New Approach for Solving Symmetric Fuzzy Linear Programming Problem

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Abstract

Several authors have used ranking function for solving linear programming problem. In This paper is proposed two ranking function for solving fuzzy linear programming and compare these two approach with trapezoidal fuzzy number .The proposed approach is very easy to understand and it can applicable, also the data were chosen from general company distribution of dairy (Canon company) was proposed test approach and compare; This paper prove that the second proposed approach is better to give the results and satisfy the minimal cost using Q.M. Software

Keyword: Fuzzy Linear Programming, Proposed ranking function, Trapezoidal fuzzy number, Crisp Linear Programming problem .

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1-Introduction

The decision maker has to deal with uncertainly. Representing this parameter a fuzzy data and dealing with them using the concept of fuzzy theory seems to be the most suitable way to face the problem of uncertainly the ranges in degree between 0 and 1 .The formulation fuzzy linear programming problem (FLPP) by Zimmerman. Many researchers proposed various types of (FLPP) and solve these problem using different method, but as a way of processing data by allowing partied set membership rather from crisp set membership. Our proposed methods for solving fuzzy linear programming by the use of two ranking function also solver the fuzzy linear programming by reducing in to Crisp Linear Programming problem (CLP).

2- General Introduction

2-1 basic definition

(i) A fuzzy set Ã on R is called trapezoidal fuzzy number if its membership function is defined as follow

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-(a-\alpha)}{\alpha} & a-\alpha\le x<a\\ 1 & a\le x\le b \\ \dfrac{(b+\alpha)-x}{\alpha} & b<x\le b+\alpha\end{cases}\quad\ldots\ldots\ldots(1)$$

We denote a trapezoidal fuzzy number Ã by Ã={a,b,α,α}

(ii) A fuzzy set Ã={a,b,α,β} is called trapezoidal fuzzy number if its membership function is defined as follow

$$\mu_{\tilde{A}}(x)=\begin{cases}1-\dfrac{(a-x)}{\alpha} & a-\alpha\le x\le a\\ 1 & a<x<b \\ 1-\dfrac{(x-b)}{\beta} & b\le x\le b+\beta\end{cases}\quad\ldots\ldots\ldots(2)$$

2-2 Arithmetic operation[(1)]

The arithmetic operation on two trapezoidal fuzzy number $\tilde{A}_1$= {a,b,α,α} & $\tilde{A}_2$={c,d,β,β} then

(i) Addition: $\tilde{A}_1 \oplus \tilde{A}_2$= (a+c,b+d,α+β,α+β)

(ii) Subtraction: $\tilde{A}_1 \ominus \tilde{A}_2$= (a-c,b-d,α-β,α-β)

(iii) Multiplication : $\tilde{A}_1\otimes\tilde{A}_2=\left[\left(\frac{a+b}{2}\right)\left(\frac{c+d}{2}\right)-t,\left(\frac{a+b}{2}\right)\left(\frac{c+d}{2}\right)+t,|b\beta-d\alpha|,|b\beta+d\alpha|\right]$

And $t=\dfrac{t_2-t_1}{2}$ where

$$t_1=Min[ac,ad,cb,bd]$$
$$t_2=Max[ac,ad,cb,bd]$$

(iv) Scalar multiplication

$$\lambda\tilde{A} = \begin{cases} (\lambda\lambda a\lambda b, \lambda\alpha, \lambda\alpha) & \lambda \geq 0 \\ (\lambda\lambda a\lambda b, -\lambda\alpha, -\lambda\alpha) & \lambda < 0 \end{cases}$$

2-3 Traditional Ranking Function

Let Ã={a,b,α,β}be a fuzzy numbers ,then the ranking function is[1]:

$$R(\tilde{A}) = \frac{1}{4}[a + b + \alpha + \beta] \text{........}(3)$$

2-4 Proposed Ranking Function

(i) The researchers are proposed a new technique for ordering fuzzy sets in which ranking function R(Ã) is calculated for fuzzy number Ã={a,b,α,α} from λ-cut $\tilde{A}_\lambda$ = [b-(b- a)λ] According to the following formula[4]

$$R(\tilde{A}) = \int_0^1 [b - (b-a)\lambda] d\lambda = \left[b - \frac{(b-a)}{2}\right] = \frac{a+b}{2} \text{........}(4)$$

