Comparing Bayes Estimators With others , for scale parameter and Reliability function of two parameters Frechet distribution

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Abstract

This Paper deals with comparing maximum likelihood estimator , and the second one is proposed Bayes estimator under General Entropy loss Function using Prior $g_1(x)$,while the third estimator is also Bayes under quadratic loss Function and using proposed prior $g_2(x)$, After estimator θ ,we also estimate Reliability Function R (t_i) and considering the shape parameter λ is Known constant .The Comparison is done using Simulation procedure taking different values of λ and of n, the results are compared using Statistical Measure Mean Square error MSE. All the results' is explained in tables.

Keyword: Maximum likelihood estimator (MLE) Bayes (1) and Bayes (2) Mean square error (E) λ is shape parameter θ is scale parameter $\hat{\theta}_{Bayes}$ under Quadratic loss function $\hat{\theta}_{GE}$ under General Entropy loss function

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Introduction

The Reliability of any machine or device, is the possibility of work continuity of the machine without breaking down or Failure during a period [0,t), and without reducing reliability applications in industrial Field, Reliability have been studied under various application by Moore, A.H. and Bilikam [14] , (1978) and Charek, D.J.(1985) Rytgaard (1990) Estimated Reliability Function of Pareto [17] , Many other researcher like Husain, A.M , Zimmer, W. J(2000) and Erefaie, A. and Parisian (2005) estimated parameters and Reliability function of Pareto [5] and also Saleh, S.M(2006) estimated a Reliability function also of Pareto. Farhan, Hta Salman (2007) estimated survival Function of Lomax distribution in case of censored data (II). and Nadarajah S, and Kotz .S(2008) [16] .

Also Pandey H. and Raol A.K (2009) give different Bayes estimator. Also Singh G., B. P., S. K., U., and R.D. (2011) estimated shape and Reliability function when this shape parameter is unknown [19] . Also Setiya Parul, Kumar Uinol (2013) estimated shape Parameter, Reliability and hazard function of Pareto type (I) Model.

Here we continue the work under Reliability Field by comparing different Bayes estimators of scale Parameter(θ) and Reliability Function For two Parameter Frechet distribution The Comparison between Maximum Likelihood and Bayes estimator using Squared error loss function and also General Entropy Function ,is done through simulation.

Definition

The p.d.f of the random Variable X that have two Parameter Frechet:

x ∿ Frechet (λ,θ)

Is defined by

$$f(x,\lambda,\theta) = \frac{\lambda}{\theta}\left(\frac{\theta}{x}\right)^{\lambda+1} e^{-\left(\frac{\theta}{x}\right)^{\lambda}} \qquad x > 0 \qquad (1)$$

While the c. d. f is

$$F(x) = e^{-\left(\frac{\theta}{x}\right)^{\lambda}} \qquad x, \lambda, \vartheta > 0 \qquad (2)$$

Also

4) $R(t) = 1-F(t)=\int_t^{\infty} f(u)du$

$$= \int_t^{\infty} \frac{\lambda}{\theta}\left(\frac{\theta}{x}\right)^{\lambda+1} e^{-\left(\frac{\theta}{x}\right)^{\lambda}} dx$$

$$R(t) = 1- e^{-\left(\frac{\theta}{t}\right)^{\lambda}} \qquad (3)$$

Now we explain some properties of Reliability Function R(t)

Definition of Reliability Function:

The reliability Function R(t) is defined as:

R(t) = Pr (T > t) (3)

Which means that the item is Still working at time t, and it is defined on the domain t$\in [0,\infty)$ so **R(t)**$\in [0,1]$ **.**

<u>And some of its properties are:</u>

1. **R(0) =1**
2. $\lim_{t\to\infty} R(t) = 0$
3. **R(ta) ≥ R(tb)**$\Leftrightarrow ta \le tb$

<u>Method of estimation:</u>

Here we consider the estimation of parameter θ **by Method of Maximum likelihood. [13]**

The log-likelihood for a random sample $x_1, x_2, \ldots, x_n$

From (1) is:

$$L(X,\lambda,\theta) = \lambda^n \theta^{-n} \pi \left(\frac{\theta}{x_i}\right)^{\lambda+1} e^{-\Sigma\left(\frac{\theta}{x_i}\right)^{\lambda}} \quad (4)$$

$$\text{Log } L = n \log\lambda - n \log\theta + (\lambda+1)\Sigma \log\left(\frac{\theta}{x_i}\right) - \Sigma\left(\frac{\theta}{x_i}\right)^{\lambda} \quad (5)$$

