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Analysis of Reliability for the Electrical Industries Company in Diyala using Modified Extension Weibull Distribution (An Application and Simulation study)

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Abstract:

In this paper defining modified extension Weibull distribution, reliability function and hazerd function, we explain the maximum likelihood method under type one censoring sample. The censored samples play an important role in reliability analysis .In this paper we estimate and derived modified extension Weibull distribution whith three parameters under the censored samples type one by maximum likelihood estimation method depend on Newton-Raphson method. Then we applied simulation procedure using Monte-Carlo technique to estimate reliability function under different samples sizes and various initial values for the parameters for all estimated parameters of modified extension Weibull model by used MATLAB program. Finally we find the probability density function f(t), reliability function R(t),and hazard function h(t) for real data which get it from (Diyala province Company) is one of the formations of the Ministry of Industry and Minerals.

Paper type: Research paper
Key words: Modified Extension Weibull Distribution, Censored sample, Reliability function, Simulation.

1. Introduction

The reliability function is one of most commonly used function in life data analysis, it is a function of time, it gives the probability of an item operating for certain amount of time without failure, the censored samples are most frequently in the life, in many cases when lifetime data are analyzed some units which enter the experiments must be failed but the other ones not failed, these units exceed the experiment time without failed, there are many types censored data, left censored data, interval censored data, right censored data the right censored data divides in to three type which are as follows: a-type one censoring data, b-type two censoring data, C-type three (progressively) censoring data .The research contain many section .Second section we mentioned material and methods Third section explain discussion of results. Fourth section discussed the conclusion. And in last section we mentioned the references .

There are some studies that have discussed this topic. Xie and Goh (2002) presented a modified Weibull extension, model with three parameters they obtained the mathematical and statistical properties for this proposed distribution. Nada rajah (2005) derived explicit algebraic formulas for the kth moments for this distribution. Peng et al. (2012) proposed comparative analysis to prove that the modified Weibull extension distribution is more accurate and flexible in modeling the satellite reliability than the classical Weibull distribution. Sarhan and Apaloo (2013) proposed a new model of a lifetime distribution that mainly generalizes two distributions, one of them is modified Weibull extension, they named as exponentiated modified Weibull extension distribution. They showed that the exponentiated modified Weibull extension distribution can be used quite effectively in fitting and analyzing real lifetime data. Kadhum and Hasan (2014) used the Rayleigh distribution to study type I censored samples. Helen and Uma (2015) introduced a new operation for addition, subtraction and multiplication of different type of pentagonal fuzzy numbers on the basis of alpha cut sets of fuzzy numbers and a new approach for ranking of pentagon fuzzy numbers using indenter of the centroids. Al-Noor and Al-Sultany (2017) used Approximation classical computational methods to estimate inverse Weibull parameters and reliability function with fuzzy data, as well as they provided compared numerically through Monte-Carlo simulation study, then using mean squared error values and integrated mean squared error values respectively. After that, they utilized the ranking function to get a crisp number. Robaiay (2019) they studied-Duwry and Al-Al Non-parametric methods were compared under the use of type I censored sample and simulation technique. Abadi and AL- Kanani.(2020) submit study to estimate and derive the three parameters which contain two scales parameters and one shape parameter of a new mixture distribution for the singly type one censored data which is the branch of right censored sample .AL kanani and Yass (2021) preformed Maximum likelihood method and Rank set sampling method for modified Weibull extension distribution by utilizing simulation technique they found that MLE is the best method depend on mean squares error procedure. Feroze et al. (2022) explored the suitability of the Modified Weibull Extension model in modeling censored medical datasets. The analysis has been carried out using Bayesian methods under different loss functions and informative priors. They concluded that Bayesian methods performed better as compared to maximum likelihood estimates. All distribution with three parameters under type one censored samples by using maximum likelihood utilizing the Monte-Carlo technique by per forming mean squares error procedure for different samples sizes and various initial values. Then applying these producers and processes for real data.

The problem of this research is to estimate the reliability function through a more flexible and adaptive distribution using type-one censored sample.

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2. Material and Methods

2.1. Modified Extension Weibull Distribution

The two researchers Xie and Goh in (2002) introduced the modified extension weibull distribution.

The reliability function of this distribution is as follows :

Where Ω is parameters space , β and λ are shapes parameters, α is scale parameter.