(ii) Another proposed technique ranking function R(Ã) of Ã={a,b,α,β} from λ-cut $\tilde{A}_\lambda$ = [b-(b- a)λ,(α+β) λ+ α]According to the following formula

$$R(\tilde{A}) = \frac{1}{2}\left[\int_0^1 [b - (b-a)\lambda] d\lambda + \int_0^1 [\alpha + (\alpha+\beta)\lambda] d\lambda\right] = \frac{1}{2}\left[b - \frac{(b-a)}{2}\right] + \frac{1}{2}\left[\alpha - \frac{(\alpha+\beta)}{2}\right]$$

$$R(\tilde{A}) = \frac{1}{4}[a + b + \alpha - \beta] \text{........}(5)$$

3- Formulation of fuzzy linear programming problem

A fuzzy linear programming problem with constraint & variable can be written as

$$\text{Max or Min } \tilde{Z} = \sum_{j=1}^{j=n} \tilde{C}_j \otimes \tilde{X}_j$$

$$\text{S.T.} \quad \sum_{j=1}^{j=n} a_{ij}\tilde{X}_{ij} (\leq, =, \geq) \tilde{b}_i \quad , i = 1,2,\ldots\ldots, m$$

$$\sum_{j=1}^{j=n} \tilde{X}_{ij} \geq \tilde{0} \quad , j = 1,2,\ldots\ldots, n$$

$$\text{Where } \tilde{X}_j = (X_j, Y_j, \alpha_j, \alpha_j)$$

$$\tilde{C}_j = (P_j, Q_j, \beta_j, \beta_j)$$

$$\tilde{b}_j = (b_j, g_j, \eta_j, \eta_j)$$

Are symmetric trapezoidal fuzz y number

4 - proposed technique for solving fuzzy linear programming

The steps of proposed technique are following

Step (1): Formulate the selected problem in to fuzzy linear programming as show above

$\tilde{X}_j = (X_j, Y_j, \alpha_j, \alpha_j), \tilde{C}_j = (P_j, Q_j, \beta_j, \beta_j), \tilde{b}_j = (b_j, Q_j, \eta_j, \eta_j)$ **Step (2): Substituting the values of**

In fuzzy linear programming problem in step (1) we get

$$\begin{aligned}
&Max\ or\ Min\quad \tilde{Z} = \sum_{j=1}^{j=n} (P_j, Q_j, \beta_j, \beta_j) \otimes (X_j, Y_j, \alpha_j, \alpha_j)\\
&S.T.\quad \sum_{j=1}^{j=n} a_{ij}(X_j, Y_j, \alpha_j, \alpha_j)(\le, =, \ge)(b_j, g_j, \eta_j, \eta_j)\quad, i = 1,2,......., m\\
&\sum_{j=1}^{j=n} (X_j, Y_j, \alpha_j, \alpha_j) \ge \tilde{0}\quad, j = 1,2,......., n
\end{aligned}$$

Step (3): Assuming the fuzzy linear programming problem $a_{ij}(X_j, Y_j, \alpha_j, \alpha_j) = (\tilde{X}_j, \tilde{Y}_j, \tilde{\alpha}_j, \tilde{\alpha}_j)$

Obtain in step (2) can be written as

$$\begin{aligned}
&Max\ or\ Min\quad \tilde{Z} = \sum_{j=1}^{j=n} (P_j, Q_j, \beta_j, \beta_j) \otimes (X_j, Y_j, \alpha_j, \alpha_j)\\
&S.T.\quad \sum_{j=1}^{j=n} (\tilde{X}_j, \tilde{Y}_j, \tilde{\alpha}_j, \tilde{\alpha}_j)(\le, =, \ge)(b_j, g_j, \eta_j, \eta_j)\quad, i = 1,2,......., m\\
&\sum_{j=1}^{j=n} (X_j, Y_j, \alpha_j, \alpha_j) \ge \tilde{0}\quad, j = 1,2,......., n
\end{aligned}$$