Therefore, the Maximum likelihood estimator of θ **is:**

$$\hat{\theta}_{ML} = \left(\frac{n}{k}\right)^{\frac{1}{\lambda}} \quad (6)$$

$$K = \sum_{i=1}^{n} \left(\frac{1}{x_i}\right)^{\lambda}$$

The second estimator of θ**is the proposed Bayes estimator of** θ **under General Entropy loss Function [3] which defined by**

$$L_G E(\hat{\theta},\theta) = w\left[\left(\frac{\hat{\theta}}{\theta}\right)^{c} - C\ln\left(\frac{\hat{\theta}}{\theta}\right) - 1\right] \qquad w > 0 \quad (7)$$

Since the risk Function is expected loss

$$R_{GE}(\hat{\theta},\theta) = EL_{GE}(\hat{\theta},\theta)$$

$$= \int_0^\infty \left(\frac{\hat{\theta}}{\theta}\right)^{c} - c\ln\left(\frac{\hat{\theta}}{\theta}\right) - 1] h(\theta|x)\, d\theta \qquad w = 1$$

$$R_{GE}(\hat{\theta},\vartheta) = \int \left[\frac{(\hat{\theta})^{c}}{\theta} - c\ln(\hat{\theta}) + c\ln(\theta) - 1\right] h(\theta|x)$$

$$\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}} = \int \left[\frac{c(\hat{\theta})^{c-1}}{\theta^{c}} - \frac{c}{\hat{\vartheta}}\right] h(\theta|x) d\theta$$

$\hat{\theta}$ **Bayes estimator under General Entropy loss function**

$$\hat{\theta}_{GE} = \left[\frac{1}{E(\hat{\theta}|x)}\right]^{\frac{1}{c}} \quad (8)$$

Therefore under prior distribution of θ **i.e**

$g_1(\theta) = K\theta^{c}$ (9)

Of $h_1(\theta|x)$ distribution we find the posterior

$$h_1(\theta|x) = \frac{\lambda\theta^{n\lambda+c}e^{-\theta^{\lambda}T}T^{(n+\frac{c+1}{\lambda})}}{\Gamma\left(n+\frac{c+1}{\lambda}\right)} \quad (10).$$

We solve the Expectation to find the formula of $\hat{\theta}_{GE}$

$$E\left(\frac{1}{\theta^{c}}|x\right) = \int_0^{\infty} \frac{1}{\theta^{c}} h_1(\theta|x)d\theta$$

$$= \frac{T^{\frac{c}{\lambda}}\Gamma(n+\frac{1}{\lambda})}{\Gamma\left(n+\frac{c+1}{\lambda}\right)}$$

$$T=\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\lambda}$$

The Bayes estimator of θ under Entropy loss function [8] is:

$$\hat{\theta}_{Bayes(GE)} = \left[\frac{\Gamma\left(n+\frac{c+1}{\lambda}\right)}{T^{\frac{c}{\lambda}}\Gamma\left(n+\frac{1}{\lambda}\right)}\right]^{\frac{1}{c}} \quad (11)$$

Where λ is known constant and c is also known

$$T = \sum_{i=1}^{n}\left(\frac{1}{x_i}\right)^{\lambda}$$

The third estimator Bayes estimator according to prior distribution

$$\Pi_2(\theta) = K\left(\theta^{\lambda}\right)^{B-1}e^{-B\theta^{\lambda}} \qquad \theta > 0 \quad (12)$$

λ *known* B *constant*

Sing the p .d .f of Frechet defined in equation (1), we find the second posterior $h_2(\theta/x)$

$$h_2(\theta|x) = \frac{\lambda\left(\theta^{\lambda}\right)^{n+B-1}T^{n+B+\frac{1}{\lambda}-1}e^{-\theta^{\lambda}T}}{\Gamma\left(n+B+\frac{1}{\lambda}-1\right)} \quad (13)$$

$$T = B+\sum_{i=1}^{n}\left(\frac{\theta}{x_i}\right)^{\lambda}$$

Therefor the Bayes estimator of θ is the posterior mean

$$\hat{\theta}_{Bayes\ 2} = E(\theta|x) = \frac{n+B}{T} \quad (14)$$

T = B+$\sum \left(\frac{\theta}{x_i}\right)^{\lambda}$

Where $\Pi_2(\theta) = K(\theta^{\lambda})^{B-1} e^{-B\theta^{\lambda}}$ **(15)**