$\Omega = \{(\alpha, \beta, \lambda); \alpha, \beta, \lambda > 0\}$

$$R(t) = e^{\lambda\alpha\left[1-e^{\left(\frac{t}{\alpha}\right)^{\beta}}\right]} \qquad \alpha, \beta, \lambda > 0\,, t > 0\,. \qquad \text{(Xie and Goh in 2002)}$$

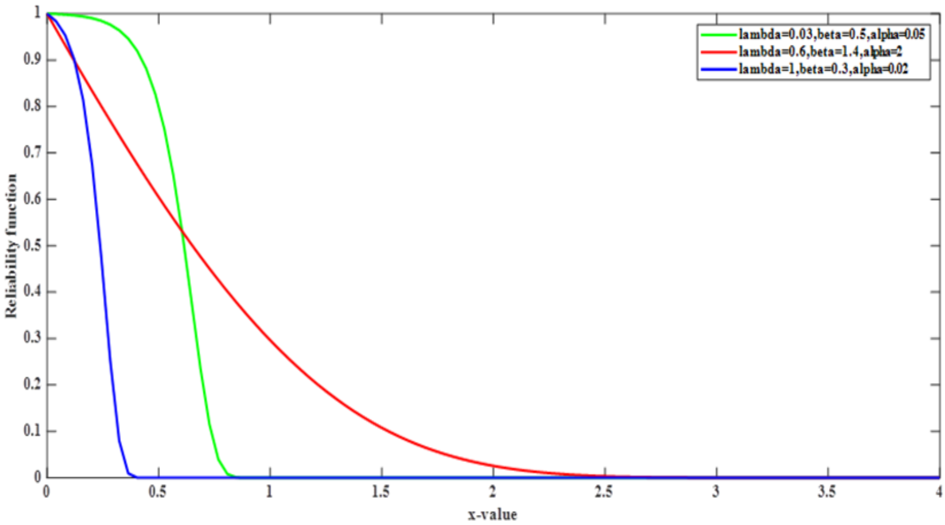

Figure 1: The reliability function of Modified Extension Weibull Distribution.

The corresponding failure rate function has the following form

$$h(t) = \lambda\beta\left(\frac{t}{\alpha}\right)^{\beta-1} e^{\left(\frac{t}{\alpha}\right)^{\beta}}$$

The cumulative distribution and probability density function are as follows :

$$F(t) = 1 - R(t) = 1 - e^{\lambda\alpha\left[1-e^{\left(\frac{t}{\alpha}\right)^{\beta}}\right]}$$

$$f(t) = \lambda\beta\left(\frac{t}{\alpha}\right)^{\beta-1} e^{\left[\left(\frac{t}{\alpha}\right)^{\beta} + \lambda\alpha\left(1-e^{\left(\frac{t}{\alpha}\right)^{\beta}}\right)\right]} \qquad \text{(Xie and Goh 2002)}$$

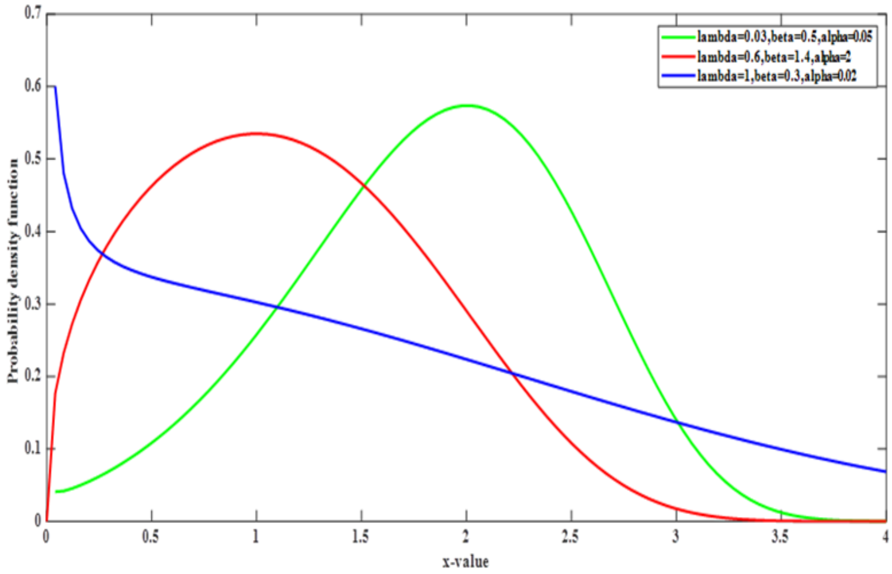

Figure2: The probability density function of Modified Extension Weibull Distribution

2.2. Maximum Likelihood Method (MLEM) for Type-One Censoring Sample:

Let $t_1 , t_2 , ... t_n$ be a random sample for modified extension Weibull distribution, then the likelihood of this distribution is as follows: (Yass and Al Kanani, 2022).
m =the number of items which are failed is random variable. On the experimental side, the value of m was determined in the quality control room by censored samples
n =the number of the all items which are fixed.
n-m = the number of items which are not failed.