Step (4): convert fuzzy linear programming (FLP) into crisp linear programming (CLP) by using proposed ranking function in equations (3) and (4) we get

$$R(P_j, Q_j, \beta_j, \beta_j) \otimes (X_j, Y_j, \alpha_j, \alpha_j) = R(P_j, Q_j, \beta_j, \beta_j)\, R(X_j, Y_j, \alpha_j, \alpha_j)$$

Then the (CLP) can be written from step (3)

$$\begin{aligned}
&Max\ or\ Min\quad \tilde{Z} = \sum_{j=1}^{j=n} R(P_j, Q_j, \beta_j, \beta_j)\, R(X_j, Y_j, \alpha_j, \alpha_j))\\
&S.T.\quad \sum_{j=1}^{j=n} R(\tilde{X}_j, \tilde{Y}_j, \tilde{\alpha}_j, \tilde{\alpha}_j)(\le, =, \ge)\, R(b_j, g_j, \eta_j, \eta_j)\quad, i = 1,2,......., m\\
&R(X_j, Y_j, \alpha_j, \alpha_j) \ge 0\quad, j = 1,2,......., n\\
&Y_j - X_j \ge 0\quad, \eta_j, \alpha_j \ge 0
\end{aligned}$$

Step (5): Solve the (CLP) obtained in step (4) and substitute their value in

$\tilde{X}_j = (X_j, Y_j, \alpha_j, \alpha_j)$

Step (6): Find the fuzzy optimal value of fuzzy linear programming by substituting the value in

$$\tilde{Z} = \sum_{j=1}^{j=n} \tilde{C}_j \otimes \tilde{X}_j$$

5-Application

Will be applied to the methods used in the theoretical side of the General Company for dairy production, namely (Canon Company) where this company distributes various products for their clients Who in the following governorates (Baghdad-Diwaniyah-Karbala) through the transported of its warehouses to a group of clients in three the provinces so that the transportation cost of a nature Quartet Fuzzy numbers calculated in US dollars as follows

Provinces / Warehouses	Baghdad	Diwaniyah	Karbala	Fuzzy Supply
warehouse1	20,30,40,40	21 ,75 ,50,50	60 ,100 ,90,90	30 ,130 ,150,150
warehouse2	30 ,50 ,50,50	100,100 ,50,50	75 ,50 ,30 ,30	80 ,170 ,100,100
warehouse3	٣٠,90 ,240 ,240	100 ,100 ,190,190	60 ,140 ,230,230	60 ,90 ,250,250
warehouse4	60,100,190,190	30 ,130 ,220 ,220	75,120 ,40,40	60 ,210 ,300,300
Fuzzy Demand	90 ,120 ,350 ,350	80 ,380,260 ,260	60 ,100 ,190,190	230,600,800,800 / 230,600, 800,800

Now convert the transportation problem into crisp linear programming problem as follows using

1-Traditinal Ranking Function $\tilde{R}(A,b,C) = \dfrac{a+b+\alpha+\beta}{4}$ **then**

$$Min\,\tilde{Z} \approx 32.5\tilde{X}_{11} + 49\tilde{X}_{12} + 85\tilde{X}_{13} + 45\tilde{X}_{21} + 75\tilde{X}_{22} + 46.25\tilde{X}_{23} + 150\tilde{X}_{31} + 145\tilde{X}_{32} + 165\tilde{X}_{33} + 135\tilde{X}_{41} + 150\tilde{X}_{42} + 68.75\tilde{X}_{43}$$

S.T.

