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Solving Multi-Objective Machine Scheduling Problem Using the Meerkat Clan Algorithm

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Abstract:
Machine scheduling problems have become increasingly complex and dynamic. The complexity and size of the problems require the development of methods and solutions whose efficiency is measured by their ability to find acceptable results within a reasonable amount of time. Therefore, this paper addresses to propose a new mathematical model for multi objective function based on Single-machine scheduling problems by minimizing the discounted total weighted completion time $\sum_{j=1}^{n} w_j\left(1-e^{-rC_j}\right)$, the number of tardy jobs $\sum_{j=1}^{n} U_j$, the maximum earliness E_{max} and the maximum weighted tardiness $T_{\max}^{w}$ with release date r_j denoted $\left(1 / r_j / \sum_{j=1}^{n} w_j\left(1-e^{-rC_j}\right)+\sum_{j=1}^{n} U_j+E_{max}+T_{\max}^{w}\right)$ which are an NP-hard. To achieve efficient solutions, a metaheuristic method (Meerkat clan algorithm (MCA)) is used to solve the mathematical model and compare it with branch and bound (BAB) method. Computational results show that MCA provides efficient solutions in terms of accuracy and calculation speed compared to BAB. In addition, the BAB can solve up to 10 problems, while MCA can resolve problems up to 1000 for multi objective.
Keywords: Scheduling; Single machine; Multi-Objective; Branch and Bounded; Meerkat Clan

1. Introduction:

Scheduling is one of the well-known problems in operations research and production management, involving allocating resources over time to perform a collection of tasks. The complexity of scheduling problems stems from the combinatorial nature of arranging tasks, especially as the number of tasks and constraints increases. The scheduling theory addresses the machine scheduling problem (MSP), which applies to various sectors, including production facilities. The solution to the MSP, known as the schedule, is the best optimal method to minimize the multi-objective function (Hoogeveen, 1996).These objective functions produce several optimum solutions instead of just one (Fasihi et al., 2023)& (Atiya et al., 2016) .There are multiple optimal solutions for every objective function, making it impossible to consider one as better than others. The optimum solutions here are called non-dominated solutions (Shao et al., 2021).

Multi-objective single-machine scheduling problems are crucial because they reflect real-world scenarios where decision-makers must balance various performance metrics to achieve optimal outcomes (Neufeld et al., 2023). In addition, they extend the classical problems by incorporating multiple, often conflicting, objectives. This helps improve operational efficiency and meets diverse stakeholder requirements, making it a vital area of study (Chachan & Hameed, 2019) & (Ezugwu, 2024) . Most studies on scheduling theory assume that order processing takes the same amount of time over the entire planning horizon. While traditional optimization methods can be powerful tools for solving well-defined scheduling problems, their application to multi-objective single machine scheduling problems often faces significant challenges due to the complexity of handling multiple objectives, computational and scalability issues, and the need for robust and diverse solutions. These challenges usually necessitate specialized multi-objective optimization techniques, metaheuristic algorithms, or hybrid approaches to effectively address the complexities of multi-objective scheduling problems (Yin et al., 2024).

The Meerkat Clan Algorithm (MCA) employs several specific strategies to be effective. These strategies are designed to find near-optimal solutions by leveraging the social behavior of meerkats and balancing exploration and exploitation (Srinivasan et al., 2021). Hence, the Meerkat Clan Algorithm was used to solve the new multi-objective mathematical model for single machine scheduling problem. Therefore, this paper made several contributions that can be summarized as follows:

- Proposed multi-objective model for minimizing the total weighted discounted completion time $\sum_{h=1}^{m} w_h(1-e^{-\alpha C_h})$, the number of tardy jobs $\sum_{h=1}^{m} U_h$, maximum earliness $\mathrm{E}_{\max}$ and the maximum weighted tardiness T_{max}^{w} for a single machine scheduling problem.
- For the first time, the Meerkat Clan Algorithm was introduced to solve the multi-objective model for the single-machine scheduling problem.