$L(\lambda,\alpha,\beta) = \frac{n!}{(n-m)!}[R(t_m)]^{n-m}\prod_{i=1}^{m} f(t_i)$, Let $K = \frac{n!}{(n-m)!}$

$$L=Ke^{(n-m)\lambda\alpha\left(1-e^{\left(\frac{t_m}{\alpha}\right)^{\beta}}\right)}\lambda^m\alpha^m\prod_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta-1}e^{\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta}}e^{\lambda\alpha\sum_{i=1}^{m}\left(1-e^{\left(\frac{t_i}{\alpha}\right)^{\beta}}\right)}$$

Taking the natural logarithm for the two side

$$\ln L = \text{In } K + (n-m)\lambda\alpha\left(1-e^{\left(\frac{t_m}{\alpha}\right)^{\beta}}\right) + m\ln\lambda + m\ln\alpha$$
$$+(\beta-1)\sum_{i=1}^{m}\ln\left(\frac{t_i}{\alpha}\right) + \sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} + \lambda\alpha\sum_{i=1}^{m}\left(1-e^{\left(\frac{t_i}{\alpha}\right)^{\beta}}\right)$$

Let A=n-m

$$f(\lambda) = A\,\alpha\left(1-e^{\left(\frac{t_m}{\alpha}\right)^{\beta}}\right) + \frac{m}{\lambda} + \alpha\sum_{i=1}^{m}\left(1-e^{\left(\frac{t_i}{\alpha}\right)^{\beta}}\right) \quad (1)$$

$$g(\alpha) = A\lambda\beta\left(\frac{t_m}{\alpha}\right)^{\beta}e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} + A\lambda\left(1-e^{\left(\frac{t_m}{\alpha}\right)^{\beta}}\right) + \frac{m}{\alpha} - \frac{(\beta-1)m}{\alpha} - \frac{\beta}{\alpha}\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} \quad (2)$$
$$+\lambda\beta\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta}e^{\left(\frac{t_i}{\alpha}\right)^{\beta}} + \lambda\sum_{i=1}^{m}\left(1-e^{\left(\frac{t_i}{\alpha}\right)^{\beta}}\right)$$

$$h(\beta) = -A\,\lambda\,\alpha\left(\frac{t_m}{\alpha}\right)^{\beta}\ln\left(\frac{t_m}{\alpha}\right)e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} + \sum_{i=1}^{m}\ln\left(\frac{t_i}{\alpha}\right) + \sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta}\ln\left(\frac{t_i}{\alpha}\right) \quad (3)$$
$$-\lambda\,\alpha\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta}\ln\left(\frac{t_i}{\alpha}\right)e^{\left(\frac{t_i}{\alpha}\right)^{\beta}}$$

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$$\begin{bmatrix}\lambda_{k+1}\\ \alpha_{k+1}\\ \beta_{k+1}\end{bmatrix} = \begin{bmatrix}\lambda_{k}\\ \alpha_{k}\\ \beta_{k}\end{bmatrix} - J^{-1}\begin{bmatrix}f(\lambda)\\ g(\alpha)\\ h(\beta)\end{bmatrix} \quad , \; J = \begin{bmatrix}\frac{\partial f}{\partial\lambda} & \frac{\partial f}{\partial\alpha} & \frac{\partial f}{\partial\beta}\\ \frac{\partial g}{\partial\lambda} & \frac{\partial g}{\partial\alpha} & \frac{\partial g}{\partial\beta}\\ \frac{\partial h}{\partial\lambda} & \frac{\partial h}{\partial \alpha} & \frac{\partial h}{\partial\beta}\end{bmatrix}$$

$$\frac{\partial f}{\partial\lambda} = \frac{-m}{\lambda^2} \quad (4)$$

$$\frac{\partial f}{\partial\alpha} = A\,\beta\left(\frac{t_m}{\alpha}\right)^{\beta} e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} + A\left(1 - e^{\left(\frac{t_m}{\alpha}\right)^{\beta}}\right) + \beta\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}} + \sum_{i=1}^{m}\left(1 - e^{\left(\frac{t_i}{\alpha}\right)^{\beta}}\right) \quad (5)$$

$$\frac{\partial f}{\partial\beta} = -A\,\alpha\left(\frac{t_m}{\alpha}\right)^{\beta} \ln\left(\frac{t_m}{\alpha}\right) e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} - \alpha\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} \ln\left(\frac{t_i}{\alpha}\right) e^{\left(\frac{t_i}{\alpha}\right)^{\beta}} \quad (6)$$

$$\frac{\partial g}{\partial\lambda} = A\beta\left(\frac{t_m}{\alpha}\right)^{\beta} e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} + A\left(1 - e^{\left(\frac{t_m}{\alpha}\right)^{\beta}}\right) + \beta\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}} + \sum_{i=1}^{m}\left(1 - e^{\left(\frac{t_i}{\alpha}\right)^{\beta}}\right) \quad (7)$$

$$\frac{\partial g}{\partial\alpha} = -A\frac{\lambda\beta^2}{\alpha}\left(\frac{t_m}{\alpha}\right)^{2\beta} e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} - A\frac{\lambda\beta^2}{\alpha}\left(\frac{t_m}{\alpha}\right)^{\beta} e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} + A\frac{\lambda\beta}{\alpha}\left(\frac{t_m}{\alpha}\right)^{\beta} e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} + \frac{(\beta-2)m}{\alpha^2} \quad (8)$$
$$+\frac{\beta^2}{\alpha^2}\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} + \frac{\beta}{\alpha^2}\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} - \frac{\lambda\beta^2}{\alpha}\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{2\beta} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}} + \frac{\lambda\beta^2}{\alpha}\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}}$$
$$+\frac{\lambda\beta}{\alpha}\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}}$$