$$\tilde{X}_{11} + \tilde{X}_{12} + \tilde{X}_{13} \approx 115$$

$$\tilde{X}_{21} + \tilde{X}_{22} + \tilde{X}_{23} \approx 112.5$$

$$\tilde{X}_{31} + \tilde{X}_{32} + \tilde{X}_{33} \approx 162.5$$

$$\tilde{X}_{41} + \tilde{X}_{42} + \tilde{X}_{43} \approx 217.5$$

$$\tilde{X}_{11} + \tilde{X}_{21} + \tilde{X}_{31} + \tilde{X}_{41} \approx 227.5$$

$$\tilde{X}_{12} + \tilde{X}_{22} + \tilde{X}_{32} + \tilde{X}_{42} \approx 245$$

$$\tilde{X}_{13} + \tilde{X}_{23} + \tilde{X}_{33} + \tilde{X}_{43} \approx 135$$

$$\tilde{X}_{11}, \tilde{X}_{12}, \tilde{X}_{13}, \tilde{X}_{21}, \tilde{X}_{22}, \tilde{X}_{23}, \tilde{X}_{31}, \tilde{X}_{32}, \tilde{X}_{33}, \tilde{X}_{41}, \tilde{X}_{42}, \tilde{X}_{43} \geq 0$$

Then Basic feasible solution as illustrated below

BV	$\tilde{X}_{11}$	$\tilde{X}_{12}$	$\tilde{X}_{13}$	$\tilde{X}_{21}$	$\tilde{X}_{22}$	$\tilde{X}_{23}$	$\tilde{X}_{31}$	$\tilde{X}_{32}$	$\tilde{X}_{31}$	$\tilde{X}_{41}$	$\tilde{X}_{42}$	$\tilde{X}_{43}$	R	R_2	R_3	R_4	R_5	R_6	R_7	$\tilde{R}(b)$
R_1	١	١	١										١							115
R_2				١	١	١								١						112.5
R_3							١	١	١						١					162.5
R_4										١	١	١				١				217.5
R_5	١			١			١			١							١			227.5
R_6		١			١			١			١							١		245
R_7			١			١			١			١							١	135
$R(\tilde{Z})$	٣٢.٥	٤٩	٨٥	٤٥	٧٥	٤٦.٢٥	١٥٠	١٤٥	١٦٥	١٣٥	١٥٠	٦٨.٧٥								٠

And the optimal solution is achieved using (Q.M.) software as illustrated below

Availability Amount		Units Shipped	Cost/ Unit ($)
Warehouses	**Provinces**		
Warehouse1	**Baghdad**	**115**	**32.5**
Warehouse2	**Baghdad**	**112.5**	**45**
Warehouse3	**Diwaniyah**	**٢٧.٥**	**١٤٥**
Warehouse3	**Karbala**	**١٣٥**	**١٦٥**
Warehouse٤	**Diwaniyah**	**١٠٥**	**150**
Warehouse4	**Karbala**	**١٢٢.٥**	**٦٨.٧٥**

With total cost is 58547

2-Proposed Ranking Function $\tilde{R}(A,b,C)=\dfrac{a+b}{2}$ then

$$Min\,\tilde{Z}\approx 25\tilde{X}_{11}+48\tilde{X}_{12}+80\tilde{X}_{13}+40\tilde{X}_{21}+100\tilde{X}_{22}+62.5\tilde{X}_{23}+60\tilde{X}_{31}+100\tilde{X}_{32}+100\tilde{X}_{33}+80\tilde{X}_{41}+80\tilde{X}_{42}+97.5\tilde{X}_{43}$$

S.T.

$$\begin{aligned}
&\tilde{X}_{11}+\tilde{X}_{12}+\tilde{X}_{13} \approx 80\\
&\tilde{X}_{21}+\tilde{X}_{22}+\tilde{X}_{23} \approx 125\\
&\tilde{X}_{31}+\tilde{X}_{32}+\tilde{X}_{33} \approx 75\\
&\tilde{X}_{41}+\tilde{X}_{42}+\tilde{X}_{43} \approx 135\\
&\tilde{X}_{11}+\tilde{X}_{21}+\tilde{X}_{31}+\tilde{X}_{41} \approx 105\\
&\tilde{X}_{12}+\tilde{X}_{22}+\tilde{X}_{32}+\tilde{X}_{42} \approx 230\\
&\tilde{X}_{13}+\tilde{X}_{23}+\tilde{X}_{33}+\tilde{X}_{43} \approx 80\\
&\tilde{X}_{11},\tilde{X}_{12},\tilde{X}_{13},\tilde{X}_{21},\tilde{X}_{22},\tilde{X}_{23},\tilde{X}_{31},\tilde{X}_{32},\tilde{X}_{33},\tilde{X}_{41},\tilde{X}_{42},\tilde{X}_{43}\geq 0
\end{aligned}$$