For this prior the posterior is $h_2(\theta/x)$

$$h_2(\theta|x) = \frac{\lambda(\theta^{\lambda})^{n+B-1}\left[B+\sum\left(\frac{\theta}{x_i}\right)^{\lambda}\right]^{n+B+\frac{1}{\lambda}-1} e^{-\theta^{\lambda}T}}{\Gamma\left(n+B+\frac{1}{\lambda}-1\right)} \quad (16)$$

T = B+$\sum_{i=1}^{n}\left(\frac{\theta}{x_i}\right)^{\lambda}$

$$\hat{\theta}_{Bayes(2)} = \frac{n+B}{T} \quad (17)$$

t_i: **Is the observation**

C: constant known

Shape parameter also is known:λ Finally we use the simulation procedure to compare different.

Simulation in Procedure

The estimation of θ and of R(t) is alone through simulation procedure using initial values.

λ	2	4
θ	1.5	2
c	+1	2
B	+1	2

And the sample size taken are n=30, 60, 90,120,150.

The results are compared using statistical Measure Means square error (MSE).

Table (1): Estimation of Scale Parameters (θ) and mean square error

($\lambda = 2$, $\theta = 1.5$, c=1, B=1)

$\lambda = 2$ $\theta = 1.5$ c =1 B =1					
n		*Bayes*	*Bayes − GE*	*ML*	*Best*
	$\hat{\theta}$	1.13291	1.514634	1.518295	*GE*

30	$MSE(\hat{\theta})$	0.144925	0.018283	0.018459	
60	$\hat{\theta}$	1.130092	1.513016	1.509867	ML
	$MSE(\hat{\theta})$	0.142406	0.009615	0.009504	
90	$\hat{\theta}$	1.130042	1.509364	1.507269	ML
	$MSE(\hat{\theta})$	0.140634	0.006356	0.006303	
120	$\hat{\theta}$	1.129558	1.502871	1.505302	GE
	$MSE(\hat{\theta})$	0.140373	0.004836	0.005106	
150	$\hat{\theta}$	1.130781	1.501963	1.505707	GE
	$MSE(\hat{\theta})$	0.138906	0.003734	0.004211	

Table (2): Estimation of Scale Parameters (θ) and mean square error

($\lambda = 2$, $\theta = 1.5$, c=2, B=1)

$\lambda = 2$		$\theta = 1.5$	$c = 2$	$B = 1$	
n		$Bayes$	$Bayes-GE$	ML	$Best$
30	$\hat{\theta}$	1.135307	1.534281	1.521653	ML
	$MSE(\hat{\theta})$	0.144295	0.021816	0.020772	
60	$\hat{\theta}$	1.131705	1.493707	1.507439	GE
	$MSE(\hat{\theta})$	0.141201	0.008467	0.009555	
90	$\hat{\theta}$	1.130507	1.491012	1.506832	GE
	$MSE(\hat{\theta})$	0.140732	0.006206	0.006894	
120	$\hat{\theta}$	1.129365	1.506909	1.503779	ML
	$MSE(\hat{\theta})$	0.140407	0.005089	0.005035	
150	$\hat{\theta}$	1.129394	1.501829	1.504324	GE
	$MSE(\hat{\theta})$	0.139664	0.002545	0.003804	

Table (3): Estimation of Scale Parameters (θ) and mean square error

($\lambda = 2$, $\theta = 1.5$, c = 2, B = 2)

$\lambda = 2$	$\theta = 1.5$		$c = 2$	$B = 2$	
n		$Bayes$	$Bayes - GE$	ML	$Best$
30	$\hat{\theta}$	1.13096	1.514317	1.521689	GE
	$MSE(\hat{\theta})$	0.146905	0.017285	0.021232	
60	$\hat{\theta}$	1.128754	1.50651	1.509235	GE
	$MSE(\hat{\theta})$	0.143282	0.008397	0.009761	
90	$\hat{\theta}$	1.128882	1.511475	1.507294	ML
	$MSE(\hat{\theta})$	0.141497	0.006792	0.006677	
120	$\hat{\theta}$	1.131907	1.512842	1.5097	ML
	$MSE(\hat{\theta})$	0.138051	0.004589	0.0045	
150	$\hat{\theta}$	1.127201	1.504769	1.502267	ML
	$MSE(\hat{\theta})$	0.141092	0.003612	0.003583	

