



Comparison between estimates of beta regression model using MSE

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Abstract:

The multiple linear regression analysis shows the effects of explanatory variables on the dependent variable. Its function is to represent data to understand the shape and nature of the relationship between explanatory variables and the response variable. One of the main problems faced by linear regression is the nature of the data, as sometimes it does not follow a normal distribution. This has led to the emergence of different types of linear regression models such as the beta regression model, where the data of the dependent variable is less than one and confined within the period (0,1), and the data is relative and decimal. Non-traditional methods will be used to estimate beta regression parameters because some traditional methods do not provide suitable and accurate results when dealing with one of the main problems encountered in regression, which is multicollinearity. To address this issue, we used some methods include The Maximum Likelihood Estimation (MLE), ridge regression, modified Liu, and Ozkale and Kaciranlar method. Shrinking parameters are proposed for each method, and a comparison between the methods is conducted using the Mean Squared Error comparison criterion through a simulation experiment with two approaches, the simulation results showed that the ridge regression method is the best for the first and second methods of simulation.

Paper type Research Paper

Keywords: Beta Regression Model, Multicollinearity, Maximum Likelihood Estimation, Ridge Regression, Modified Liu, Ozkale and Kaciranlar Method, Simulation.

1.Introduction:

Linear regression is considered one of the topics used in statistics, in addition to its importance in the medical, economic, social and many fields of life (Rao et al, 2008; Thompson, 2003; Mohamed and Ahmed, 2022). Among the problems that linear regression suffers from is that the data do not follow a normal distribution and the emergence of different regression models such as the Poisson regression model, gamma regression model and exponential regression model (Irshayid and Saleh, 2022; Radam and Hameed, 2023). James and Stein (1961) studied regression models in which random errors are distributed in spherical distributions when there is a problem of multicollinearity. They found a modern method to estimate model parameters and to address the problem of multicollinearity. They also explained the characteristics of a good estimator. Extracted from models suffering from multicollinearity. Hoerl and Kennard (1970) presented a proposal for a method that addresses the problem of multicollinearity in linear regression models by adding a small positive quantity (k) to the diagonal elements of the $(x'x)$ matrix, and they called it the Ridge Regression Estimator Method (Abonazel and Taha,2023; Kadhim and Suslim, 2002; Mohammad and Ahmed, 2022). They showed theoretically that the estimator resulting from this addition is a biased estimator and is more efficient than the estimator produced by the least squares method. Therefore, the beta regression model in which the random errors follow a beta distribution will be studied, the beta distribution is considered one of the most important continuous probability distributions whose data are within the period $(0, 1)$, and it is worth noting that the beta distribution is derived from the beta function (Hirmez, 1990; Yang and Chang, 2010). The beta distribution is one of the important and basic statistical distributions in its ability to adapt and model continuous random variables (Abonazel et al,2022; Abonazel et al,2023)., because its values are decimal numbers confined between the period $(1, 0)$ (Hirmez, 1990), one of the most important problems that regression models suffer from is the problem of multicollinearity between the explanatory variables when a correlation is formed between the variables (Swindel, 1976; Sadullah and Selahattin, 2008), such as the connection of two or more variables (Abdulah et al, 2019; Ahmed et al, 2020), to reduce its effect on linear regression models, many methods have been developed to estimate model parameters and detect the problem (Ahmed et al, 2020; Kareem and Hashim, 2021).

1.1 Literature review :

Ferrari and Cribari-Neto(2004) presented a new study that includes a beta regression model when random errors follow a beta distribution and the distribution parameters are composed of the mean and Precision parameters. They also demonstrated the importance and usefulness of the beta regression model when The data is continuous and within the period $(0, 1)$.

Ospina et al(2006) calculated the second-order deviations of the maximum likelihood estimators of the beta regression model and used them by defining (bias-adjusted) estimators as an alternative to the analytically corrected maximum likelihood estimators, using the (Parametric bootstrap) method, in addition to suggesting different strategies for estimating intervals.

Simas et al (2010) expanded the formula of the beta regression model and studied it when the regression and the Precision parameter is non-linear, they compared three estimators free of deviations, the simulation results indicate a better result by estimating the model directly in its non-linear form, in addition to choosing the appropriate dispersion variances for the regression model.