The remaining sections of this paper are organized as follows: Section 2 provides the related works of the proposed study. Section 3 presents the methodology of our studuy that containt the mathematical formulation of the problem, metaheuristic algorithms (Meerkat Clan Algorithm), Branch and Bound method. Computational study and results are presented in Section 4. Section 5 presents the potential challenges and limitations. Finally, conclusions and suggestions for future work are given in Section 6.

2.Literature Review:

Most previous studies on scheduling have focused on a single method of measuring performance (T'kindt et al., 2024). Several researchers have extensively studied and (Mahnam et al., 2013) used the BAB method to solve the problem $1/r_j/E_{max} + T_{max}$. (Abdul-Razaq & Akram, 2018) presented a multi objective function $1//\sum(C_j + T_j) + T_{max} + E_{max}$ and used branch and bound to minimize this problem. (Abbass, 2019) used the BAB method to solve the problem $1//\sum_{j=1}^{n}(U_j + C_j + T_j + T_{max})$ for $n \leq 20$. (Chachan & Jaafar, 2020) considered the problem $1/r_j/\sum_{j=1}^{n}(C_j + T_j + E_j + U_j + V_j)$ used (BAB) method up to 16 jobs. (Amin & Ramadan, 2021) showed the problem $1 // E_{max} + T_{max}$ to find near optimal solution. (Yousif & Ali, 2024) studied the problem $1 // E_{max} + R_L$ solved this problem by the branch and bound method. (Neamah & Kalaf, 2024b) used the exact method to solve the problem $1 // \sum C_j, \sum V_j, E_{max}$. Several researchers have extensively studied and documented (Atiya et al., 2016)& (Khraibet & Ghafil, 2021) &(Abbas & Ghayyib, 2024) & (Kalaf et al., 2024) &(Fakhruddin Saleh et al., 2024) & (Ragheb Abdulrazak Adlia* & Hatim Mahmoud, 2024) & (Atiya Wardil * & Khaleel Ibrahim, 2024) & (Shaker Salman*, 2024)

SMSP problems have become increasingly sophisticated and NP-hard during the last few decades. Therefore, metaheuristic algorithms are proposed to obtain optimal or near-optimal solutions for the problem under consideration. (Zlobinsky & Cheng, 2018) used Simulated Annealing to solve the problem of minimizing the weighted earliness and tardiness. (Ali & Ahmed, 2020) used Bee Algorithm and Particle Swarm Optimization to solve the problem $\left(1 // \sum C_j + R_L + T_{max}\right)$. (Moharam et al., 2022) introduced chimp optimization algorithm to minimize the Tardy/Lost (TL) penalties. (Costa & Fernandez-Viagas, 2022)studied a single-machine scheduling problem, where the objective is minimum total tardiness used the Harmony search. (Zhang et al., 2019) used the Tabu search to find a near optimum solution for minimizing the make span. Moreover, some researchers are interested in metaheuristic algorithms (Srinivasan et al., 2021)& (Mohammadi & Moaddabi, 2021).

3.Methodology:

3.1 Mathematical Model:

The scheduling problem is a set of n jobs on a single machine. Even job $j, j \in N$, where $N = \{1,2,\ldots,n\}$ has integer a processing time p_j and a positive weighted w_h on the machine and ideally would be completed at its due date d_j and release date r_j. Discounted total weighted completion time $\sum_{j=1}^{n} w_j\left(1 - e^{-rC_j}\right)$ where C_j be a completion time for job j, given by the relationship: $C_j = \sum_{j=1}^{n} p_j$ and $C_1 = r_1 + p_1$, $C_j = max\{r_j, C_{j-1}\} + p_j$ for $j = 2,3,\ldots,n$. The number of tardy jobs $\sum_{j=1}^{n} U_j$ is completed after it's due date ($C_j > d_j$ $then$ $U_j = 1$), ($C_j \leq d_j$ $then$ $U_j = 0$). The lateness time of the job j is defined by $L_j = C_j - d_j$. The maximum earliness E_{max} of job j is defined by $E_{max} = max\{0, -L_j\} = max\{d_j - C_j, 0\}$, tardiness for job j, $T_j = max\{0, L_j\}$, slack time for job j is defined by $S_j = d_j - p_j$. The maximum weighted tardine $T_{max}^{w} = max\{w_jT_j\} = max\{w_j(c_j - d_j), 0\}$.

