) (Zhu, 2001) (Tangirala, 2015) :

<u>Step 1:</u>

Estimating θ using the least squares method and the corresponding transformation function is denoted by $\hat{G}^{(1)}(L)$.

$$\hat{G}^{(1)}(L) = \frac{\hat{B}^{(1)}(L)}{\hat{A}^{(1)}(L)}$$

<u>Step 2:</u>

Generate the instruments as follows:

$\chi^{(1)}(t) = \hat{G}^{(1)}(L)\, x(t)$ ... (11)

$\zeta^{(1)}(t) = [-\chi^{(1)}(t-1) \dots -\chi^{(1)}(t-n_a)\, x_1(t-1) \dots x_1(t-n_b) \dots x_r(t-1) \dots x_r(t-n_b)]^T$ ... (12)

Then, the parameters should be estimated using the (IV) method ،by using these instruments $\zeta^{(1)}(t)$ and denote the estimate $\hat{\theta}^{(2)}$ and the corresponding transfer function estimate.

$$\hat{G}^{(2)}(L) = \frac{\hat{B}^{(2)}(L)}{\hat{A}^{(2)}(L)}$$

Step 3:
Let it be:
$\widehat{W}^{(2)}(t) = \widehat{A}^{(2)}(L)\, y(t) - \widehat{B}^{(2)}(L)\, x(t)$ ... (13)
Assuming an autoregressive model AR with the order $(na + nb)$ for $\widehat{W}^{(2)}(t)$, as shown below.
$Z(L)\, \widehat{W}^{(2)}(t) = e(t)$ ... (14)
The Z(L) should be estimated using least squares (LS) and the result is denoted $\hat{Z}(L)$.
Z(L): linear filter.
Step 4:
Let $\chi^{(2)}(t)$ be defined very similar to (11), and let
$\zeta^{(2)}(t) = [-\chi^{(2)}(t) \ldots -\chi^{(2)}(t-n_a) x1(t-1) \ldots x1(t-n_b) \ldots x_r(t-1) \ldots x_r(t-n_b)]^T$

Using these instruments, the final estimates of the parameters can be arrived at, shown in the formula below.
$\varphi_F(t) = \hat{Z}(L)\, \varphi(t)$
$y_F(t) = \hat{Z}(L)\, y(t)$
$\hat{\theta}_{IV4} = \left[\sum_{t=1}^{n} \zeta^{(2)}(t) \varphi_F^T(t)\right]^{-1} \left[\sum_{t=1}^{n} \zeta^{(2)}(t) y_F(t)\right]$... (15)

3.3.2.2 Regularized Least Square (RELS):
The regularized least squares method is considered one of the important methods used in estimating the model parameters (MISO ARX), which deals with poor conditioning and has an important role in controlling overfitting, especially when modelling high-dimensional models, i.e., containing many parameters. Therefore, this problem is addressed by allowing some bias through the regularization term, Which is given by the following mathematical formulas (Chen & Ljung, 2013) (Chen et al., 2012) (Yusof et al., 2013) (Ljung, 2013) (Chen, 2018):
$y(t) = \varphi^T(t)\, \theta + v(t)$
Then it is rewritten using matrices and in the following form:
$Y = \phi\theta + \nu$ ... (16)
$Y = [y(1)\ y(2) \ldots y(N)]^T$
$\phi = [\varphi(1)\ \varphi(2) \ldots \varphi(N)]^T$
$\nu = [\nu(1)\ \nu(2) \ldots \nu(N)]^T$
Where the estimation of parameter $\boldsymbol{\theta}$ using the least squares method is given by the following formula:
$\hat{\theta}^{LS} = \arg_{\theta} \min \ \|Y - \phi\theta\|^2$... (17)
$\hat{\theta}^{LS} = \left(\phi^T\phi\right)^{-1} \phi^T Y$ ... (18)
By implementing the RELS method, we add the regularization term $J(\theta)$ to the LS method equation, which is given by the following formula:
$\hat{\theta}^{RE} = \arg_{\theta} \min \ \|Y - \phi\theta\|^2 + \psi\, J(\theta)$... (19)
$J(\theta) = \theta^T P^{-1} \theta$
ψ= represents the regularization parameter and is a positive number $\psi \geq 0$
P = represents the (regularization matrix), which is a symmetric and positive semi-definite (p.s.d) matrix and is known as the kernel matrix in machine learning.
The presence of the matrix (P) improves the estimate's numerical properties and reduces its variance. However, some bias is introduced, and the optimal choice of the matrix (P) makes the MSE as low as possible.
The following formula gives the estimation of the regular least squares method:
$\hat{\theta}^{RE} = \left(\phi^T\phi + \psi P^{-1}\right)^{-1} \phi^T Y$ ... (20)

1- Regularized Least Square Method with High-Frequency Stable Spline Kernel (RELS – HF)

It is known as the high-frequency stable chip core and is symbolized by the symbol (HF) when ρ= -√Ɣ, where the following image gives the regulation matrix (Chen & Ljung, 2013).

$P_{i,j}^{HF}\ (\alpha) = C\ \ (-1)^{i-j}\min(Ɣ^{j}, Ɣ^{i}) \ldots (21)$

$\alpha = [C \quad Ɣ]^{T}$ ، $C\ \geq 0\ ,\ 0 \leq\ Ɣ < 1$

2- Regularized Least Square Method with Squared Exponential Kernel (RELS – SE)

It is known as the quadratic exponential kernel and is symbolized by the symbol (SE), where the regularization matrix is given in the following form (Zhu, 2001).