Liu (2013) proposed a new method to reduce and address the problem of multicollinearity in multiple linear regression and they relied on the shrinkage parameters of the ridge regression method and the Liu method, to estimate the parameters of the multiple linear regression model and call it the Modified Liu Method.

Algamal and Abonazel (2021) presented the Modified Liu Method for the beta regression model to address the problem of multicollinearity. The estimator of the Modified Liu method was compared with the maximum likelihood method, Liu method and the ridge regression method, which depends on the mean squares error.

Qasim et al (2021) presented a new Ridge Regression estimator method for the beta regression model in order to reduce the instability of the maximum likelihood method for the beta regression model. They also proposed new formulas for calculating the shrinkage parameter for the Ridge Regression method, as well as proposing Median Squared Error(SE) This criterion can provide compelling evidence supporting the proposed method in the Monte Carlo simulation study.

Abonazel et al (2022) developed a new biased estimator and it was called (Two-Parameter Estimator) to estimate the parameters of the beta regression model. This estimator depends on several methods, namely the Ridge Regression method, the Liu method, the Modified Liu method, and the Maximum Likelihood Estimators method, because the new estimator depends on the parameters of these methods.

Geissinger et al (2022) presented a new case of beta regression in the natural sciences, meaning it is continuous data. They used two types of data related to the natural sciences, which is biochemistry. And environmental components, using the Maximum Likelihood method.

The multiple linear regression model has many types and hypotheses. Among those types, when the random error is distributed in a normal distribution, the model parameters can be estimated by using the ordinary least squares method and the maximum likelihood method, as its capabilities are accurate and good.

However, when the random error is distributed with other probability distributions such as the beta distribution and in the presence the problem of multicollinearity among its explanatory variables will be resorted to the Ridge Regression method (Yousif and Hussein, 2015), the modified Liu method, and the Ozkale and Kaciranlar method to estimate the model parameters and to obtain accurate and good results.

The objective of this research is to make a comparison between estimation methods using the MSE comparison standard, by estimating the Beta Regression model when there is a problem of multicollinearity using the Ridge Regression method, the modified Liu method, and the Ozkale and Kaciranlar method, using two simulation methods and using the proposed shrinkage parameters.

2. Material and Methods:

2.1 Beta Regression Model:

The probability density function of the beta distribution (Ospina et al, 2006; Simas et al, 2010) can be written after performing transformations in the equation (1):

$$f(y; \mu, \varphi) = \frac{\Gamma(\varphi)}{\Gamma(\mu\varphi)\Gamma((1-\mu)\varphi)} y^{\mu\varphi-1} (1-y)^{(1-\mu)\varphi-1}; 0 < y < 1 \quad (1)$$

where : $0 < \mu < 1$, $\varphi > 0$

φ : Precision Parameter (Abonazel et al, 2022; Yasin, 2023).

μ : Mean of beta probability distribution.

2.2 Maximum Likelihood Method:

To estimate the parameters of the beta regression model using the maximum likelihood estimator (MLE) method (Cribari-Net and Vasconcellos, 2002; Smithson and Verkuilen, 2006), and by maximizing the observations of the beta regression equation and taking the natural logarithm, the equation is as follows:

$$L = \sum_{t=1}^n \{ \log \Gamma(\varphi) - \log \Gamma(\mu_t(\varphi)) - \log \Gamma((1 - \mu_t)(\varphi)) + (\mu_t(\varphi) - 1) \log(y_t) + ((1 - \mu_t)(\varphi) - 1) \log(1 - y_t) \} \quad (2)$$

The derivation of the maximum likelihood method equation goes through two stages:
The first stage is according to the equation (3):

$$\beta^{(r+1)}_{ML1} = \beta^{(r)} + \left(I_{\beta\beta}^{(r)} \right)^{-1} S_{\beta}^{(r)}(\beta) \quad ; r = 1, 2, \dots, R \quad (3)$$

$$\beta^{(1)} = \hat{\beta}_{ols} = (X'X)^{-1}X'Y \quad (4)$$

Where:

$\hat{\beta}_{ols}$: Ordinary Least Square Method (OLS).