Then Basic feasible solution as illustrated below

BV	$\tilde{X}_{11}$	$\tilde{X}_{12}$	$\tilde{X}_{13}$	$\tilde{X}_{21}$	$\tilde{X}_{22}$	$\tilde{X}_{23}$	$\tilde{X}_{31}$	$\tilde{X}_{32}$	$\tilde{X}_{31}$	$\tilde{X}_{41}$	$\tilde{X}_{42}$	$\tilde{X}_{43}$	R_1	R_2	R_3	R_4	R_5	R_6	R_7	$\tilde{R}(b)$
R_1	١	١	١										١							٨٠
R_2				١	١	١								١						١٢٥
R_3							١	١	١						١					٧٥
R_4										١	١	١				١				١٣٥
R_5	١			١			١			١							١			١٠٥
R_6		١			١			١			١							١		٢٣٠
R_7			١			١			١			١							١	٨٠
$R(\tilde{Z})$	٢٥	٣٨	٨٠	٣٠	١٠٠	٦٢.٥	٦٠	١٠٠	١٠٠	٨٠	٨٠	٩٧.٥								٠

And the optimal solution is achieved using (Q.M.) software as illustrated below

Availability Amount		Units Shipped	Cost/ Unit ($)
Warehouses	**Provinces**		
Warehouse1	Diwaniyah	٨٠	٤٨
Warehouse2	Baghdad	٤٥	٤٠
Warehouse2	Karbala	٨٠	٦٢.٥
Warehouse3	Baghdad	٦٠	٦٠
Warehouse3	Diwaniyah	١٥	100
Warehouse4	Diwaniyah	١٣٥	٨٠

With total cost is 26540

2 Proposed Ranking Function $\tilde{R}(A,b,C)=\dfrac{a+b+\alpha-\beta}{4}$ **then**

$$Min\,\tilde{Z} \approx 12.5\tilde{X}_{11}+24\tilde{X}_{12}+40\tilde{X}_{13}+20\tilde{X}_{21}+50\tilde{X}_{22}+31.25\tilde{X}_{23}+30\tilde{X}_{31}+50\tilde{X}_{32}+50\tilde{X}_{33}+40\tilde{X}_{41}+40\tilde{X}_{42}+48.75\tilde{X}_{43}$$

S.T.

$$\tilde{X}_{11}+\tilde{X}_{12}+\tilde{X}_{13} \approx 40$$
$$\tilde{X}_{21}+\tilde{X}_{22}+\tilde{X}_{23} \approx 62.5$$
$$\tilde{X}_{31}+\tilde{X}_{32}+\tilde{X}_{33} \approx 37.5$$
$$\tilde{X}_{41}+\tilde{X}_{42}+\tilde{X}_{43} \approx 67.5$$
$$\tilde{X}_{11}+\tilde{X}_{21}+\tilde{X}_{31}+\tilde{X}_{41} \approx 52.5$$
$$\tilde{X}_{12}+\tilde{X}_{22}+\tilde{X}_{32}+\tilde{X}_{42} \approx 115$$
$$\tilde{X}_{13}+\tilde{X}_{23}+\tilde{X}_{33}+\tilde{X}_{43} \approx 40$$
$$\tilde{X}_{11}, \tilde{X}_{12}, \tilde{X}_{13}, \tilde{X}_{21}, \tilde{X}_{22}, \tilde{X}_{23}, \tilde{X}_{31}, \tilde{X}_{32}, \tilde{X}_{33}, \tilde{X}_{41}, \tilde{X}_{42}, \tilde{X}_{43} \geq 0$$