$\mathrm{P}_{\mathrm{i,j}}^{\mathrm{SE}}(\alpha) = \mathrm{C}\ \mathrm{e}^{\frac{-(\mathrm{i}-\mathrm{j})^2}{2Ɣ^2}} \ldots (22)$

$\alpha = [C \quad Ɣ]^{T}, C\ \geq 0\ , 0 \leq\ Ɣ < 1$

3.3.2.3 Comparison of estimation methods (MISO ARX) model

Two measures were used to compare the estimation methods of the model (MISO ARX), namely Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE), where the measure that gives the lowest estimated values is the best, which is given in the following formula (Liu & Shi, 2013) (Yusof et al., 2013) (Fakhri & Mohammed, 2016).

$$\mathrm{MAE} = \frac{1}{n}\sum_{t=1}^{n}\left|y_{(t)} - \hat{y}_{(t)}\right| \ldots (23)$$

$$\mathrm{MAPE} = \left(\frac{1}{n}\sum_{t=1}^{n}\left|\frac{y_{,t)} - \hat{y}_{(t)}}{y_{(t)}}\right|\right) \times 100\% \ldots (24)$$

$y_{(t)}$: is real value.

$\hat{y}_{(t)}$: is an estimated value.

n: is the size of data.

3.3.3 The third stage:

After determining the appropriate model and estimating its coefficients, comes the stage of verifying the quality of the model suitable for the prediction process. This is done by testing the presence of the (ARCH) effect to ensure the validity of the (GARCH-X) model for application or not. This is done through the tests given in the following mathematical formulas (Bollerslev, 1986).

3.3.3.1 ARCH Test:

This test was proposed by the scientist Engle in 1982 AD. This test is characterized by ease and simplicity of calculation. It is used to find out whether there is a problem of heterogeneity of the variance of the random error, which depends on this test to investigate the presence of an effect (throne), meaning whether the series of errors follows the ARCH process or not.

Whereas the hypothesis for testing the presence of the ARCH effect under the null hypothesis and the alternative hypothesis is written in the following formula (Wang et al., 2005) (Lee, 1991):

H_0:αi=0 (There is no ARCH effect)

H_1:αi≠0 (There is an ARCH effect)

i=1, 2, 3, …, d

Where the Lagrange multiplier LM is calculated, and thus the test statistic is calculated after finding the coefficient of determination $\hat{R}^2$ as follows:

$LM\ =\ ARCH\ Test\ =\ n * \hat{R}^2 \sim\chi^2_{(P)} \ldots\ (25)$

n: represents the sample size.

P: represents the number of estimated model parameters.

$\boldsymbol{\hat{R}^2}$: represents the coefficient of determination estimated.

Where:

$$\widehat{R}^2 = \frac{SSR}{SST} \dots (26)$$

SSR: Represents the regression sum of squares.
SST: Represents the total sum of squares.
Where the probability value of the test statistic for (ARCH) is compared with the significance level (0.05), if it is ($p-value < 0.05$), we reject the null hypothesis and accept the alternative hypothesis, which states the existence of an effect (GARCH-X), i.e. the existence of a problem of Heteroscedasticity.

3.3.4 The Fourth stage:

Building a forecasting model (GARCH-X) is done by following the traditional stages of building time series models. Still, in hybrid models, the beginning of building these models is by studying the residual series resulting from the (MISO ARX) model, as follows:

3.3.4.1 Model Order Determination:

The stage of determining the orders of the model (GARCH-X) is one of the important stages through which the optimal and appropriate model is chosen for the hybridization process, as choosing orders lower or higher than the actual order leads to inconsistency in the model parameters, which leads to a decrease in the model's performance in the prediction process, as one of the most important criteria used in choosing the order of the model (GARCH-X) is (Akaike Information Criterion (AIC) and_Bayesian Information Criterion (BIC)) and these criteria are known mathematically by the following formula (Akaike, 1974) (Ismail & Mohammed, 2021):

$$AIC = [-2\log(L) + 2(F)]/n \dots (27)$$

$$BIC = [-2\log(L) + (F\ln(n))]/n \dots (28)$$

log (L): Logarithm of the maximum likelihood function.
F: Number of model parameters.
n: Sample size.

3.3.4.2 Quasi Maximum Likelihood (QMLE):

The mechanism for estimating hybrid models depends on a two-stage methodology: the first stage is estimating the model (MISO ARX), and the second stage is estimating the parameters of the model (GARCH-X) by using (QMLE) method As follows (Francq & Zakoian, 2007) (Linton, 2010) (Han & Kristensen, 2014):

$$vt = \eta t\sqrt{h_t} \quad , \quad \eta t \cong i.i.d\ N(0,1) \dots (29)$$

$$ht = \propto_0 + \sum_{i=1}^{p} \propto_i v_{t-i}^2 + \sum_{j=1}^{q} \theta_j h_{t-j} + \sum_{i=1}^{r}\sum_{k=1}^{bi} \lambda_{ik} x_{i(t-k)}^2 \dots (30)$$

η : A series of random variables that are independent and identical and that are assumed to follow the standard normal distribution with mean (0) and variance (1)
Where Θ belongs to the model parameter space given by

$$\Theta \subset (0,+\infty) \times [0,\infty)^{p+q}$$

Assume that observations are ($\nu_1, \dots, \nu_n, x_1, \dots, x_n$) constitute a realization (of length n).
Where:

$$\vartheta = \left(\propto_0, \propto_i, \theta_j, \lambda_{ik}\right)' \ , i = 1,2,\cdots,p \ , j = 1,2,\cdots,q \ , k = 1,2,\cdots,b$$