Then, the problem can be represented mathematically as:

$$T = min_{\sigma \in S}\{M_{(\sigma)}\} = min_{\sigma \in S}\left\{\sum_{j=1}^{n}(w_j(1 - e^{-\alpha c_j}) + U_j) + E_{max} + T_{max}^{w}\right\}$$

$s.t$

$$C_{\sigma(j)} \geq r_{\sigma(j)} + p_{\sigma(j)} \qquad j = 1,2, \dots, n$$

$$C_{\sigma(j)} \geq C_{\sigma(j-1)} + p_{\sigma(j)} \qquad j = 2,3 \dots, n$$

$$O < \alpha < 1 \qquad j = 1,2, \dots, n$$

$$U_{\sigma(q)} = \begin{cases} 1 & if\ C_{\sigma(j)} > d_{\sigma(j)} \\ 0 & o.w \end{cases} \qquad j = 1,2, \dots, n$$

$$E_{\sigma(j)} \geq d_{\sigma(j)} - C_{\sigma(j)} \qquad j = 1,2, \dots, n$$

$$C_{\sigma(j)} > p_{\sigma(j)} \qquad j = 1,2, \dots, n$$

$$T_{\sigma(j)} \geq C_{\sigma(j)} \qquad j = 1,2, \dots, n$$

$$T_{\sigma(j)} \geq 0 \qquad j = 1,2, \dots, n$$

$$r_{\sigma(j)} \geq 0, W_{\sigma(j)} \geq 1, d_{\sigma(j)} > 0,\ E_{\sigma(j)} \geq 0, p_{\sigma(j)} > 0 \quad j = 1,2, \dots, n$$

3.2 Meerkat Clan Algorithm:

The Meerkat Clan Algorithm is an inspirational algorithm derived from the behavior of meerkats searching for food in the desert, as proposed in (Sadiq Al-Obaidi et al., 2018). This algorithm utilizes effective methods to address optimization problems and achieve optimal solutions. Meerkats are social animals that live in groups ranging from five to thirty individuals, with each group occupying a specific territory.As friendly creatures, meerkats exhibit remarkable cooperation in performing various tasks such as guarding and parental supervision. Each group is characterized by the presence of a dominant alpha female and a leading alpha male, who exert significant influence within the group. Groups relocate to different areas when food becomes scarce or when another, more dominant group replaces them.This algorithm is based on three key components inspired by the social behavior of meerkats. The first component involves assigning individuals as guards or lookouts while others hunt or play, with the guards alerting the group in case of danger (Srinivasan et al., 2021). Additionally, meerkats employ diverse strategies for foraging, taking different paths daily and avoiding revisiting the same area for at least a week to allow food supplies to replenish. Moreover, meerkats must balance between caring for the group and hunting to achieve the best outcomes (Saleh & Sadiq, 2024). In multi-objective single-machine scheduling problems, the objectives of minimizing total weighted discounted completion time, minimizing the number of tardy jobs, and minimizing maximum weighted tardiness often interact in complex ways. The MCA effectively balances these objectives. MCA operates within a multi-objective optimization framework aiming to approximate the Pareto front. The Pareto front represents a set of non-dominated solutions where no objective can be improved without worsening another. MCA searches for diverse solutions along this front, allowing decision-makers to choose the best trade-offs based on their preferences. In addition, MCA divides the solution space into different clans, each focusing on various regions and aspects of the problem. This diversity allows the algorithm to explore different trade-offs through the objective functions. After that, each clan is guided by a leader who helps refine the solutions within that clan. This process helps balance the trade-offs between objectives by focusing on different aspects of the solution space. MCA evaluates solutions based on a combination of objectives.

The fitness of a solution is determined by how well it balances the trade-offs between the objective functions. Then, MCA employs mechanisms to maintain diversity among the solutions in the population. This prevents the algorithm from converging too quickly on a suboptimal region of the solution space and ensures that various trade-offs are explored. This helps in finding solutions that offer a good balance between the different objectives.