Table (4): Estimation of Scale Parameters (θ) and mean square error

($\lambda = 4$, $\theta = 2$, c = 1, B = 1)

$\lambda = 4$	$\theta = 2$		$c = 1$	$B = 1$	
n		$Bayes$	$Bayes - GE$	ML	$Best$
30	$\hat{\theta}$	1.130642	2.030767	2.022323	ML
	$MSE(\hat{\theta})$	0.766383	0.035143	0.03441	
60	$\hat{\theta}$	1.131144	2.005386	2.013188	GE
	$MSE(\hat{\theta})$	0.759875	0.013291	0.0151	
90	$\hat{\theta}$	1.130895	2.000607	2.008815	GE
	$MSE(\hat{\theta})$	0.759085	0.001935	0.011048	
120	$\hat{\theta}$	1.131282	2.011057	2.008963	ML
	$MSE(\hat{\theta})$	0.757649	0.008988	0.008928	
150	$\hat{\theta}$	1.128676	2.005638	2.003967	ML
	$MSE(\hat{\theta})$	0.761441	0.006576	0.006549	

Table (5): Estimation of Scale Parameters (θ) and mean square error

$(\lambda = 4, \ \theta = 2,$ **c = 2, B = 2)**

Estimators of Reliability function as follows;

$\lambda = 4$	$\theta = 2$	$c = 2$	$B = 2$		
n		***Bayes***	***Bayes − GE***	***ML***	***Best***
30	$\hat{\theta}$	**1.126376**	**2.039564**	**2.022777**	***ML***
	$MSE(\hat{\theta})$	**0.774017**	**0.039646**	**0.037975**	
60	$\hat{\theta}$	**1.126371**	**2.019355**	**2.010994**	***ML***
	$MSE(\hat{\theta})$	**0.768559**	**0.017358**	**0.016964**	
90	$\hat{\theta}$	**1.13008**	**2.004135**	**2.011555**	***GE***
	$MSE(\hat{\theta})$	**0.760494**	**0.009324**	**0.011103**	
120	$\hat{\theta}$	**1.127876**	**2.001171**	**2.004998**	***GE***
	$MSE(\hat{\theta})$	**0.763334**	**0.003258**	**0.008364**	
150	$\hat{\theta}$	**1.127109**	**2.001155**	**2.001821**	***GE***
	$MSE(\hat{\theta})$	**0.764346**	**0.006367**	**0.00702**	

Table (6): Reliability estimator when $(\lambda = 2, \ \theta = 1.5, \ \ c = 1 \ \ B = 1)$

$\lambda = 2$		$\theta = 1.5$		$c = 1$		$B = 1$			
n	t_i	***Real***	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{GE}$	$MSE(\hat{R}_{GE})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	***Best***
30	**0.5**	**0.999877**	**0.991205**	**0.000138**	**0.999706**	**3.15E-07**	**0.999688**	**3.50E-07**	***GE***
	1	**0.894601**	**0.717159**	**0.035348**	**0.894336**	**0.001618**	**0.892412**	**0.001656**	***GE***
	1.5	**0.632121**	**0.436617**	**0.041708**	**0.643372**	**0.004531**	**0.64036**	**0.004476**	***Ml***
	2	**0.430217**	**0.273893**	**0.026271**	**0.439253**	**0.0036**	**0.436581**	**0.003535**	***Ml***
	2.5	**0.302324**	**0.186411**	**0.01433**	**0.311076**	**0.002133**	**0.308955**	**0.002079**	***Ml***
60	**0.5**	**0.999877**	**0.992767**	**7.17E-05**	**0.999816**	**5.11E-08**	**0.99981**	**6.51E-08**	***GE***
	1	**0.894601**	**0.721374**	**0.032265**	**0.896123**	**0.000928**	**0.895159**	**0.000935**	***GE***
	1.5	**0.632121**	**0.433912**	**0.041119**	**0.638111**	**0.002215**	**0.636591**	**0.002199**	***Ml***
	2	**0.430217**	**0.275292**	**0.024912**	**0.437003**	**0.001745**	**0.435663**	**0.001722**	***Ml***
	2.5	**0.302324**	**0.185238**	**0.014196**	**0.306961**	**0.001083**	**0.305907**	**0.001068**	***Ml***
90	**0.5**	**0.999877**	**0.993145**	**5.97E-05**	**0.999835**	**3.33E-08**	**0.999831**	**3.49E-08**	***GE***
	1	**0.894601**	**0.718898**	**0.032211**	**0.894722**	**0.000531**	**0.89407**	**0.000535**	***GE***
	1.5	**0.632121**	**0.432471**	**0.040975**	**0.634532**	**0.001356**	**0.633516**	**0.001352**	***Ml***
	2	**0.430217**	**0.274011**	**0.025022**	**0.433827**	**0.001084**	**0.432935**	**0.001076**	***Ml***
	2.5	**0.302324**	**0.185085**	**0.014074**	**0.305645**	**0.000717**	**0.304943**	**0.00071**	***Ml***