The Gaussian quasi-likelihood is given by and multiply the function n times.

$$L_n(\vartheta) = L_n(\vartheta, \nu_1, \dots, \nu_n, x_1, \dots, x_n) = \prod_{t=1}^{n}\left\{\frac{1}{\sqrt{2\pi h_t}} exp\left(-\frac{V_t^2}{2h_t}\right)\right\} \dots (31)$$

A QMLE is thus a measurable solution of the equation:

$$\hat{\vartheta}_n = \underset{\vartheta \in \Theta}{\arg\max}\, L_n(\vartheta)$$

Where (ϑ) represents the vector of unknown parameters to be estimated And taking the logarithm, it turns out that maximizing the probability is equivalent to minimizing, for ϑ:

$$\tilde{I}_n(\vartheta) = \frac{1}{n}\sum_{t=1}^{n} \tilde{\ell}_t \qquad \text{where} \qquad \tilde{\ell}_t = \tilde{\ell}_t(\vartheta) = \frac{v_t^2}{h_t} + log(h_t)$$

Thus, the QMLE is a measurable solution to the equation

$$\hat{\vartheta}_n = \underset{\vartheta\, \epsilon \Theta}{\arg\min}\, \tilde{I}_n(\vartheta)$$

3.3.5 Hybrid Model:

$[(\text{MISO ARX}(n_a, n_{b1}, n_{b2}, \dots, n_{br}, n_k) - \text{GARCH}(q,p) - X(b_1, b_2, \dots, b_r))]$

The use of time series models for forecasting and decision-making has become necessary, and hence, the accuracy of forecasting plays an important role in interpreting and analyzing phenomena. On this basis, the decision is made to overcome the problems facing time series. Researchers have resorted to hybrid models, and one of the most important goals of this research is to employ the hybrid (Zhang, 2011)) methodology to build the hybrid model, which is represented by merging the linear time series model (MISO-ARX) and the non-linear time series model (GARCH-X) into one model known as $(\text{MISO ARX}(n_a, n_{b1}, n_{b2}, \dots, n_{br}, n_k) - \text{GARCH}(q,p) - X(b_1, b_2, \dots, b_r)$).This method deals with data that suffers from fluctuations that occur over time and is characterized by giving highly efficient results in the forecasting process, which are given in the following formula (Parwati et al., 2023) (Janczura & Pu´c, 2023) (Liu & Shi, 2013) (Hickey et al., 2012) :

$$y_t = \frac{1}{A(L)}\left[\sum_{i=1}^{r} Bi(L)\; x_i(t-nk) + v(t)\right] \dots (32)$$

$$v_t = \eta_t \sqrt{h_t} \dots (33)$$

$$ht = \alpha_0 + \sum_{i=1}^{p} \alpha_i\, v_{t-i}^2 + \sum_{j=1}^{q} \theta_j\, h_{t-j} + \sum_{i=1}^{r}\sum_{k=1}^{bi} \lambda_{ik}\, x_{i(t-k)}^2 \dots (34)$$

The (Zhang) methodology assumes that time series are a mixture of two components, linear ($\mathbf{L_t}$) and non-linear ($\mathbf{N_t}$).

To obtain the predictions of the hybrid model, this is done by modelling the non-linear residuals obtained from the MISO ARX model and then using them as inputs in building the GARCH-X model, as described in the following mathematical equations:

$$v_t = y_t - \hat{L}_t \dots (35)$$

$\hat{\mathbf{L}}_t$: Represents the estimated prediction value in time (t) for the model (MISO ARX)

$$\hat{L}_t(\ell) = \hat{y}(t+\ell) = w_\ell(L)\, G(L)\, x(t) + \left[1 - w_\ell(L)\right] y(t) \dots (36)$$

$$y(t) = G(L)\, x(t) + \varepsilon(t)$$

$$w_\ell(L) = \bar{H}_\ell(L) H^{-1}(L)$$

$$\bar{H}_\ell(L) = \sum_{k=0}^{\ell-1} h(k)\, L^{-k}$$

$\hat{\mathbf{N}}_t$: Represents the estimated prediction value in time (t) for the model (GARCH-X)

$$\hat{N}_t(\ell) = \hat{h}_t(\ell) = \hat{\alpha}_o + \sum_{i=1}^{n}\sum_{k=1}^{bi}\left(\hat{\alpha}_i + \hat{\vartheta}_i + \hat{\lambda}_{ik}\right)\hat{h}_{t+\ell-i\backslash t} + \sum_{i=\ell}^{m}\sum_{k=1}^{bi}(\hat{\alpha}_1\, \hat{v}^2{}_{t+\ell-i} + \hat{\vartheta}_1 \hat{h}_{t+\ell-i\backslash t} + \hat{\lambda}_{1k}\hat{x}^2{}_{t+\ell-i\backslash t}) \dots (37)$$

By combining the linear predictions ($\hat{\mathbf{L}}_t$) of the model MISO ARX with the non-linear predictions ($\hat{\mathbf{N}}_t$) of the model GARCH-X, we obtain the predictive values of the hybrid model as in the following equation:

$$\hat{y}_t(\ell) = \underbrace{\hat{L}_t(\ell)}_{\text{MISO ARX}} + \underbrace{\hat{N}_t(\ell)}_{\text{GARCH}-X} \dots (38)$$