The following steps of this study show how the MCA solve the single-machine scheduling problems

1. Initialize the population of meerkats, each representing a possible solution (i.e., a sequence of jobs), define parameters such n: Clan count falls between thirty to fifty; m : Size for foraging when $m < n$, c: Care size is equal to $n - m - 1$, Fr: The worst foraging rate, Cr: The lowest possible care rate, k: Finding a neighbor solution.
2. Calculate the fitness function ($min_{\sigma \in S}\{\sum_{j=1}^{n}\left(w_h(1-e^{-\alpha c_j})+U_j\right)+E_{max}+T_{max}^{w}\}$)
3. Repeat until a stopping criterion

- **Exploration**: Explore new solutions. This involves generating new sequences of jobs.
- **Exploitation**: Exploit the best-known solutions by making small adjustments to improve the current best solutions.
- **Information Sharing**: This can involve updating a shared memory or a set of best-known solutions.
- **Selection**: Select the next generation based on fitness, favoring solutions with better fitness values.
- **Update**: Update the population for the next iteration, including any new solutions generated during exploration and exploitation.

4. When the stopping condition is met, output the best solution found, which represents the optimal or near-optimal schedule for the single machine.

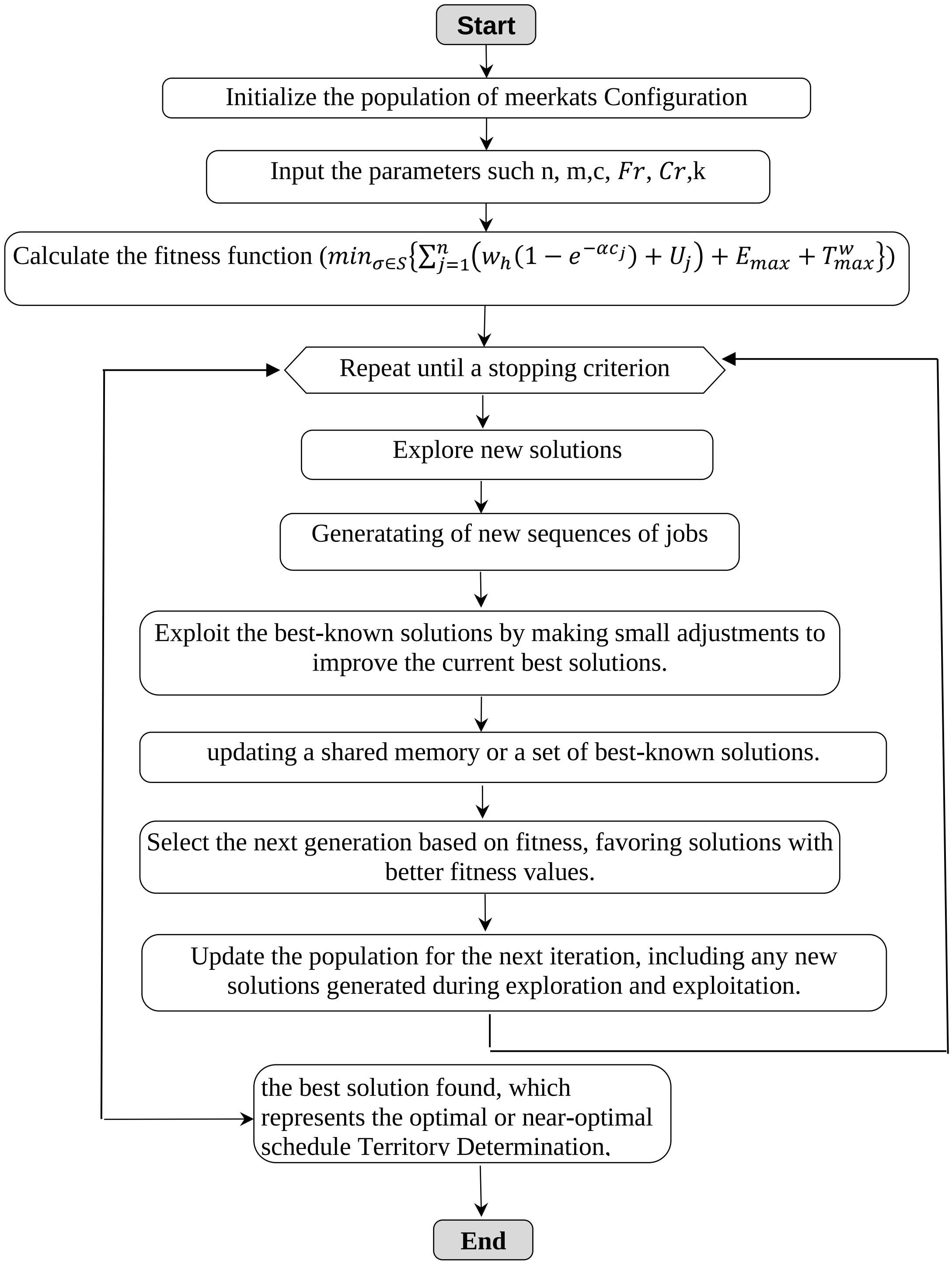

Figure 1 : Flowchart of the MCA

3.3 Branch And Bounded:

The Branch and Bound (BAB) method developed with the forward sequence branching rule. If the jobs are strung together at the first k places in the search tree, the nodes at level k are representative of the initial partial order. The derived lower bound (LB) determines the cost of the unscheduled orders, and the objective function determines the cost of scheduling the orders at a particular node. The BAB technique is dominant if the node has $LB \geq UB$ at each level. The backtracking method is then used to repeat the process until all nodes have been considered. Backtracking is the step in the BAB method that leads from the lowest to the highest level. Some researchers have worked out a method BAB (Abdul-Razaq & Akram, 2018)& (Forget et al., 2022) & (Neamah & Kalaf, 2024a)