120	0.5	0.999877	0.993445	4.99E-05	0.999851	1.48E-08	0.999849	1.54E-08	***GE***
	1	0.894601	0.719892	0.031701	0.895261	0.000481	0.894773	0.000483	***GE***
	1.5	0.632121	0.433545	0.040263	0.635339	0.00099	0.634576	0.000986	***Ml***
	2	0.430217	0.27327	0.025175	0.432721	0.000973	0.432054	0.000968	***Ml***
	2.5	0.302324	0.184895	0.014065	0.304776	0.000579	0.304251	0.000575	***Ml***
150	0.5	0.999877	0.993442	4.89E-05	0.999853	1.26E-08	0.999851	1.30E-08	***GE***
	1	0.894601	0.717508	0.032175	0.893518	0.00035	0.893123	0.000353	***GE***
	1.5	0.632121	0.431646	0.040928	0.633007	0.000898	0.632396	0.000897	***Ml***
	2	0.430217	0.272779	0.025175	0.431358	0.000687	0.430825	0.000685	***Ml***
	2.5	0.302324	0.185214	0.01394	0.304782	0.000476	0.304362	0.000474	***Ml***

Table (7): Reliability estimator when ($\lambda = 2,\ \theta = 1.5,\quad c = 2 \quad B = 1$)

$\lambda = 2$			$\theta = 1.5$		$c = 2$	$B = 1$			
n	t_i	*Real*	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{GE}$	$MSE(\hat{R}_{GE})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	*Best*
30	0.5	0.999877	0.991793	1.18E-04	0.999758	2.11E-07	0.999727	2.62E-07	***GE***
	1	0.894601	0.717772	0.035148	0.89686	0.001597	0.893078	0.001661	***GE***
	1.5	0.632121	0.434423	0.042456	0.643412	0.004575	0.637437	0.004483	***Ml***
	2	0.430217	0.276397	0.025528	0.445201	0.003832	0.439871	0.003654	***Ml***
	2.5	0.302324	0.18719	0.014261	0.314343	0.002474	0.310117	0.002344	***Ml***
60	0.5	0.999877	0.99272	7.45E-05	0.999815	5.29E-08	0.999802	6.06E-08	***GE***
	1	0.894601	0.719803	0.03256	0.896974	0.000807	0.895053	0.000818	***GE***
	1.5	0.632121	0.433624	0.041162	0.638228	0.002227	0.6352	0.002201	***Ml***
	2	0.430217	0.274056	0.025384	0.437037	0.001848	0.434371	0.001806	***Ml***
	2.5	0.302324	0.185711	0.014086	0.307191	0.001103	0.305093	0.001075	***Ml***
90	0.5	0.999877	0.99307	6.06E-05	0.999837	2.70E-08	0.999829	2.98E-08	***GE***
	1	0.894601	0.717477	0.032833	0.893956	0.000605	0.892649	0.000617	***GE***
	1.5	0.632121	0.432763	0.040989	0.635549	0.001501	0.633523	0.001492	***Ml***
	2	0.430217	0.273467	0.025218	0.434048	0.001152	0.432271	0.001136	***Ml***
	2.5	0.302324	0.185256	0.014073	0.306216	0.000789	0.304817	0.000775	***Ml***
120	0.5	0.999877	0.993335	5.24E-05	0.999849	1.74E-08	0.999844	1.88E-08	***GE***
	1	0.894601	0.719724	0.031747	0.895287	0.000472	0.894311	0.000477	***GE***
	1.5	0.632121	0.431783	0.041037	0.634404	0.001141	0.632881	0.001136	***Ml***
	2	0.430217	0.274136	0.024863	0.434635	0.000909	0.433299	0.000896	***Ml***
	2.5	0.302324	0.184831	0.01407	0.304788	0.000573	0.30374	0.000566	***Ml***
150	0.5	0.999877	0.993374	5.05E-05	0.999852	1.41E-08	0.999848	1.50E-08	***GE***