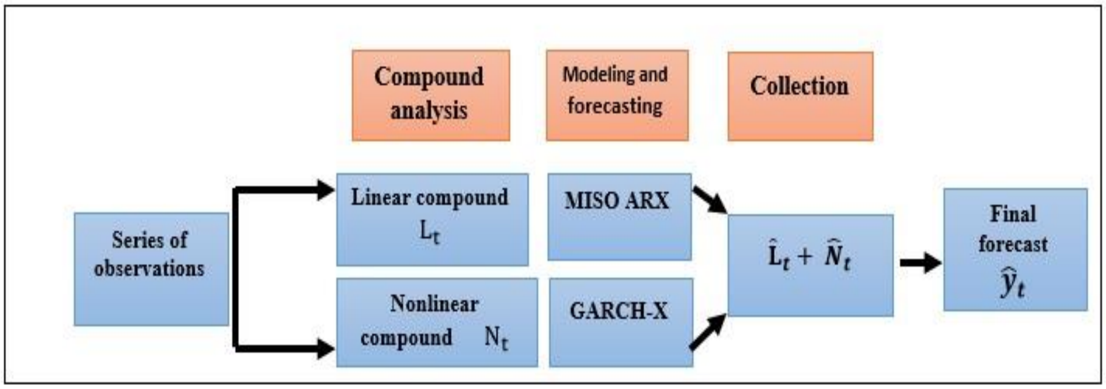

Diagram (2): Prediction process of the hybrid model (MISO ARX – GARCH –X)
Source: Prepared by the researchers

4. Results And Discussions:

The data of this study represents a time series consisting of an output series represented by (dollar exchange rates) (Allah & Shalaka, 2013) And an input series represented by (cash sales and transfer Sales), as daily measurements and with a sample size of (480) from the year (2020 to 2022) as a training data set to estimate the parameters and then build the model. Therefore, in this section, we will study building a linear (MISO ARX) model based on actual data and building a non-linear (GARCH-X) model based on a residual time series extracted from the appropriate linear (MISO ARX) model. Figures (1), (2), and (3) show the time series plot of dollar exchange rates, cash sales, and Transfer Sales.