The branch and bound method's efficiency largely hinges on the effectiveness of the bounding strategies. Calculating upper and lower bounds accurately prunes solution space, leading to faster and more efficient optimal solutions in various combinatorial and optimization problems.

3.3.1 Upper Bound (UB)

This subsection introduces the three upper bounds, the best one will be chosen as follows:

$$UB = min\{UB_1, UB_2, UB_3\}$$

1- UB_1: Where the n jobs are ordered in (WDSPT) rule, that is sequencing the jobs in non-decreasing order of

$$\frac{w_1 e^{-\alpha p_1}}{1-e^{-\alpha p_1}} \leq \frac{w_2 e^{-\alpha p_2}}{1-e^{-\alpha p_2}} \leq \cdots \leq \frac{w_m e^{-\alpha p_n}}{1-e^{-\alpha p_n}}$$

2- UB_2: Where the n jobs are ordered in EDD rule, that is sequencing the jobs in increasing order of due dates $d_1 \leq d_2 \leq \cdots \leq d_n$ and then the cost is calculated.

3- UB_3: Where the n jobs are ordered in SPT rule, that is sequencing the jobs in increasing order of a processing time $p_1 \leq p_2 \leq \cdots \leq p_n$ *a*nd then the cost is calculated.

3.3.2 Lower Bound (LB)

The lower bound is one of the most important constraints in determining a satisfactory solution to a problem. Obtaining lower bounds for an NP-hard multi-objective problem is clearly difficult. In order to determine a lower bound for the problem, it is divided into three lower bounds as illustrated in:

$$\left.\begin{array}{ll}
LB_1 = min_{\sigma(j)} = \left\{\sum_{j=1}^{n} w_j(1-e^{-\alpha c_{\sigma(j)}})\right\} & \\
Subject\ to & \\
C_{\sigma(j)} \geq r_{\sigma(j)} + p_{\sigma(j)} & j = 1,2,\dots,n \\
C_{\sigma(j)} \geq C_{\sigma(j-1)} + p_{\sigma(j)} & j = 2,3\dots,n \\
0 < \alpha < 1 & j = 1,2,\dots,n \\
r_{\sigma(j)} \geq 0, w_{\sigma(j)} \geq 1, p_{\sigma(j)} > 0, d_{\sigma(j)} > 0 & j = 1,2,\dots,n
\end{array}\right\}$$