Then Basic feasible solution as illustrated below

BV	$\tilde{X}_{11}$	$\tilde{X}_{12}$	$\tilde{X}_{13}$	$\tilde{X}_{21}$	$\tilde{X}_{22}$	$\tilde{X}_{23}$	$\tilde{X}_{31}$	$\tilde{X}_{32}$	$\tilde{X}_{31}$	$\tilde{X}_{41}$	$\tilde{X}_{42}$	$\tilde{X}_{43}$	R_1	R_2	R_3	R_4	R_5	R_6	R_7	$\tilde{R}(b)$
R_1	١	١	١										١							40
R_2				١	١	١								١						٦٢.٥
R_3							١	١	١						١					٣٧.٥
R_4										١	١	١				١				٦٧.٥
R_5	١			١			١			١							١			٥٢.٥
R_6		١			١			١			١							١		١١٥
R_7			١			١			١			١							١	٤٠
$R(\tilde{Z})$	١٢.٥	٢٤	٤٠	٢٠	٥٠	٣١.٢٥	٣٠	٥٠	٥٠	٤٠	٤٠	٤٨.٧٥								٠

And the optimal solution is achieved using (Q.M.) software as illustrated below

Availability Amount		Units Shipped	Cost/ Unit ($)
Warehouses	Provinces		
Warehouse1	Diwaniyah	٤٠	٢٤
Warehouse2	Baghdad	٢٢.٥	٢٠
Warehouse2	Karbala	٤٠	٣١.٢٥
Warehouse3	Baghdad	٣٠	٣٠
Warehouse3	Diwaniyah	٧.٥	٥٠
Warehouse4	Diwaniyah	٦٧.٥	٤٠

With total cost is 6635

6-Conclusions:

In this paper two techniques are proposed for solving fuzzy optimal solution of fuzzy linear programming problem (FLP). The (FLP) problem is converted into crisp linear programming (CLP) using two proposed ranking function for calculating the trapezoidal weight of criteria and compare between these methods with the Traditional and shows the First proposed technique is batter from Traditional and the second technique is batter from First proposed technique as illustrated below

techniques	Ranking function	Total cost
Traditional	$\tilde{R}(A,b,C) = \frac{a+b+\alpha+\beta}{4}$	$R(\tilde{Z}) = 58547$
Proposed 1	$\tilde{R}(A,b,C) = \frac{a+b}{2}$	$R(\tilde{Z}) = 26540$
Proposed ٢	$\tilde{R}(A,b,C) = \frac{a+b+\alpha-\beta}{4}$	$R(\tilde{Z}) = 6635$

7-References

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طرائق جديدة لحل مشاكل البرمجة الخطية الضبابية المتماثلة

المستخلص

تم استعمال و تمكين دالة الرتب قبل الكثير من الباحثين لحل مشكلة البرمجة الخطية الضبابية المتماثلة والمتمثلة بالأعداد او الارقام الضبابية التي تأخذ الشكل الشبة المنحرف التي تحقق دالة الانتماء ، وفي هذا البحث تم اقتراح منهجيتين أو طريقتين لدالة الرتب لحل مشاكل البرمجة الخطية الضبابية ذات الشبه المنحرف بعد ان يتم تحويلها الى انموذج البرمجة الخطية المتموج .وتمتاز هاتين المنهجيتين بالمرونة والسهولة في التطبيق، وتم اختيار عينة عشوائية من البيانات من الشركة العامة لتوزيع منتجان الالبان وتم مقارنة هاتين المنهجيتين مع المنهجية التقليدية او الكلاسيكية في حل نماذج البرمجة الخطية الضبابية وتبين بان المنهجية المقترحة الثانية افضل من المنهجية المقترحة الاولى ، وان المنهجية المقترحة الاولى كانت افضل من المنهجية التقليدية وبمعنى اخر ان هاتين المنهجيتين المقترحتين هما الافضل في اعطاء النتائج من المنهجية التقليدية ، وتم ادخال البيانات وتحليلها وتفسيرها باستعمال البرنامج الجاهز (POM-QM for Windows) الاصدار الثالث.

المصطلحات الرئيسية للبحث /البرمجة الخطية الضبابية ، دالة الرتب المقترحة ، الاعداد الضبابية ذات الشكل المنحرف ، البرمجة الخطية المتموجة .