$$\frac{\partial g}{\partial\beta} = A\,\lambda\beta\left(\frac{t_m}{\alpha}\right)^{2\beta} e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} \ln\left(\frac{t_m}{\alpha}\right) + A\lambda\left(\frac{t_m}{\alpha}\right)^{\beta} e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} + A\lambda\beta\left(\frac{t_m}{\alpha}\right)^{\beta} e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} \ln\left(\frac{t_m}{\alpha}\right) \quad (9)$$
$$-A\lambda\left(\frac{t_m}{\alpha}\right)^{\beta} e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} \ln\left(\frac{t_m}{\alpha}\right) - \frac{m}{\alpha} - \frac{\beta}{\alpha}\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} \ln\left(\frac{t_i}{\alpha}\right) - \frac{1}{\alpha}\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta}$$
$$+\lambda\beta\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{2\beta} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}} \ln\left(\frac{t_i}{\alpha}\right) + \lambda\beta\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}} \ln\left(\frac{t_i}{\alpha}\right) + \lambda\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}}$$
$$-\lambda\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}} \ln\left(\frac{t_i}{\alpha}\right)$$

$$\frac{\partial h}{\partial\lambda} = -A\,\alpha\left(\frac{t_m}{\alpha}\right)^{\beta} e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} \ln\left(\frac{t_m}{\alpha}\right) - \alpha\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} \ln\left(\frac{t_i}{\alpha}\right) e^{\left(\frac{t_i}{\alpha}\right)^{\beta}} \quad (10)$$

$$\frac{\partial h}{\partial\alpha} = A\,\lambda\,\alpha\left(\frac{t_m}{\alpha}\right)^{2\beta} \ln\left(\frac{t_m}{\alpha}\right) e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} + A\,\lambda\,\beta\,\ln\left(\frac{t_m}{\alpha}\right) e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} + A\,\lambda\left(\frac{t_m}{\alpha}\right)^{\beta} e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} \quad (11)$$
$$-A\,\lambda\left(\frac{t_m}{\alpha}\right)^{\beta} \ln\left(\frac{t_m}{\alpha}\right) e^{\left(\frac{t_m}{\alpha}\right)^{\beta}} - \frac{m}{\alpha} - \frac{1}{\alpha}\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} - \frac{\beta}{\alpha}\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} \ln\left(\frac{t_i}{\alpha}\right)$$
$$+\lambda\beta\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{2\beta} \ln\left(\frac{t_i}{\alpha}\right) e^{\left(\frac{t_i}{\alpha}\right)^{\beta}} + \lambda\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}} - \lambda\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}} \ln\left(\frac{t_i}{\alpha}\right)$$
$$+\lambda\beta\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}} \ln\left(\frac{t_i}{\alpha}\right)$$

$$\frac{\partial h}{\partial\beta} = -A\,\lambda\,\alpha\left(\frac{t_m}{\alpha}\right)^{2\beta} e^{\left(\frac{t_m}{\alpha}\right)^{\beta}}\left(\ln\left(\frac{t_m}{\alpha}\right)\right)^2$$
$$-A\,\lambda\alpha\left(\frac{t_m}{\alpha}\right)^{\beta} e^{\left(\frac{t_m}{\alpha}\right)^{\beta}}\left(\ln\left(\frac{t_m}{\alpha}\right)\right)^2 + \sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta}\left(\ln\left(\frac{t_i}{\alpha}\right)\right)^2$$
$$-\lambda\alpha\sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{2\beta} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}}\left(\ln\left(\frac{t_i}{\alpha}\right)\right)^2 - \lambda\alpha\sum_{i=1}^{m} e^{\left(\frac{t_i}{\alpha}\right)^{\beta}}\left(\frac{t_i}{\alpha}\right)^{\beta}\left(\ln\left(\frac{t_i}{\alpha}\right)\right)^2 \quad (12)$$

$$\begin{bmatrix}\epsilon_{\lambda}^{k+1}\\ \epsilon_{\alpha}^{k+1}\\ \epsilon_{\beta}^{k+1}\end{bmatrix} - \begin{bmatrix}\epsilon_{\lambda}^{k}\\ \epsilon_{\alpha}^{k}\\ \epsilon_{\beta}^{k}\end{bmatrix} = \begin{bmatrix}0.0001\\ 0.0001\\ 0.0001\end{bmatrix}$$

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2.3. Simulation Application :

Simulation procedure is now widely used in many branches of statistics. They can be used to evaluate the behavior of models as well as for some random variables. A simulation is defined as a numerical scientific method that uses logical mathematical methods to describe the behavior of a certain approved system. There are different techniques has been conducted.