	1	0.894601	0.718267	0.032001	0.894493	0.000383	0.893708	0.000387	***GE***
	1.5	0.632121	0.431711	0.040895	0.633764	0.000881	0.632544	0.000879	***Ml***
	2	0.430217	0.272686	0.025194	0.432336	0.000689	0.431269	0.000683	***Ml***
	2.5	0.302324	0.184785	0.014017	0.304726	0.000432	0.303887	0.000427	***Ml***

Table (8): Reliability estimator when ($\lambda = 2, \quad \theta = 1.5, \quad c = 2, \quad B = 2$)

$\lambda = 2$		$\theta = 1.5$		$c = 2$	$B = 2$				
n	t_i	$Real$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{GE}$	$MSE(\hat{R}_{GE})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	$Best$
30	0.5	0.999877	0.991589	1.24E-04	0.999747	2.24E-07	0.999714	2.80E-07	***GE***
	1	0.894601	0.716565	0.035359	0.898467	0.001636	0.894722	0.001689	***GE***
	1.5	0.632121	0.431643	0.043489	0.643899	0.004697	0.637927	0.004598	***Ml***
	2	0.430217	0.272515	0.026564	0.442368	0.003652	0.437056	0.003506	***Ml***
	2.5	0.302324	0.185882	0.014495	0.314385	0.002522	0.310158	0.002391	***Ml***
60	0.5	0.999877	0.992701	7.52E-05	0.999817	5.53E-08	0.999804	6.32E-08	***GE***
	1	0.894601	0.71728	0.033419	0.896412	0.000827	0.894486	0.000841	***GE***
	1.5	0.632121	0.43246	0.04162	0.638292	0.002274	0.635266	0.002247	***Ml***
	2	0.430217	0.273564	0.02548	0.436933	0.001829	0.434267	0.001788	***Ml***
	2.5	0.302324	0.184833	0.014284	0.307795	0.00113	0.305694	0.0011	***Ml***
90	0.5	0.999877	0.992958	6.16E-05	0.999833	2.79E-08	0.999826	3.08E-08	***GE***
	1	0.894601	0.720567	0.031728	0.896538	0.000617	0.89525	0.000623	***GE***
	1.5	0.632121	0.431647	0.041418	0.635588	0.001573	0.633562	0.001564	***Ml***
	2	0.430217	0.273002	0.02535	0.434908	0.001176	0.433129	0.001157	***Ml***
	2.5	0.302324	0.184745	0.014153	0.306365	0.000765	0.304965	0.000751	***Ml***
120	0.5	0.999877	0.993138	5.47E-05	0.999845	1.73E-08	0.999839	1.87E-08	***GE***
	1	0.894601	0.7175	0.032515	0.894585	0.000483	0.893606	0.000489	***GE***
	1.5	0.632121	0.43265	0.040699	0.635929	0.001178	0.634406	0.001169	***Ml***
	2	0.430217	0.273254	0.025102	0.43427	0.000871	0.432934	0.000859	***Ml***
	2.5	0.302324	0.185543	0.013862	0.306756	0.000519	0.305703	0.000508	***Ml***
150	0.5	0.999877	0.993464	4.82E-05	0.999859	1.07E-08	0.999855	1.14E-08	***GE***
	1	0.894601	0.719225	0.031693	0.895066	0.000384	0.894283	0.000387	***GE***
	1.5	0.632121	0.433891	0.04002	0.636034	0.000895	0.634814	0.000887	***Ml***
	2	0.430217	0.272322	0.025307	0.432412	0.000687	0.431345	0.000681	***Ml***
	2.5	0.302324	0.184124	0.014155	0.30403	0.000406	0.303192	0.000402	***Ml***