$$\left.\begin{array}{ll}
LB_2 = min_{\sigma(j)} = \left\{\sum_{j=1}^{n} U_{\sigma(j)}\right\} & \\
Subject\ to & \\
C_{\sigma(j)} \geq r_{\sigma(j)} + p_{\sigma(j)} & j = 1,2,\dots,n \\
C_{\sigma(j)} \geq C_{\sigma(j-1)} + p_{\sigma(j)} & j = 2,3,..,n \\
U_{\sigma(j)} \in \{0,1\} & j = 1,2,\dots,n \\
r_{\sigma(j)} \geq 0, p_{\sigma(j)} > 0, d_{\sigma(j)} > 0 & j = 1,2,\dots,n
\end{array}\right\}$$

$$\left.\begin{array}{ll} LB_3 = min_{\sigma\in S}\left\{max\{E_{\sigma(j)}\}\right\} & \\ Subject\ to & \\ C_{\sigma(j)} \geq r_{\sigma(j)} + p_{\sigma(j)} & j = 1,2,\dots,n \\ C_{\sigma(j)} \geq C_{\sigma(j-1)} + p_{\sigma(j)} & j = 2,3,\dots,n \\ E_{\sigma(j)} \geq d_{\sigma(j)} - C_{\sigma(j)} & j = 1,2,\dots,n \\ r_{\sigma(j)} \geq 0, E_{\sigma(j)} \geq 0, p_{\sigma(j)} > 0, d_{\sigma(j)} > 0 & j = 1,2,\dots,n \end{array}\right\}$$

The following steps of Algorithm:

Step1. Enter: $n, r_k p_k, d_k$ & w_k where h from 1 to k.

Step2. Order the jobs by using (WDSPT)- rule.

Step3. Calculate the value for each job h that schedules the jobs in non-decreasing order of ratio: $\frac{w_h e^{-\alpha p_h}}{1-e^{-\alpha p_h}}$ and compute $\sum_{h=1}^{m} w_h(1-e^{-\alpha c_h})(WDSPT) = \sum_{h=1}^{m} U_h(WDSPT) = E_{max}(WDSPT) = T_{max}^{w}(WDSPT)$ the WDSPT gives optimal solution.

Step4. Obtaining that $LB_1 = WDSPT$.

Step5. Order the jobs in (EDD)-rule.

Step6. Calculate the value for each job that schedules the jobs in non-decreasing order of of due dates d_h and compute $(\sum_{h=1}^{m} w_h(1-e^{-\alpha c_h})(MA)) = \sum_{h=1}^{m} U_h(MA) = E_{max}(MA) = T_{max}^{w} = (MA)$ the MA gives optimal solution.

Step7. Obtaining that $LB_2 = MA$.

Step8. Calculate the value for each job h that schedules the jobs in non-decreasing order of of due dates S_h and compute $(\sum_{h=1}^{m} w_h(1-e^{-\alpha c_h})(MST)) = \sum_{h=1}^{m} U_h(MST) = E_{max}(MST) = T_{max}^{w} =$ (MST) the MST gives optimal solution.

Step9. Obtaining that $LB_3 = MST$.

Step10. $LB = LB_1 + LB_2 + LB_3$.

Obtaining

4. Results And Discussion

Simulation used to verify and evaluate the performance of the Meerkat Clan Algorithm for solving the multi-objective model based on a single-machine scheduling problem. Various problems with medium to large sizes of 3 to 1000 are studied. The results also compare the performance of the MCA and BAB methods to evaluate their efficiency in solving these problems. The processing time is uniformly distributed across in $U[1, 10]$ and weights were generated from the set $\{1,2,\dots,10\}$. It is now a standard method for creating single machine scheduling problems with due dates. The due dates are uniformly distributed within the range $[P(1-TF-RDD/2), P(1-TF+RDD/2)]$; where $P = \sum_{j=1}^{n} p_j$, which is influenced by the relative range of due date (RDD) and the average tardiness factor (TF). The TF value is extracted from the set of values 0.1, 0.2, 0.3, 0.4, and 0.5, while the RDD value is obtained from the set of values 0.8, 1.0, 1.2, 1.4, 1.6, and 1.8. According to an analysis of the algorithms' performances, the MATLAB programming language encoded and resolved these examples comparably. As the stopping criterion, each algorithm, including MCA and BAB was executed for 1,000 iterations. The Table below showed the results of the BAB method for the problem with different values of n ($n = 3\ to\ 10$), the optimal value, the upper bound, the initial lower bound, the computing time in seconds (Time) that the BAB is stopped after a fixed period of time, here after 1800 seconds (i.e., after 30 minutes). While, using metaheuristic was the Meerkat Clan Algorithm delivered and solved up to 1000 jobs.