F(t)=1-R(t)=1-
$$e^{\lambda \alpha\left[1-exp\left(\frac{t}{\alpha}\right)^{\beta}\right]} \quad (13)$$

u=1- $e^{\lambda \alpha\left[1-e^{\left(\frac{t}{\alpha}\right)^{\beta}}\right]}$

u~[0,1] uniform distribution

$\ln(1-u)$= { λα[1- $e^{\left(\frac{t}{\alpha}\right)^{\beta}}$]}

$\frac{\ln(1-u)}{\lambda\alpha}$ = [1- $e^{\left(\frac{t}{\alpha}\right)^{\beta}}$]

$$\ln\left(1-\frac{\ln(1-u)}{\lambda\alpha}\right)=\left(\frac{t}{\alpha}\right)^{\beta}$$

$$t=\alpha\left[\ln\left(1-\frac{\ln(1-u)}{\lambda\alpha}\right)\right]^{\frac{1}{\beta}} \quad \lambda,\alpha,\beta>0 \quad (14)$$

*Selecting the samples size (n=10,25,30,50,100)

* Selecting the initial values (α=0.25,0.50,0.75) ,(β=0.5,1,1.5), (λ=1,2,3).

*find the estimator for parameters and probability density function for all samples size and for all initial values

*find the mean squares error which formulated as follows:

MSE[f(t)]=
$$\sum_{i=1}^{n}\frac{\left[\hat{f}(t)-f(t)\right]^{2}}{n-3} \quad (15)$$

Where 3 is the number of parameters

2.4. Numerical Results :

The numerical results for the simulation approach are presented in the following tables after applying the simulation technique to identify the parameter estimate, probability density function, and mean square error procedure.

With a 1000 iteration rate, the MATLAB software employed the Monte Carlo methodology for the simulation and the Newton- Raphson method for the numerical solution.

As shown in table (1,2,3)and figure (3,4,5)

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Table 1: Represent the parameters estimation and probability density function and mean square error when (α_0=0.25)

β_0	λ_0	n	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$f(t)$	$Msef(t)$
0.5	1	10	0.21014	0.890257	0.972289	0.80515	9.72E-05
		25	0.302436	0.56913	1.006355	0.805456	9.21E-06
		30	0.43732	0.466657	1.006917	0.80543	6.86E-07
		50	0.27858	0.482417	1.002647	0.805428	3.30E-07
		100	0.24999	0.516906	1.005792	0.805411	1.71E-07
	2	10	0.264697	0.603484	2.020691	0.658249	4.82E-05
		25	0.413883	0.46622	2.007197	0.6548	1.81E-05
		30	0.250049	0.467965	2.006925	0.651059	7.68E-08
		50	0.249787	0.507962	2.001645	0.650319	7.45E-08
		100	0.250023	0.494711	2.003739	0.650072	3.87E-08
	3	10	0.193946	0.456694	2.987517	0.550327	8.74E-05
		25	0.512662	0.414499	3.001517	0.548987	5.96E-06
		30	0.249564	0.468721	3.002246	0.545494	6.61E-07
		50	0.249921	0.473466	3.008064	0.54081	3.05E-07
		100	0.249918	0.527168	3.001697	0.540132	2.58E-07
1	1	10	0.314991	1.094581	0.978989	0.543679	2.08E-03
		25	0.335137	1.070222	1.001255	0.541074	8.48E-06
		30	0.246443	1.043102	1.004179	0.540542	2.87E-06
		50	0.261761	1.017404	1.005221	0.540302	1.57E-06
		100	0.282558	0.975435	1.004674	0.540122	1.55E-06
	2	10	0.356584	0.890172	2.003477	0.370333	1.10E-04
		25	0.212108	1.229055	1.999053	0.363784	2.17E-05
		30	0.249441	0.893498	2.004085	0.362722	6.89E-07
		50	0.336748	0.875087	2.003157	0.362701	1.96E-07
		100	0.251664	1.030627	2.001555	0.36234	1.03E-07
	3	10	0.208381	0.819974	3.01669	0.27251	1.58E-04
		25	0.27485	1.131224	3.000922	0.265632	4.75E-07
		30	0.245582	1.076981	3.003101	0.265114	2.97E-07
		50	0.291422	1.066996	3.001266	0.264626	3.75E-08
		100	0.252726	0.987988	3.007264	0.264198	2.02E-08
1.5	1	10	0.284381	1.833797	1.003907	0.362619	2.83E-04
		25	0.25956	1.631063	1.006887	0.347838	1.65E-05
		30	0.253429	1.373672	1.008489	0.345147	2.32E-06
		50	0.272348	1.725549	1.004265	0.345022	1.20E-06
		100	0.287426	1.615482	1.008226	0.344791	5.11E-07
	2	10	0.321663	1.057694	2.00668	0.296911	8.11E-04
		25	0.253481	1.404742	2.006107	0.250202	1.71E-05
		30	0.27798	1.508395	2.00219	0.247318	2.26E-07
		50	0.244937	1.744611	2.000741	0.212601	3.65E-09
		100	0.248399	1.609914	2.006686	0.212149	3.38E-09
	3	10	0.266896	1.184853	2.971596	0.16122	4.31E-04
		25	0.248965	1.407372	3.008938	0.149482	5.88E-06
		30	0.252335	1.630866	3.007716	0.147238	4.12E-07
		50	0.246179	1.517223	3.002862	0.147185	5.06E-10
		100	0.26067	1.501135	3.004597	0.14710	2.87E-10