R : Number of iterations.

Estimation equation (3) is considered the first stage in estimating the parameters of the beta regression model according to the maximum likelihood method.

Also, $I_{\beta\beta}^{(r)}$ represents Fisher's equation

$$I_{\beta\beta}^{(r)} = -E \left(\frac{\partial^2 \ell(\beta, \varphi)}{\partial \beta \partial \beta'} \right) = \varphi X' W X$$

$$S(\beta) = \varphi X' A (y^* - \mu^*) \quad (5)$$

The second stage of estimating the parameters of the beta regression model is according to the equation as following:

$$\beta^r_{ML2} = (x' \hat{w} x)^{-1} x' \hat{w} \hat{z} \quad (6)$$

$$\hat{z} = \hat{\eta} + \hat{w}^{-1} \hat{A} (y^* - \mu^*) \quad (7)$$

$$\hat{w}_t = \varphi \{ \psi'(\mu_t \varphi) + \psi'((1 - \mu_t) \varphi) \} \frac{1}{[g'(\mu_t)]^2}$$

X : is an (n x k) matrix of regressors.

$$A = \text{diag} \left\{ \frac{1}{g'(\mu_1)}, \dots, \frac{1}{g'(\mu_t)} \right\}$$

$$y^* = \text{Log} \frac{y_t}{1-y_t} \quad ; \left\{ \text{Log} \frac{y_1}{1-y_1}, \dots, \text{Log} \frac{y_n}{1-y_n} \right\}$$

(8)

$$\mu^* = \{ \psi(\mu_t \varphi) - \psi((1 - \mu_t) \varphi) \}$$

$$= \{ \{ \psi(\mu_1 \varphi) - \psi((1 - \mu_1) \varphi) \}, \dots, \{ \psi(\mu_n \varphi) - \psi((1 - \mu_n) \varphi) \} \} \quad (9)$$

η_t : Linear Predictor (Asar, 2018).

$$A = \text{diag} \left\{ \frac{1}{g'(\mu_1)}, \dots, \frac{1}{g'(\mu_t)} \right\}$$

$$y^* = \text{Log} \frac{y_t}{1-y_t} \quad ; \left\{ \text{Log} \frac{y_1}{1-y_1}, \dots, \text{Log} \frac{y_n}{1-y_n} \right\}$$

(8)

$$\mu^* = \{ \psi(\mu_t \varphi) - \psi((1 - \mu_t) \varphi) \}$$

$$= \{ \{ \psi(\mu_1 \varphi) - \psi((1 - \mu_1) \varphi) \}, \dots, \{ \psi(\mu_n \varphi) - \psi((1 - \mu_n) \varphi) \} \} \quad (9)$$

η_t : Linear Predictor (Asar, 2018).

2.3 Ridge Regression (RR):

There are two types of multicollinearity, which are complete and semi-complete (Algamal et al, 2022). When the multicollinearity is semi-complete (semi multicollinearity) and the determinant value of the information matrix is very small, this leads to the variance of the estimated information being large in very large proportions (Yalian and Yang, 2012; Hussein, 2019), we conclude from this that some explanatory variables are sometimes insignificant in the linear regression model, and one of the most important methods for dealing with this problem is

the ridge regression method (Kadhim and suslim, 2002; Khalaf and Shukur, 2005). The ridge regression method will be used for estimation the beta regression model in presence the problem of multicollinearity (Muniz and Kibria, 2009; Hussein,2016), in addition to proposing the shrinkage parameter (k) and the mean square error criterion based on Monte Carlo simulation.

Let $\hat{\beta}$ be an estimate of the vector β and the Weighted Sum of Squared Error (WSSE) is defined as follows (Kibria, 2003; Qasim et al, 2021):

$$\begin{aligned} \phi &= (y - \hat{\beta}_{RR})'(y - \hat{\beta}_{RR}) \\ &= (y - X\hat{\beta}_{ML})'(y - X\hat{\beta}_{ML}) + (\hat{\beta}_{RR} - \hat{\beta}_{ML})'X'WX(\hat{\beta}_{RR} - \hat{\beta}_{ML}) \end{aligned}$$

Where:

ϕ : Minimum value.