Table 1: comparison results BAB and MCA with (n = 3 to 1000).

n	BAB				MCA	
	Av. of UB	**Av. of LB**	**Av. of BAB**	**Av. of time**	**Av. of MCA**	**Av. of time**
3	27.40408516	26.56758881	27.40408516	0.0009225	27.40408516	0.2248035
4	23.74089241	17.13867378	23.74089241	5.2035062	23.74089241	0.095335
5	28.26225281	27.00437546	28.10505104	8.3713162	23.74089241	0.08825
6	51.09547424	38.74572754	50.26849365	121.7023699	28.26225281	0.085324
7	113.8022232	101.7890549	110.1090851	156.5885616	51.09547424	0.088017
8	145.7970428	79.47692108	135.8821716	180.6513175	84.92289734	0.0997496
9	173.5439148	90.24220276	151.1593628	750.8574263	97.62181091	0.1113103
10	1380.373772	760.2249756	1155.07428	1800	146.2519836	0.1216842
20	-------	-------	-------	-------	152.1480408	0.1240352
40	-------	-------	-------	-------	285.2049866	0.3257853
80	-------	-------	-------	-------	420.8836365	0.3950244
100	-------	-------	-------	-------	1005.529846	0.6918425
200	-------	-------	-------	-------	1651.296631	1.0878116
300	-------	-------	-------	-------	1814.060791	2.2276778
400	-------	-------	-------	-------	1995.63853	3.0160037
500	-------	-------	-------	-------	2344.873291	3.0898451
600	-------	-------	-------	-------	2948.749756	5.5196646
700	-------	-------	-------	-------	3758.495605	7.1349164
800	-------	-------	-------	-------	4510.522461	8.0532439
900	-------	-------	-------	-------	5489.3012	9.1349164
1000	-------	-------	-------	-------	9518.09375	10.6533901

5. Potential Challenges and Limitations

Although the Meerkat Clan Algorithm has proven effective in solving multi-objective scheduling problems, it faces some potential challenges. Its efficiency depends heavily on parameter tuning, which may require multiple trials to select the optimal values significantly as the size of the problems increases. Also, the performance of MCA may suffer in some cases when dealing with problems with strict constraints or complex variables, whereas other algorithms, such as branch and bound, may be more accurate. In addition, MCA does not always guarantee reaching the optimal solution, but it does provide a solution close to the ideal with high computational efficiency. The performance of MCA could be improved in the future by combining it with machine learning techniques or developing automatic parameter tuning strategies.

6. Conclusion And Future

This study used a new multi-objective model for a single machine scheduling problem, and Meerkat Clan Algorithm as a novel metaheuristic approach for solving the model. Additionally, it enhanced the Branch and Bound method. The results demonstrate that this integration significantly improves the performance of the BAB method, achieving outcomes in up to 10. The Meerkat Clan Algorithm regarding accuracy and computational efficiency for multi-objective single-machine scheduling problems. MCA excels with its diverse solution representation, adaptive search strategies, and ability to handle multi-objective optimization effectively and solve up to 1000 jobs. In addition, some extensions of the problems can be studied in the future, such as

1- $1/r_j/\sum_{h=1}^{m}(w_h(1-e^{-\alpha C_h}), V_{max}), L_{max}^{w}$

2- $1//Lex(\sum_{h=1}^{m} w_h C_j, T_{max}, \sum_{h=1}^{m} V_h)$

Authors Declaration:

Conflicts of Interest: None

-We Hereby Confirm That All The Figures and Tables In The Manuscript Are Mine and Ours. Besides, The Figures and Images, which are Not Mine, Have Been Permitted Republication and Attached to The Manuscript.

- Ethical Clearance: The Research Was Approved by The Local Ethical Committee in The University.

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