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Table 2: Represent the parameters estimation and probability density function and mean square error when (α_0=0. 50)

β_0	λ_0	n	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$f(t)$	$Msef(t)$
0.5	1	10	0.715263	0.538934	0.975794	0.816553	5.79E-04
		25	0.701651	0.466958	1.00342	0.806209	3.05E-06
		30	0.500027	0.487229	1.003651	0.806064	6.68E-07
		50	0.498739	0.466562	1.004183	0.805898	5.12E-07
		100	0.500022	0.478337	1.005845	0.805058	4.33E-07
	2	10	2.560622	0.454253	1.940861	0.731625	8.04E-04
		25	0.491957	0.552721	2.001695	0.706117	6.80E-06
		30	0.588622	0.512475	2.004527	0.705849	1.36E-07
		50	0.499928	0.458685	2.006423	0.705823	1.26E-07
		100	0.499936	0.564512	2.002926	0.705472	1.13E-07
	3	10	0.981653	0.661628	2.98213	0.622103	1.47E-03
		25	0.779221	0.507223	3.005026	0.601726	1.14E-05
		30	0.825415	0.551053	3.00182	0.601174	5.14E-07
		50	0.499346	0.395083	3.006921	0.601104	3.24E-07
		100	0.514266	0.512837	3.005884	0.601077	3.15E-07
1	1	10	0.580526	0.895585	1.013171	0.661185	4.80E-05
		25	0.498414	0.951719	1.013065	0.657841	1.46E-05
		30	0.532105	1.285063	1.003841	0.656729	6.49E-06
		50	0.482785	1.076673	1.003376	0.65048	4.56E-07
		100	0.564846	0.960795	1.002708	0.6496	3.08E-07
	2	10	0.482917	1.459033	2.01617	0.458687	6.63E-05
		25	0.496023	0.939034	2.006699	0.441827	1.31E-05
		30	0.496738	1.120875	2.005399	0.440367	1.18E-06
		50	0.74426	1.024763	2.004794	0.439662	1.95E-07
		100	0.544068	1.01648	2.002024	0.439316	1.91E-07
	3	10	0.5896	1.058128	2.985863	0.333636	6.23E-04
		25	0.549823	1.262748	3.002299	0.330752	5.85E-06
		30	0.701548	0.817985	3.004576	0.328199	4.25E-08
		50	0.549773	1.016175	3.004783	0.323965	2.55E-08
		100	0.511385	0.976121	3.002289	0.322094	1.88E-08
1.5	1	10	0.50502	1.376611	1.051639	0.492858	6.37E-05
		25	0.642496	1.715571	1.003735	0.49227	1.06E-05
		30	0.595543	1.48079	1.005818	0.491494	6.66E-06
		50	0.513206	1.422829	1.007943	0.491271	6.13E-06
		100	0.531287	1.661557	1.004989	0.490801	1.10E-07
	2	10	0.764805	1.573051	1.999684	0.300769	4.81E-05
		25	0.504018	1.608636	2.006302	0.300685	2.26E-06
		30	0.504586	1.412887	2.006552	0.300072	1.28E-06
		50	0.541493	1.586044	2.00632	0.300014	4.69E-07
		100	0.550973	1.371423	2.005074	0.300002	4.61E-07
	3	10	0.524769	1.471158	3.011567	0.212707	1.41E-04
		25	0.523874	1.594025	3.008412	0.205996	8.09E-07
		30	0.600014	1.659449	3.006759	0.203001	5.39E-07
		50	0.534819	1.421294	3.006253	0.202879	3.03E-07
		100	0.497586	1.53729	3.003607	0.202796	2.05E-07

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Table 3: Represent the parameters estimation and probability density function and mean square error when (α_0=0.75)