$\Theta(\hat{\beta}) > 0$ and B_{RR} It is obtained by minimizing the range of $\widehat{\beta}_{RR}'\widehat{\beta}_{RR}$ while setting the constraint(Kibria and Banik, 2016; Qasim et al, 2021):

$$Q = \hat{\beta}'_{RR}\hat{\beta}_{RR} + \left(\frac{1}{k}\right)\{(\hat{\beta}_{RR} - \hat{\beta}_{ML})'X'WX(\hat{\beta}_{RR} - \hat{\beta}_{ML}) - \phi_0\} \quad (10)$$

Where :

$\left(\frac{1}{k}\right)$: It represents the Lagrange Multiplier

After simplifying, we obtain the $\hat{\beta}_{BRR}$ estimator (Awwad et al,2022)

$$\hat{\beta}_{BRR} = (X'WX + k_{PR}I)^{-1}X'WX\hat{\beta}_{ML} \quad (11)$$

$k > 0$

K : It is the shrinkage parameter (Khalaf and Shukur, 2005).

I : Identity matrix ($P \times P$).

The proposed shrinkage k is:

$$k_{PR} = \frac{1}{\varphi \text{Min}(\alpha_j)^2} \quad (12)$$

2.4 Modified Liu Method (MLR):

Algamal and Abonazel (2021) developed another method to reduce the problem of multicollinearity by developing the Liu method to the Modified Liu method by estimating the two shrinkage parameters and integrating the shrinkage parameter for Liu method and the shrinkage parameter for the Ridge Regression method (Liu, 1993; Asar and Genc, 2016; Algamal and Abonazel, 2021), the Modified Liu Regression (MLR) method is as follows (Liu, 2003; Omara, 2019):

$$\hat{\beta}_{MLR} = (X'\widehat{W}X + k_{PR}I)^{-1}(X'\widehat{W}X - d_{PL}I)\hat{\beta}_{ML} \quad (13)$$

$k > 0, -\infty < d < \infty$

The bias vector for modified liu method for beta regression is (Liu, 2004; Abonazel et al, 2022):

$$\text{Bias}(\hat{\beta}_{MLR}) = -(d_{PL} + k_{PR})(X'\widehat{W}X + k_{PR}I)^{-1}\beta_{ML} \quad (14)$$

The proposed shrinkage parameters for the Modified Liu method for beta regression are according to the following equations:

$$k_{PR} = \frac{1}{\varphi \text{Min}(\alpha_j)^2} \quad (15)$$

$$d_{PL} = \text{Median} \left(\sqrt{\frac{\alpha_j^2}{\lambda_j}} \right) \quad (16)$$

2.5 Ozkale and Kaciranlar Method (O.K.):

This is the method proposed by (Ozkale and Kaciranlar, 2007) for the beta regression model to address the problem of multicollinearity and to obtain more stable and accurate results, and the estimation equation is as follows (Kurnaz and Akay, 2015; Lukman et al, 2019; Omara, 2019; Abonazel et al, 2022):

$$\hat{\beta}_{TPBR} = (X' \widehat{W} X + k_R I)^{-1} (X' \widehat{W} X + k_R d_M I) \hat{\beta}_{ML} \quad (17)$$

Where:

$$k > 0 ; 0 < d < 1$$

The bias vector for two beta regression parameters is as follows (Abonazel et al, 2022):

$$Bias(\hat{\beta}_{TPBR}) = k_R (d_M - 1) (X' \widehat{W} X + k_R I)^{-1} \beta_{ML} \quad (18)$$

The shrinkage functions are as follows:

The shrinkage parameter (k) is according to the equation (19):

$$k_R = \frac{1}{\varphi \sum_{j=1}^p \hat{\alpha}_j^2} \quad (19)$$

The shrinkage parameter (d) is according to the equation (20):

$$d_M = \frac{\sum_{j=1}^p \left[\frac{\left(\frac{1}{\varphi} - k_M \hat{\alpha}_j^2 \right)}{(\lambda_j + k_M)^2} \right]}{\sum_{j=1}^p \left[\frac{\left(\frac{1}{\varphi} + \lambda_j \hat{\alpha}_j^2 \right)}{\lambda_j (\lambda_j + k_M)^2} \right]} \quad (20)$$

3. Discussion of Results:

3.1 Simulation:

The comparison process between the estimation methods was carried out based on the mean square error in generating random data on the MATLAB program.