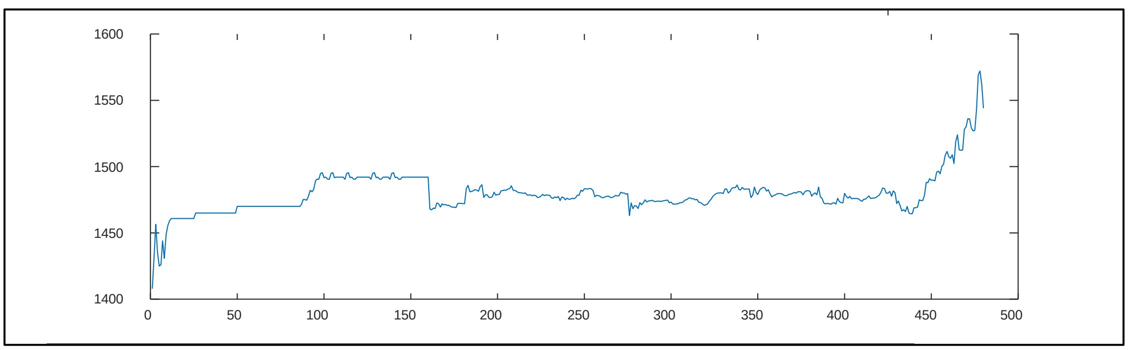

Figure (1): The dollar exchange rates.
Source: Prepared by the researchers

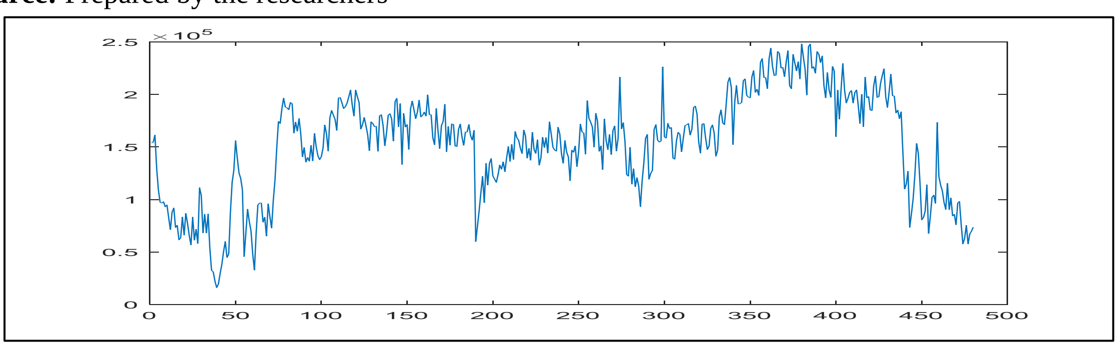

Figure (t): Transfer sales.
Source: Prepared by the researchers

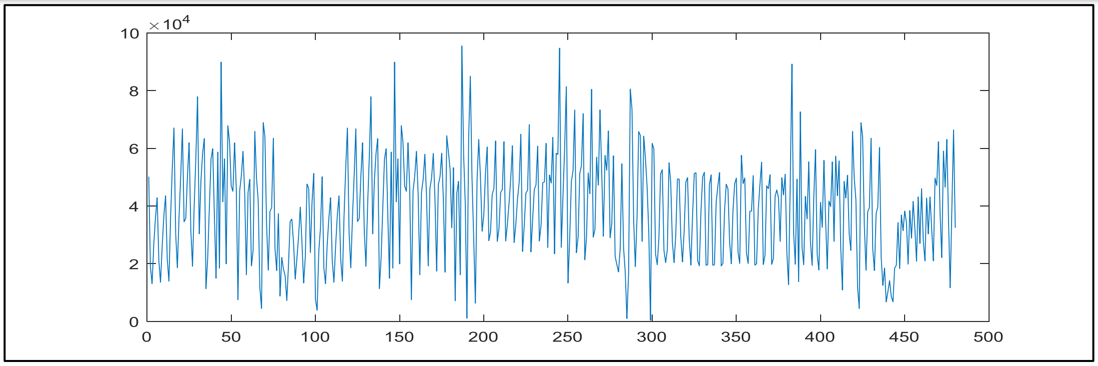

Figure (3): Cash sales

Source: Prepared by the researchers

4.1 Building a linear model (MISO ARX):

After initializing the input and output series, the appropriate model (MISO ARX) is chosen based on the lowest values of the statistical criteria AIC, FPE, and MDL with different ranks and estimated according to the equations (8), (9), and (10). The results showed that the optimal model that represents the phenomenon is the model:

Table (1): Diagnostic Scales of the MISO ARX Model

MODEL		AIC	FPE	MDL
MISO ARX (1 5 3 5 3)		2.7569	2.6846	2.8715

4.1.1 Estimation of model parameters (MISO ARX):

After completing the diagnosis stage and obtaining the orders of the optimal model that were diagnosed based on the previous diagnosis criteria, the stage of estimating the parameters of the model begins (MISO ARX (1 5 3 5 3)) using three estimation methods, which are (IV4, RELS-HF, and RELS-SE) which are given through the following equations (15), (21), (22). Based on these methods, the estimated results were obtained, which are given in the following table:

Table (2): Estimation of the Parameters of the MISO ARX Model

Estimation methods	Estimated values of parameters MISO ARX [1 5 3 5 3] model								
	$\hat{a}_1$	$\hat{b}_{11}$	$\hat{b}_{12}$	$\hat{b}_{13}$	$\hat{b}_{14}$	$\hat{b}_{15}$	$\hat{b}_{21}$	$\hat{b}_{22}$	$\hat{b}_{23}$
IV4	-9.894e-01	1.185e-04	3.79e-05	3.595e-05	5.655e-05	-2.112e-06	6.234e-06	-2.878e-05	6.105e-05
RELS (HF) Kernel	-1.002e+0	-8.479e-06	-4.98e-06	-2.443e-05	-7.509e-06	-1.625e-05	-1.348e-05	-1.295e-05	1.979e-05
RELS (SE) Kernel	-1.002e+0	-8.179e-06	-1.016e-05	-1.121e-05	-1.09e-05	-9.488e-06	-9.746e-06	-6.379e-06	9.6e-06

4.1.2 Comparison of estimation methods:

After estimating the model parameters (MISO ARX) using three estimation methods (IV4, RELS-HF, and RELS-SE), the stage of comparing these methods comes to determining the best method, depending on

The following comparison criteria (MAE, MAPE) according to the equations (23), (24), and the results are given through the following table:

Table (3): Comparison Measures for Model Methods (MISO ARX)

Estimation Methods	Comparison Measures	
	MAE	MAPE
IV4	3.2184	0.2170
RELS (HF)	1.7845	0.1200
RELS (SE)	1.7303	0.1164

From Table (3) above, we observe that the best method for estimating the parameters of Model (MISO ARX) is Method (RELS (SE)) because the values of the statistical measures (MAE = 1.7303) and (MAPE = 0.1164) are the lowest possible compared to the values of these measures for the other methods.

4.1.3 Model validity tests (ARCH Effects):

Before starting the process of estimating the parameters of the model (GARCH-X), the stage of examining the accuracy of the model comes by detecting the presence of non-linear characteristics in the series of residuals of the model (MISO ARX), and this is done by using the following tests.

4.1.3.1 Arch Test:

To test the presence of ARCH effects, Engle's ARCH test, according to the equation (25), was performed before estimating the GARCH-X model to detect the presence of heteroscedasticity. The result of this test is expressed in the following table:

Table (4): shows the (ARCH) test for the series of residuals of the MISO ARX model

Null Hypothesis	Lag	Test Statistic	$p-\text{value}$
False	10	5.96367e+01	4.2456e-09
False	15	8.14651e+01	3.7655e-11
False	20	8.50866e+01	5.2892e-10
False	30	8.91264e+01	8.9034e-08

It is clear from table (4) that the probability value (p-value) was smaller than the significance level (0.05), which means don’t reject the alternative hypothesis, which is the presence of serial correlation in the residuals of the linear model MISO ARX at time shifts (k=10,15,20,30) and thus a case of heterogeneity of variance in the residuals.