Table (9): Reliability estimator when ($\lambda = 4,\ \theta = 2,\ \ c = 1,\ B = 1$)

$\lambda = 4$		$\theta = 2$			$c = 1$	$B = 1$			
n	t_i	$Real$	$\widehat{R}_{Bayes}$	$MSE(\widehat{R}_{Bayes})$	$\widehat{R}_{GE}$	$MSE(\widehat{R}_{GE})$	$\widehat{R}_{Mle}$	$MSE(\widehat{R}_{Mle})$	$Best$
30	0.5	1	0.991606	0.00012	0.999999	2.07E-11	0.999999	2.45E-11	GE
	1	0.981684	0.717502	0.074007	0.979825	0.000195	0.979199	0.000206	GE
	1.5	0.830987	0.435335	0.159844	0.835512	0.002682	0.833098	0.002701	GE
	2	0.632121	0.277099	0.128157	0.642292	0.004937	0.639286	0.00489	Ml
	2.5	0.472708	0.185772	0.083266	0.482108	0.003826	0.479303	0.003761	Ml
60	0.5	1	0.992781	7.53E-05	1	1.53E-12	1	1.69E-12	GE
	1	0.981684	0.720816	0.070172	0.981055	8.99E-05	0.98075	9.26E-05	GE
	1.5	0.830987	0.43167	0.161233	0.832122	0.001422	0.830889	0.00143	GE
	2	0.632121	0.274537	0.128847	0.637102	0.002276	0.635582	0.002264	Ml
	2.5	0.472708	0.185501	0.082922	0.478177	0.001705	0.47677	0.001687	Ml
90	0.5	1	0.993234	5.77E-05	1	2.19E-13	1	2.37E-13	GE
	1	0.981684	0.720657	0.069573	0.981298	5.84E-05	0.981094	5.96E-05	GE
	1.5	0.830987	0.433984	0.158801	0.832521	0.00093	0.831697	0.000933	GE
	2	0.632121	0.2743	0.128661	0.637063	0.001475	0.636047	0.001466	Ml
	2.5	0.472708	0.185334	0.082912	0.476339	0.001246	0.475401	0.001238	Ml
120	0.5	1	0.993238	5.62E-05	1	2.42E-13	1	2.56E-13	GE
	1	0.981684	0.720055	0.069484	0.981283	4.36E-05	0.98113	4.42E-05	GE
	1.5	0.830987	0.433323	0.159006	0.832135	0.000702	0.831515	0.000703	GE
	2	0.632121	0.271883	0.130278	0.632236	0.001173	0.631473	0.001173	Ml
	2.5	0.472708	0.185391	0.082812	0.476211	0.00101	0.475508	0.001004	Ml
150	0.5	1	0.993457	5.07E-05	1	1.06E-13	1	1.12E-13	GE
	1	0.981684	0.720086	0.069277	0.98147	3.46E-05	0.981348	3.50E-05	GE
	1.5	0.830987	0.431899	0.159999	0.830922	0.000568	0.830424	0.00057	GE
	2	0.632121	0.273584	0.128926	0.634809	0.000889	0.634198	0.000887	Ml
	2.5	0.472708	0.184567	0.083221	0.474446	0.000741	0.473884	0.000738	Ml

Table (10): Reliability estimator when ($\lambda = 4,\ \theta = 2,\quad c = 2,\quad B = 2$)