1. the random error variable (Y_i) was generated in a beta regression model according to a beta distribution with two parameters (μ_i) and (φ).

Where:

$$y_i \sim \text{Beta}(\mu_i, \varphi)$$

$$\varphi = \{1, 2\}$$

As for (μ_t) its value is according to the equation (21):

$$\mu_t = \frac{e^{x_t' \beta}}{1 + e^{x_t' \beta}} \quad (21)$$

2. The explanatory variables $x_i = (x_{i1}, \dots, x_{ip})'$ are subjected to correlational relationships to form the problem of multicollinearity between the explanatory variables of the model using the following equation (Arashi et al, 2021):

$$x_{ij} = (1 - \rho^2)^{0.5} Z_{ij} + \rho Z_{ij} \quad (22)$$

$$i = 1, 2, \dots, n \quad , j = 1, 2, \dots, p$$

ρ : It represents the simple correlation coefficient between each pair of explanatory variables.

Z_{ij} : They represent random numbers that are independent and generated in two different ways, as follows:

1. The first method of simulation : Z_{ij} is generated when it has a pseudo-standard normal distribution

2. The second method of simulation: it is done by generating Z_{ij} values according to real data, as follows:

$$Z_{ij} = L(X) + (U(X) - L(X)) \text{rand}(n, P) \quad (23)$$

Since:

$L(X)$: represents the lowest value of the explanatory variable in the real applied experiment data.

$U(X)$: represents the upper value of the explanatory variable in the real applied experiment data.

n : represents the sample size.

P : represents the number of explanatory variables.

3. Six sizes were chosen for the selected samples (20, 30, 50, 60, 100, 150) and three values for the correlation coefficients (0.90, 0.95, 0.99). The number of explanatory variables in the experiment was assumed to be (2, 5) and the experiment was repeated 1000 times.

4. The mean square error estimates for the model are calculated according to the following equation:

$$MSE(\hat{Y}) = \frac{\sum_{i=1}^n (y_i - \hat{y})^2}{n - k} / L \quad (24)$$

3.2 Results of Simulation :

The simulation results for the first method of simulation include the following results, which are shown below in Table 1:

Table 1: Represents a comparison between the estimation methods in the presence of the proposed shrinkage parameters and with respect to the number of variables (2, 5) by adopting the mean square error criterion for the first method of simulation.

P	n	ρ	ϕ	RR	MLR	O.K.
2	20	0.90	1	0.242853161169977	1.73984764404361	1.47433295517998
			2	0.0794658304821197	0.710342813069322	0.519179254523815
		0.95	1	0.236351878744249	1.92021106316904	1.59966488919547
			2	0.0744184927728133	0.747731921356797	0.530102112052892
		0.99	1	0.223681915780450	2.35970654028362	1.89185510390365
			2	0.0653917986791959	0.839121132617783	0.556622444075752
	30	0.90	1	0.236740611004212	1.20775086704980	1.07784270557839
			2	0.0815625052548073	0.508168210071924	0.410661322543856
		0.95	1	0.231305984208223	1.31000026851344	1.15480469020285
			2	0.00922671547699422	0.136909707663703	0.102024136224316
		0.99	1	0.220788160851393	1.56479459568447	1.34175369017031
			2	0.0685251495267468	0.582622160158033	0.439191410869473
	50	0.90	1	0.242288935073918	0.910472046823098	0.851643549555290
			2	0.0872390887535289	0.376166643953343	0.331477754163610
		0.95	1	0.237696612513651	0.977355716939770	0.907562126944415
			2	0.0109638885470376	0.0922055443616729	0.0775552318512533
		0.99	1	0.228945692580354	1.14032140912100	1.04157277101508
			2	0.0750903478062654	0.420654774393536	0.355194667963601
	60	0.90	1	0.239058864943827	0.822251258602106	0.777266287438765
			2	0.0872327889184683	0.339295119939626	0.304932772395718
		0.95	1	0.234740236769663	0.879598347883308	0.826394583452940
			2	0.0830244895817542	0.349564571187446	0.310674930471660
		0.99	1	0.226565503965725	1.01790869740000	0.943139084814303
			2	0.0755061536985577	0.375428714133321	0.325415962496382
	100	0.90	1	0.243705843745015	0.666307907690692	0.644265630493102
			2	0.0905544119307057	0.270174586608284	0.253314975232123
		0.95	1	0.239869752403356	0.705257836203813	0.679430799189001
			2	0.0865793654515260	0.275968935583050	0.256999349417598
		0.99	1	0.232740096498724	0.799577334421131	0.763915563050519
			2	0.0141161267129018	0.0827748785355776	0.0743965642973238
	150	0.90	1	0.242920816645181	0.564816713587232	0.552192688079442
			2	0.0916056230084848	0.228878325520578	0.219148577557707
		0.95	1	0.239356549288510	0.592589578058194	0.577917448728386
			2	0.0877738113879950	0.232017075283025	0.221135645679100
		0.99	1	0.232818839044347	0.659528329064194	0.639606021066771