β_0	λ_0	n	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$f(t)$	$Msef(t)$
0.5	1	10	1.503775	0.502839	1.001455	0.848986	2.99E-04
		25	1.034225	0.51867	1.002436	0.846732	2.05E-06
		30	1.110642	0.464126	1.006235	0.846088	4.71E-06
		50	1.256454	0.506383	1.005861	0.845618	1.45E-06
		100	0.795082	0.486028	1.008113	0.844242	3.12E-07
	2	10	0.716806	0.599369	1.989067	0.720428	4.72E-06
		25	0.747455	0.480688	2.006269	0.712636	5.95E-06
		30	0.744457	0.46433	2.004725	0.711993	9.57E-08
		50	0.812656	0.467514	2.001285	0.71064	4.64E-08
		100	0.749855	0.549451	2.005892	0.709866	1.88E-08
	3	10	-0.17805	0.417857	2.960825	0.629896	1.88E-03
		25	0.769403	0.552075	3.001388	0.621479	1.71E-06
		30	0.749899	0.397415	3.005513	0.62024	1.19E-07
		50	0.749925	0.409163	3.002375	0.61986	5.45E-08
		100	0.749978	0.504163	3.005971	0.618957	4.07E-08
1	1	10	1.301861	1.131661	0.981712	0.703065	7.26E-04
		25	0.728709	1.111835	1.00099	0.690873	6.66E-06
		30	0.932223	1.018838	1.001595	0.682179	1.91E-06
		50	0.99014	1.046329	1.006876	0.682015	1.01E-06
		100	0.933522	1.028331	1.003721	0.681854	1.53E-07
	2	10	1.006238	0.464229	1.966242	0.501786	2.62E-04
		25	0.831686	0.977024	2.008698	0.499545	5.72E-06
		30	0.986704	1.069788	2.001913	0.490581	6.38E-07
		50	0.749069	0.960137	2.001276	0.489297	1.22E-07
		100	0.795333	1.076904	2.00465	0.487011	1.18E-07
	3	10	0.700737	1.29631	2.999765	0.371478	6.37E-04
		25	0.824205	1.283346	3.002539	0.369731	3.18E-06
		30	0.740114	1.115954	3.001968	0.366395	3.28E-07
		50	0.746511	1.006809	3.00162	0.365946	3.11E-07
		100	0.823297	1.0523	3.005777	0.365923	1.09E-07
1.5	1	10	0.6145	2.32621	0.986322	0.55411	6.48E-05
		25	0.787732	1.736819	1.001067	0.546451	1.97E-05
		30	0.82131	1.408927	1.003526	0.544702	9.81E-07
		50	0.814606	1.530484	1.006271	0.544796	8.08E-07
		100	0.75988	1.523443	1.004278	0.543969	7.36E-07
	2	10	0.664182	2.023005	2.029037	0.344167	2.76E-07
		25	0.700532	2.124468	2.003853	0.343725	2.05E-07
		30	0.943934	1.100214	2.00554	0.343437	1.86E-07
		50	0.859871	1.39401	2.008308	0.343233	1.41E-07
		100	0.728101	1.498679	2.001216	0.343018	1.24E-07
	3	10	0.888148	1.755071	2.98195	0.248191	2.41E-04
		25	0.864183	1.743617	3.004929	0.235780	1.20E-07
		30	1.007961	1.388865	3.004953	0.235695	1.09E-07
		50	0.822423	1.50081	3.004964	0.235596	1.03E-07
		100	0.75819	1.519841	3.002405	0.235383	2.38E-08

2.5. Particle Real data :

In this section, the real data that was taken from the control unit of samples in (Diyala General Company for Electrical Industries) is applied, one of the formations of the Ministry of Industry and Minerals, and Number of units participating in the experiment is 186, and the units that have failure times are 40.

2.6. Chi-Square Test:

In this section using the good of fit test to know if the data distributed as modified extension Weibull distribution or not . then using the statistical program (SPSS) to…etc, it was discovered that the calculated value of X^2equals to 15.5887172643, while comparing it to the tabulated value of$X^2{}_{(K,1-\alpha)}$under level of significant of 0.05 and 10 degrees of freedom, it equal 18.31 calculating the value of X^2 is less than X^2 is tabulated value that means accepting the null hypothesis H_0 and the data is distribution according to modified extension Weibull distribution .

2.7 Apply Real data:

In this section finding the probability density function $\hat{f}(t) for\ real\ data$, Reliability function $\hat{R}$(t),and hazard function $\hat{h}$(t) for real data which get it from (Diyala province Company) the study was conducted in the field, where failure times for each unit were noted it is one of the formations of the Ministry of Industry and Minerals ,as shown in table 4, as follows:

When α_0 =0.25,β_0=0.5, λ_0 =1

And $\hat{\alpha}$=0.0461,$\hat{\beta}$=0.0425,$\hat{\lambda}$=0.3686 .

Table 4: Estimate values for function $\hat{f}$(t), $\hat{R}$(t) and $\hat{h}$(t)