4.2 Building a model (GARCH-X) suitable for the hybridization process

The nonlinear model (GARCH-X) is estimated based on the residuals of the model (MISO ARX). By matching the best nonlinear model with different orders to choose the optimal model in the prediction process, the best model (GARCH (1, 2) – X (0, 1)) was built according to the lowest values of the statistical criteria (AIC) and (BIC) and estimated according to the equations (27) and (28). The following table shows the estimation of the parameters of the model (GARCH-X) using the method (QMLE) as shown in the following table:

Table (5): Estimation of the Parameters of the GARCH-X Model

Estimation methods	Estimated Values of Parameters $(\text{GARCH}(1,2)-\text{X}(0,1))$ Model				
	$\hat{\alpha}_o$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\vartheta}_1$	$\hat{\lambda}_{21}$
QMLE	0.0000	2.3580e-01	1.0210e-01	4.2915e-01	1.71816e-05
Standard error	2.074228e-04	2.4442e-01	2.5051e-01	1.8655e-01	1.78160e-04

After estimating and diagnosing the significance of the model $(\text{GARCH}(1,2)-\text{X}(0,1))$, since all parameters are important and verification the conditions of the GARCH model, the hybrid model for this research can be written as follows:

The conditional mean equation for the hybrid model from Table (2) is written as follows:

$$y_t = 1.002y(t-1) - 8.179e-06x_1(t-5) - 1.016e-05x_1(t-6) - 1.121e-05x_1(t-7) - 1.096e-05x_1(t-8) - 9.488e-06x_1(t-9) - 9.746e-06x_2(t-3) - 6.379e-06x_2(t-4) + 9.6e-06x_2(t-5) + v_t$$

$$v_t = \eta_t\sqrt{h_t} \quad , \quad \eta_t/\mathcal{F}_{t-1} \cong \text{iid N}(0,1)$$

The equation of time Volatility for the hybrid model from table (5) is written as follows:

$$h_t = 0.2358059\, v_{t-1}^2 + 0.1021029\, v_{t-2}^2 + 0.4291555\, h_{t-1} + 1.718164e-05\; x_{2\,(t-1)}^2$$

The above equations represent the hybrid models employed in the forecasting of the dollar exchange rates.

4.2 Model validity (ARCH Effects):

After completing the estimation of the hybrid model parameters (MISO ARX-GARCH-X), comes the stage of examining the accuracy and efficiency of the hybrid model in describing the time series data under study, where the (ARCH) test was used to verify the absence of an (ARCH) effect in the residuals of the hybrid model, and the result of this test appears in the following table:

Table (6): The ARCH test for the series of residuals of the hybrid model (MISO ARX – GARCH-X)

Null Hypothesis	Lag	Test Statistic	$p-$value
True	10	0.4061	1.0000
True	15	0.6349	1.0000
True	20	0.9575	1.0000
True	30	13.2673	0.9964

From table (6), the p-value was greater than (0.05) at the time shifts taken, and this indicates Don't reject of the null hypothesis, i.e., there is no effect (ARCH).

4.3 Forecasting using the hybrid model according to the hybrid (Zhang) methodology

After completing the stages of building the hybrid model, the most important and final stage comes, which is obtaining the forecasting values of the dollar exchange rates by applying equations (36), (37), and (38) and forecasting (20) values forward, as follows (Ngailo et al., 2014):

Table (7): Forecasting values using the hybrid model [MISO ARX $(1,5,3,5,3)-$ GARCH$(1,2)-$ X$(0,1)$]

NO	$\hat{L}_t$ MISO ARX MODEL	$\hat{N}_t$ GARCH-X MODEL	$\hat{y}_t(\ell) = \hat{L}_t + \hat{N}_t$ HYBRID MODEL
1	1495.4	2.775326013	1498.175326
2	1501.2	-5.487974389	1495.712026
3	1503.3	0.250145564	1503.550146
4	1509.3	-1.256610326	1508.04339
5	1512.2	-7.26097269	1504.939027
6	1508.5	16.73530781	1525.235308
7	1506.6	4.289883843	1510.889884
8	1509.2	-12.21914658	1496.980853
9	1503.8	-4.660061055	1499.139939
10	1519.9	-0.642941563	1519.257058

11	1525	13.91991795	1538.919918
12	1513.9	1.084123563	1514.984124
13	1513.2	6.861184297	1520.061184
14	1513.5	-0.692604501	1512.807395
15	1529.3	-7.859977365	1521.440023
16	1531.2	-7.567733756	1523.632266
17	1537.3	-3.136549192	1534.163451
18	1537.1	14.95107694	1552.051077
19	1530	25.98326319	1555.983263
20	1527.3	3.919665461	1531.219665

Based on the predictive values in table (7), two series were drawn, the first representing the predictive values and the second representing the actual values, to determine the accuracy and efficiency of the hybrid model in the prediction process, as in Figure (4).

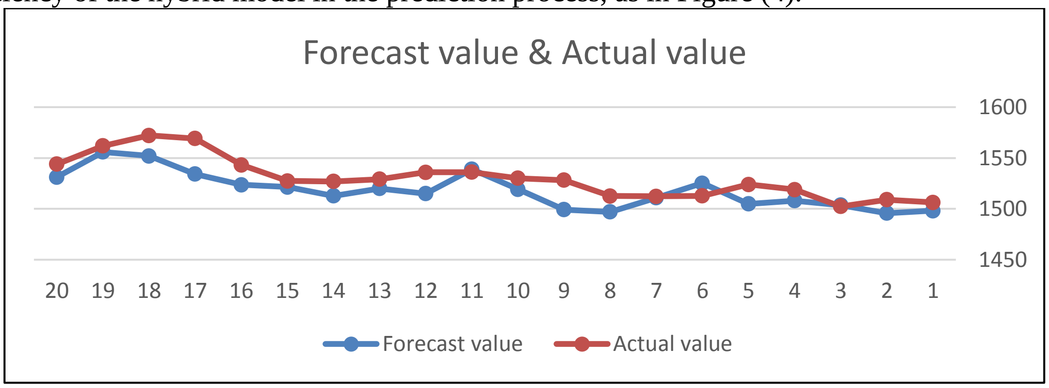

Figure (4): The forecasting values using the hybrid model with the actual value

Figure (4) shows that the predicted values were close to the actual values, which indicates that the hybrid model used can accurately predict future values.