$\lambda = 4$		$\theta = 2$		$c = 2$	$B = 2$				
n	t_i	$Real$	$\hat{R}_{Bayes}$	$MSE(\hat{R}_{Bayes})$	$\hat{R}_{GE}$	$MSE(\hat{R}_{GE})$	$\hat{R}_{Mle}$	$MSE(\hat{R}_{Mle})$	$Best$
30	0.5	1	0.991776	1.16E-04	0.999999	4.56E-11	0.999999	6.22E-11	GE
	1	0.981684	0.71598	0.074233	0.980895	1.76E-04	0.979685	1.95E-04	GE
	1.5	0.830987	0.43238	0.162466	0.83665	0.00288	0.831876	0.002928	GE
	2	0.632121	0.273191	0.130418	0.64368	0.004255	0.637698	0.00416	Ml
	2.5	0.472708	0.184533	0.083997	0.484909	0.004276	0.479343	0.004129	Ml
60	0.5	1	0.992493	7.65E-05	1	8.70E-13	1	1.08E-12	GE
	1	0.981684	0.71811	0.071426	0.981252	8.74E-05	0.980644	9.25E-05	GE
	1.5	0.830987	0.433894	0.15947	0.835167	0.001406	0.832728	0.001411	GE
	2	0.632121	0.27346	0.129526	0.638971	0.002174	0.635943	0.002143	Ml
	2.5	0.472708	0.184126	0.083744	0.478785	0.001934	0.475985	0.001897	Ml
90	0.5	1	0.993139	6.06E-05	1	3.84E-13	1	4.45E-13	GE
	1	0.981684	0.719419	0.07016	0.981627	5.38E-05	0.981225	5.58E-05	GE
	1.5	0.830987	0.432645	0.159734	0.833823	0.000898	0.832182	0.000899	Ge
	2	0.632121	0.272809	0.129743	0.636223	0.001505	0.634197	0.001492	Ml
	2.5	0.472708	0.185093	0.08305	0.478195	0.001265	0.476323	0.001243	Ml
120	0.5	1	0.993189	5.75E-05	1	2.51E-13	1	2.81E-13	GE
	1	0.981684	0.71861	0.070338	0.981303	4.57E-05	0.980997	4.71E-05	GE
	1.5	0.830987	0.432692	0.159591	0.83288	0.000779	0.831644	0.000781	GE
	2	0.632121	0.271872	0.13025	0.633351	0.001131	0.631828	0.001129	Ml
	2.5	0.472708	0.184369	0.083378	0.47559	0.000951	0.474187	0.000943	Ml
150	0.5	1	0.99343	5.02E-05	1	7.93E-14	1	8.79E-14	GE
	1	0.981684	0.717122	0.070868	0.981038	3.71E-05	0.980789	3.82E-05	GE
	1.5	0.830987	0.432562	0.159453	0.832664	0.000569	0.831672	0.000569	Ml&GE
	2	0.632121	0.27249	0.129685	0.634276	0.000813	0.633055	0.00081	Ml
	2.5	0.472708	0.18412	0.083492	0.474274	0.000798	0.473153	0.000794	Ml

Conclusion

1. The best estimator of Reliability function is maximum likelihood and then the Bayes estimator and general entropy loss function, which indicate that the prior information about $\theta$is important for improving the estimator of(scale parameter θ)and then improving estimator of Reliability function of Frechet distribution .

2. The replicate of each experiment was (R=1000), in order to obtain best estimator for(θ) and for R(t) .

3. The percentage indicate that maximum likelihood estimator of(θ) and then of R(t) ,was best with 52% comparing with Bayes estimator under General entropy is 48%.

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مقارنة مقدرات بيز مع اخرين، لمعلمة القياس ودالة المعولية لتوزيع Frechet ذي المعلمتين

المستخلص

يهتم هذا البحث بمقارنة مقدر الإمكان الأعظم ومقدر Bayes مقترح بإفتراض دالة خسارة Entropy وكذلك مقدر Bayes مقترح آخر، لتقدير معلمة القياس، ومن ثم دالة المعولية. و بواسطة المحاكاة باعتبار λ (معلمة الشكل معلومة) وسوف تتم المقارنة بواسطة المحاكاة حيث أجريت تجارب المحاكاة وكررت كل تجربة (R = 1000) مرةً، وتم تقدير معلمة القياس (θ)، ومن ثم تقدير دالة المعولية بالطرائق الثلاث: Bayes الأول، Bayes الثاني ومقدر الإمكان الأعظم، وباستخدام قيم معلمات أولية مفترضة مختلفة (λ= 2 , $\theta = 1.5, C = 2, B = 2$).

وعرضت جميع نتائج المقدرات لكل من معلمة القياس (θ) ودالة المعولية R (t_i) ، في جداول خاصة.

المصطلحات الرئيسية للبحث/ مقدر الإمكان الأعظم (MLE)، مقدر Bayes الأول، مقدر Bayes الثاني.

λ : معلمة شكلة ، θ : معلمة قياس

$\hat{\theta}_{Bayes}$: دالة خسارة تربيعية.

$\hat{\theta}_{GE}$: دالة خسارة Entropy