5	20	0.90	2	0.0810177351155632	0.241443195447440	0.227752651832593
			1	0.224262646318021	2.39324739293209	1.46162789431889
		0.95	2	0.0698904596873413	1.06938118586365	0.473271153448016
			1	0.216209944965279	2.65536007168053	1.54455323257877
		0.99	2	0.0649776317726388	1.12963323535035	0.466764619556106
			1	0.201106547970498	3.29982702121196	1.72651326852207
	30	0.90	2	0.0566604908457195	1.27184231186771	0.455021899590286
			1	0.223732132375628	1.67996272333220	1.24595933725202
		0.95	2	0.0717823376721720	0.735518239983831	0.434479263266498
			1	0.216547729153008	1.85241837304808	1.32809855004189
		0.99	2	0.0668073761265380	0.774453957437023	0.434917184797799
			1	0.202764067127237	2.26676499820914	1.51288848041864
	50	0.90	2	0.0581417754961798	0.867446002263011	0.436672151935356
			1	0.219900636255829	1.21016718391146	1.01721004087943
		0.95	2	0.0750895795204167	0.523702140596940	0.383493271651176
			1	0.214035411041681	1.31829267464633	1.08713189964915
		0.99	2	0.0703781078601558	0.546927688974045	0.387938039969871
			1	0.202703705526162	1.58128042314348	1.24920502089825
	60	0.90	2	0.0620052905954948	0.603622731361240	0.399035165572294
			1	0.219688024481672	1.09383903872220	0.947252485800825
		0.95	2	0.0767193341825638	0.470103496075259	0.362452365169379
			1	0.214220146828383	1.18754116816863	1.01214765481335
		0.99	2	0.0721197346404005	0.489548931155511	0.367396060724617
			1	0.203673203760534	1.41513274599361	1.16357783267493
100	0.90	2	0.0639169198171012	0.537174811468390	0.379697097343320	
		1	0.221822353912346	0.825711627839514	0.759284684786979	
	0.95	2	0.0810872506876259	0.352140477437405	0.301710417450121	
		1	0.217265939946426	0.886203630908605	0.807306157592234	
	0.99	2	0.0768166276148607	0.363478491986448	0.306356944302587	
		1	0.208585679104381	1.03276182780285	0.921026020924472	
150	0.90	2	0.0691902121797065	0.392026157552303	0.318496952740095	
		1	0.223003837070396	0.679743483889331	0.643200516589705	
	0.95	2	0.0833049546002092	0.286554156074498	0.258504386964927	
		1	0.218962124672076	0.723001732998766	0.679908052915333	
	0.99	2	0.0792781451042453	0.293628325861748	0.261976508331279	
		1	0.211365980442014	0.827617927107038	0.767402946162064	
			2	0.0721155341243539	0.312156421825117	0.271674775362802

As for the simulation results for the second simulation method, the following results are included in Table 2:

Table 2 : Represents a comparison between the two estimation methods in the presence of the proposed shrinkage parameters and with respect to five explanatory variables, adopting the mean squared error criterion for the second method of simulation.