5. Conclusion

The main objective of this research is to use a hybrid model that addresses the problems facing the time series and consists of a linear part and a non-linear part, modeling each part separately, and then collecting them according to the (Zhang) methodology to obtain the best predictions for the phenomenon to be studied. Based on the results obtained, we can conclude the following results:

1- Three criteria (AIC), (FPE), and (MDL) were used to diagnose the order of the linear model, and it became clear that the (MISO ARX (1, 5, 3, 5, 3)) model is the best model to represent the studied data.

2-The results of the estimation stage of the model parameters (MISO ARX (1, 5, 3, 5, 3)) showed that the (RELS—SE) method outperformed the rest of the methods used, using comparison measures represented by (MAE) and (MAPE) according to the data studied.

3-Two criteria (AIC) and (BIC) were used to diagnose the order of the nonlinear model, and it became clear that the ($\mathrm{GARCH}(1,2) - \mathrm{X}(0,1)$) model is the best.

4- After building the hybrid model ((MISO ARX (1, 5, 3, 5, 3) - $\mathrm{GARCH}(1,2) - \mathrm{X}(0,1)$) ,the results of the (ARCH) test showed that the hybrid model treated the problem of heteroscedasticity and removed the non-linearity that was in the residuals of the (MISO ARX (1, 5, 3, 5, 3)) model.

5- The hybrid model (MISO ARX (1, 5, 3, 5, 3) - $\mathrm{GARCH}(1,2) - \mathrm{X}(0,1)$) provided the best results for forecasting the dollar exchange rates, and the results were close to the original results.

Authors Declaration:

Conflicts of Interest: None

-We Hereby Confirm That All The Figures and Tables In The Manuscript Are Mine and Ours. Besides, The Figures and Images, which are Not Mine, Have Been Permitted Republication and Attached to The Manuscript.

- Ethical Clearance: The Research Was Approved by The Local Ethical Committee in The University.

References:

Ahmmed Mohammed, F. (2019). Applying Hybrid time series models for modeling bivariate time series data with different distributions for forecasting unemployment rate in the USA. *Journal of Mechanics of Continua and Mathematical Sciences*, *14*(5), 351–368. https://doi.org/10.26782/jmcms.2019.10.00026

Akaike, H. (1974). A New Look at the Statistical Model Identification. *IEEE Transactions on Automatic Control*, *19*(6), 716–723. https://doi.org/10.1109/TAC.1974.1100705

Allah, A. S. H., & Shalaka, J. K. (2013). Damage Currency Swap the Resulting From Government Departments Dealing With Trade Bank of Iraq From. *Journal of Economics and Administrative Sciences*, 19(73), 526. https://doi.org/10.33095/jeas.v19i73.997

Burnham K.P. and Anderson D.R. (2004). Multimodel Inference : Understanding AIC and BIC in Model Selection. *Sociological Methods & Research*. https://doi.org/10.1177/0049124104268644

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 31(3), 307-327.

Chen, T. (2018). Automatica On kernel design for regularized LTI system identification . *Automatica*, *90*, 109–122. https://doi.org/10.1016/j.automatica.2017.12.039

Chen, T., & Ljung, L. (2013). Implementation of algorithms for tuning parameters in regularized least squares problems in system identification . *Automatica*, *49*, 2213–2220. https://doi.org/10.1016/j.automatica.2013.03.030

Chen, T., Ljung, L., Andersen, M., Chiuso, A., Carli, F., & Pillonetto, G. (2012). Sparse multiple kernels for impulse response estimation with majorization minimization algorithms. *Proceedings of the IEEE Conference on Decision and Control*, 1500–1505. https://doi.org/10.1109/CDC.2012.6426801

Colin, K., Bako, L., Bombois, X. (2021). Data Informativity for the Closed-Loop Identification of MISO ARX Systems. *IFAC PapersOnLine*, *54*(7), 779–784. https://doi.org/10.1016/j.ifacol.2021.08.456

Dai, W., & Tsang, K. W. (2021). Hybrid resampling confidence intervals for change-point or stationary high-dimensional stochastic regression models. *Statistica Sinica*, *31*, 2239–2255. https://doi.org/10.5705/ss.202020.0439

Fakhri, M. A., & Mohammed, F. A. (2016). Applying some hybrid models for modeling bivariate time series assuming different distributions for random error with a practical application. *Journal of Economics and Administrative Sciences*, 26(117), 442-479. https://doi.org/10.33095/jeas.v26i117.1824

Francq, C., & Zakoian, J. (2007). Quasi-maximum likelihood estimation in GARCH processes when some coefficients are equal to zero. *Science Direct*, *117*, 1265–1284. https://doi.org/10.1016/j.spa.2007.01.001

Hamid, Z. H., & Mohammed, F. A. (2018). Comparison Between Ordinary Methods (LS,IV) and Robust Methods (2SWLS,LTS,RA) to estimate the Parameters of ARX(1,1,1) Model for Electric Loads. *Journal of Economics and Administrative Sciences*, 24(109), 496–514. https://doi.org/10.33095/jeas.v24i109.1563.