t_i	$\hat{f}(t)$	$\hat{R}$(t)	$\hat{h}(t)$
0.35	0.006468340	0.967010049	0.006689010
0.48	0.004847536	0.966285107	0.005016673
1.07	0.002333093	0.964352874	0.002419336
1.11	0.002256337	0.964261103	0.002339964
1.24	0.002039715	0.963982357	0.002115926
1.30	0.001953755	0.963862592	0.002027006
1.55	0.001664489	0.963412337	0.001727702
2.06	0.001284676	0.962669060	0.001334494
2.34	0.001143975	0.962329849	0.001188755
3.10	0.000885704	0.961567345	0.000921104
3.18	0.000865415	0.961497307	0.000900070
3.25	0.000848446	0.961437326	0.000882477
3.29	0.000839060	0.961403577	0.000872745
3.34	0.000827629	0.961361911	0.000860892
3.45	0.000803598	0.961272207	0.000835973
3.47	0.000799385	0.961256177	0.000831604
3.52	0.000789053	0.961216467	0.000820890
3.58	0.000777019	0.961169487	0.000808410
4.08	0.000689944	0.960803652	0.000718091
4.13	0.000682347	0.960769346	0.000710209
4.14	0.000680849	0.960762530	0.000708655
4.22	0.000669106	0.960708534	0.000696472
4.26	0.000663393	0.960681885	0.000690544
4.30	0.000657782	0.960655462	0.000684722
4.33	0.000653639	0.960635790	0.000680423
4.41	0.000642853	0.960583933	0.000669231

4.52	0.000628618	0.960514009	0.000654460
4.58	0.000621130	0.960476517	0.000646689
5.11	0.000564309	0.960174717	0.000587715
5.15	0.000558332	0.960141039	0.000581511
5.18	0.000555394	0.960124333	0.000578460
5.31	0.000543026	0.960052942	0.000565621
5.35	0.000539336	0.960031295	0.000561790
5.47	0.000528577	0.959967225	0.000550620
5.58	0.000519102	0.959909606	0.000540782
6.24	0.000468980	0.959584127	0.000488733
7.15	0.000414447	0.959183242	0.000432083
8.15	0.000368009	0.958792981	0.000383826
8.36	0.000359609	0.958716588	0.000375094
9.11	0.000332635	0.958457273	0.000347053

3.Discussion of Results :

1.From table (1,2,3) we noting :

*Noting that the probability density function f(t) are decreasing when the samples sizes (n) are increasing for all initial value of parameters.

* Noting that the probability density function are decreasing when the initial values α_0, β_0 , $\lambda_0$are increasing .

*Noting that the Mean square errors are decreasing when the samples sizes (n) are increasing for all initial value of parameters.

* Noting that the Mean squares error are decreasing when the initial values α_0, β_0 , λ_0 are increasing.

2. From table (4) Probability density function $\hat{f}(t)$estimate values are falling .whenever the failure rates t_i rise.

* When the failure time t_i increases, the estimate values of the reliability function $\hat{R}(t)$ drop.

* When the failure time t_i increases, the estimate values of the Hazard function $\hat{h}(t)$ rate function decrease.

4. Conclusion:

From tables (1,2,3), it can be shown that when mean squares error techniques are used, the estimated values of the probability density function f(t) become extremely tiny in comparison to the genuine values of the empirical probability density function f(t) and when applied to real data, it is discovered that the estimate values of the probability density function are falling when the probability density function, Reliability function, and Hazard function are found for real data .The estimate values of the reliability function fall as the failure time increases, and the estimate values of the Hazard function rate function also decrease as the failure time increases as is clear in table (4).

Authors Declaration:

Conflicts of Interest: None

-We Hereby Confirm That All The Figures and Tables In The Manuscript Are Mine and Ours. Besides, The Figures and Images, Which are Not Mine, Have Been Permitted Republication and Attached to The Manuscript.

- Ethical Clearance: The Research Was Approved By The Local Ethical Committee in The University.

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تقدير دالة المعولية لتوزيع ويبل المعدل باستخدام مراقبة العينات من النوع الاول مع المحاكاة

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مستخلص البحث:
في هذا البحث يتم تعريف توزيع ويبل المعدل ودالة المعولية ودالة الخطر، نشرح طريقة الاحتمالية القصوى ضمن عينة الرقابة من النوع الأول. تلعب العينات الخاضعة للرقابة دورًا مهمًا في أوقات الحياة. في هذه البحث تقدير واشتقاق المعلمات الثلاثة لتوزيع امتداد ويبل المعدل باستخدام العينات الخاضعة للرقابة من النوع الأول بطريقة تقدير الاحتمالية القصوى يعتمد على طريقة نيوتن- رافسون. بعد ذلك تم تطبيق إجراء المحاكاة باستخدام تقنية مونت كارلو لإيجاد دالة المعولية تحت أحجام عينات مختلفة وقيم أولية مختلفة لمعلمات جميع المعلمات المقدرة لتوزيع امتداد ويبل المعدل من خلال تطبيق القيم الأولية في برنامج MATLAB ثم مقارنة دالة الموثوقية المقدرة مع دالة الثبات التجريبية باستخدام طريقة مربعات الخطأ المتوسطة. وأخيرًا ايجاد دالة كثافة الاحتمال f(t)، ودالة المعولية R(t)، ودالة الخطر h(t) للبيانات الحقيقية التي تم الحصول عليها من (شركة ديالى العامة) هي إحدى تشكيلات وزارة الصناعة والمعادن.

نوع البحث: ورقة بحثية.
المصطلحات الرئيسة للبحث: الامتداد المعدل لتوزيع ويبل، العينة الخاضعة للرقابة، دالة الموثوقية، المحاكاة.