P	n	ρ	ϕ	RR	MLR	O.K.
5	100	0.90	1	0.00386699039904131	0.00386690803511351	0.00386704158299637
			2	0.00333038413818084	0.00333042724363634	0.00333040360031634
		0.95	1	0.00378747227886261	0.00378756811272587	0.00378752088672872
			2	0.00324208068553154	0.00324213251340113	0.00324210207719897
		0.99	1	0.00364908274521512	0.00364914156203378	0.00364912606087515
			2	0.00309346710701350	0.00309352396647516	0.00309349691050538

The results of the simulation experiment showed that the ridge regression method for the beta regression model is better than the rest of the estimation methods by comparing the results of the mean square errors for all the estimation methods used, where the ridge regression method was the smallest among all the results of the estimation methods in the presence of the proposed shrinkage parameters and when there are explanatory variables equal to (2, 5) and at two levels of Precision Parameter (1, 2) and for the first and second simulation methods..

4. Conclusion:

1. The simulation results showed that the Ridge Regression method is better than the Modified Liu method and Ozkale and Kaciranlar method in the presence of the proposed shrinkage parameters, for the first and second simulation methods.
2. When using the second method of simulation, the Ridge Regression method when there are five explanatory variables is the best in the presence of the proposed shrinkage parameters.
3. The results of the mean square errors showed that the second best method after the Ridge Regression method is the Ozkale and Kaciranlar method when there are two explanatory variables and also in the presence of 5 explanatory variables as for the first simulation method.
4. Based on the simulation results, it is recommended to use the ridge regression method in the presence of the proposed shrinkage parameter to estimate the parameters of the beta regression model as an alternative to the modified Liu method and the Ozkale and Kaciranlar method.
5. Other shrinkage parameters suitable for estimation methods can be suggested and compared.

Authors Declaration:

Conflicts of Interest: None

-We Hereby Confirm That All The Figures and Tables In The Manuscript Are Mine and Ours. Besides, The Figures and Images, Which are Not Mine, Have Been Permitted Republication and Attached to The Manuscript.

- Ethical Clearance: The Research Was Approved By The Local Ethical Committee in The University.

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مستخلص البحث:

يوضح تحليل الانحدار الخطي المتعدد التأثيرات بين المتغيرات التفسيرية والمتغير التابع، حيث تكمن وظيفته في تمثيل البيانات لفهم شكل وطبيعة العلاقة بين المتغيرات التفسيرية ومتغير الاستجابة، ومن اهم المشاكل التي يعاني منها الانحدار الخطي هي طبيعة البيانات اذ انها في بعض الأحيان لا تتبع التوزيع الطبيعي وذلك أدى الى ظهور أنواع مختلفة من نماذج الانحدار الخطي كأنموذج انحدار بيتا، اذ تكون فيه بيانات المتغير المعتمد أصغر من الواحد وتكون محصورة ضمن الفترة (0,1) وتكون بياناته نسبية وعشرية. وسوف يتم استعمال طرائق غير تقليدية لتقدير معالم انحدار بيتا وذلك لكون بعض الطرائق التقليدية لا تعطي نتائج مناسبة ودقيقة عند وجود احد اهم المشاكل التي يعاني منها الانحدار وهي مشكلة التعدد الخطي ولمعالجة هذه المشكلة يتم استعمال طريقة الإمكان الأعظم (MLE) وطريقة انحدار الحرف وطريقة ليو المعدلة وطريقة (Ozkale And Kaciranlar) مع اقتراح معالم تقلص لكل طريقة واجراء مقارنة بين الطرائق باستعمال معيار المقارنة متوسطات مربعات الخطأ وذلك بأجراء تجربة المحاكاة بأسلوبين، وقد بينت نتائج المحاكاة ان طريقة انحدار الحرف هي الأفضل وذلك بالنسبة للأسلوب الأول والأسلوب الثاني للمحاكاة.

نوع البحث: ورقة بحثية.

المصطلحات الرئيسية للبحث: انموذج انحدار بيتا، التعدد الخطي، طريقة الإمكان الأعظم، طريقة انحدار الحرف، طريقة ليو المعدلة، طريقة (Ozkale And Kaciranlar)، المحاكاة.