Han, H., & Kristensen, D. (2014). Asymptotic Theory for the QMLE in GARCH-X Models With Stationary and Nonstationary Covariates. *Journal of Business and Economic Statistics*, *32*(3), 416–429. https://doi.org/10.1080/07350015.2014.897954

Hickey, E., Loomis, D. G., & Mohammadi, H. (2012). Forecasting hourly electricity prices using ARMAX-GARCH models: An application to MISO hubs. *Energy Economics*, *34*(1), 307–315. https://doi.org/10.1016/j.eneco.2011.11.011

Hu, H.-Y. (2019). Evaluating Multi-Horizon Volatility-Forecasting Performances of GARCH-Type Models. *Master ' s Thesis Financial Economics Evaluating Multi-Horizon Volatility-Forecasting Performances of GARCH-Type Models, February*.

Ismail, Z. K., & Mohammed, F. A. (2021). Predicting Social Security Fund compensation in Iraq using the ARMAX model. *Journal of Economics and Administrative Sciences*, *27(125), 493-508. https://doi.org/10.33095/jeas.v27i125.2089*

Janczura, J., & Pu´c, A. (2023). ARX-GARCH Probabilistic Price Forecasts for Diversification of Trade in Electricity Markets — Variance Stabilizing Transformation Financial Risk-Minimizing Portfolio Allocation. *Energies*, 1–18. https://doi.org/10.3390/en16020807

Kazim, B. B., & Hassan, F. T. (2017). Forecasting the use of Generalized Autoregressive Conditional Heteroscedastic Models (GARCH) Seasonality with practical application. *Journal of Economics and Administrative Sciences*, *23(96)*, 341–362. https://doi.org/10.33095/jeas.v23i96.373.

Ketz, P. (2022). Allowing for weak identification when testing GARCH-X type models. *Economics* -, 1–28. https://doi.org/https://doi.org/10.48550/arXiv.2210.11398

Lee, J. H. H. (1991). A Lagrange multiplier test for GARCH models. *Economics Letters 37 (1991) 265-271*, *37*, 265–271.

Linton, O. B. (2010). ARCH models. In *Macroeconometrics and Time Series Analysis: Vol. IV* (Issue 313, pp. 2960–27). https://doi.org/10.1057/9780230280830_2

Liu, H., & Shi, J. (2013). Applying ARMA – GARCH approaches to forecasting short-term electricity prices. *Energy Economics*, *37*, 152–166. https://doi.org/10.1016/j.eneco.2013.02.006

Ljung, L. (1999). System Identification Theory for the User Second Edition. *Prentice Hall Information and System Sciences Series Thomas Kailath, Editor Astrom* . printed in the United States of America

Ljung, L. (2013). Some Classical and Some New Ideas for Identification. *J Control Autom Electr Syst (2013) 24:3–10*, 3–10. https://doi.org/10.1007/s40313-013-0004-7

Md Yeasin, K.N. Singh, A. L. and R. K. P. (2020). *Modelling Volatility Influenced by Exogenous Factors using an Improved Garch-X Model*. *74*(3), 209–216.

Mohammed, F. A., & Tawfeeq, H. A. (2021). Using Kernel Methods To Estimate The Parameters Of (Multiple Input- Single Output) ARX MODEL. *International Journal of Agricultural & Statistical Sciences, 2021, Vol 17, P1311*, *17*(Dc), 1311–1321.

Mohammed, F. A., & Yadkar, A. S. (2015). Using ARCH , GARCH Models in Prediction at Daily Closing Price For Iraqi Stock Exchange index. *Kirkuk University Journal of Administrative and Economic Sciences*, *2*, 237–266. https://doi.org/10.32894/1913-005-002-010

Ngailo, E., Luvanda, E., & Massawe, E. S. (2014). Time Series Modelling with Application to Tanzania Inflation Data. *Journal of Data Analysis and Information Processing*, *2*, 49–59. https://doi.org/http://dx.doi.org/10.4236/jdaip.2014.22007.

Parwati, L. S., Nugrahani, E. H., & Budiarti, R. (2023). Forecasting Stock Price Using Armax-Garchx Model During The Covid-19 Pandemic. *Mathline : Jurnal Matematika Dan Pendidikan Matematika*, *8*(2), 489–502. https://doi.org/10.31943/mathline.v8i2.413

Rachad, S., Nsiri, B., & Bensassi, B. (2015). System Identification of Inventory System Using ARX and ARMAX Models. *International Journal of Control and Automation*, *8*(12), 283–294. https://doi.org/10.14257/ijca.2015.8.12.26

Tangirala, A. K. (2015). *Principles Of System Identification Theory and Practice*.Taylor & Francis Group, LLC.

Wang, W., Van Gelder, P. M., Vrijling, J. K., & Ma, J. (2005). Testing and modelling autoregressive conditional heteroskedasticity of streamflow processes. *Nonlinear processes in Geophysics*, 12(1), 55-66.

Yusof, F., Kane, I. L., & Yusop, Z. (2013). Jurnal Teknologi Science and Engineering, Elektronische Ressource. *Jurnal Teknologi*, *63*(2), 27–34. https://jurnalteknologi.utm.my/index.php/jurnalteknologi/article/view/1908/1447

Zhang, Y. (2011). Unbiased identification of a class of multi-input single-output systems with correlated disturbances using bias compensation methods. *Mathematical and Computer Modelling*, *53*(9–10), 1810–1819. https://doi.org/10.1016/j.mcm.2010.12.059

Zhu, Y. (2001). Multivariable System Identification For Process Control. In *Elsevier Science & Technology Books* (Issue October). Elsevier Science & Technology Books. https://doi.org/10.1016/b978-0-08-043985-3.x5000-5.