Detecting Outliers In Multiple Linear Regression

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Abstract :

It is well-known that the existence of outliers in the data will adversely affect the efficiency of estimation and results of the current study. In this paper four methods will be studied to detect outliers for the multiple linear regression model in two cases : first, in real data; and secondly, after adding the outliers to data and the attempt to detect it. The study is conducted for samples with different sizes, and uses three measures for comparing between these methods . These three measures are : the mask, dumping and standard error of the estimate.

اكتشاف القيم الشاذة في الانحدار الخطي المتعدد

من المعروف ان وجود القيم الشاذة في البيانات يؤثر سلبا على كفاءة التقديرات والنتائج للدراسة الموضوعة، وفي هذا البحث سيتم دراسة (4) طرائق لاكتشاف القيم الشاذة لنموذج الانحدار الخطي المتعدد ولحالتين: لبيانات حقيقية والحالة الثانية بعد اقحام قيم شاذة للبيانات ومحاولة كشفها، وقد تمت الدراسة باحجام عينات مختلفة واعتماد (٣) مقاييس للمقارنة بين هذه الطرائق هي: القناع، الاغراق والخطأ المعياري للتقدير .

1. Introduction :

There are many methods for detecting outliers in linear regression model as:

Elashoff (1972) studied the linear regression model. She illustrated the existing outliers cause the bias in estimator and the high variance . Draper and John (1981) illustrated the benefit of using the Cook Distance.

Pena and Yohai (1999) suggested fast procedure to estimate linear regression parameters in case of existing outliers and how to detect it . Chen (2003) detected outliers in multiple linear regression model. He depended on many robust estimate methods such as (LTS) . Gal (2005) presented several methods for the detection of outliers in univariate and multivariate . Karpinski (2007) illustrated in his book the outliers and how to detect them by using several methods .

Mishra (2008) studied several robust and non-robust methods for detecting outliers in multiple linear regression ; he used a Monte Carlo method for comparison between real data and theoretical data .

Asikgil and Erar (2009) tried to determine multiple outliers by using various methods in the presence of masking and swamping effects for the linear regression model .

The multiple linear regression model is as the following equation :

$$Y = X\beta + \epsilon \qquad ....(1.1)$$

where :

Y : vertical vector (n*1) of observed response values.

X : matrix (n*p) of (p) regressors .

β : vertical vector (p*1) of regression coefficients .

€ : vertical vector (n*1) of error terms .

n : sample size .

The method of ordinary least squares (OLS) is the most widely used technique to find the best estimates of (β) which minimizes the sum of squared distance for actual observations to the regression surface under the assumption ($\epsilon \sim NID(0,\sigma^2 I)$); but if the data has outliers the assumption is not satisfied and the estimate dose not minimize the sum of squared distance and will not be optimal . In this case, we must firstly detect outliers and treat them and then apply (OLS) method or we can estimate (β) by robust methods of estimate instead of (OLS) method.

Outlier : we can define the outlier as; the observation (or subset of observations) that appear inconsistent (extreme) with the remainder of the data set and has a profound destructive influence on the statistical analysis ; and in linear regression model is not necessarily be extreme (Barnett & Lewis 1994) .

There are several types of outliers in linear regression :

i. **In X-Space : If one or more of the observation values lie far away from the group observations at the (X) axis .**
ii. **In Y-Space : If one or more of the observation values lie far away from the group observations at the (Y) axis .**
iii. **In (XY) - Space : If one or more of the observation values lie far away from the group observations at the (X) and (Y) axis.**

Care should be taken in detecting the outlier in set data to prevent masking and swamping problems ; where :
masking ; the unable of the procedure to detect the outliers , swamping; consider the clean observations as outliers (Adnan and others 2003) .

2. Methods Of Detecting Outliers :

There are various methods to detect the influential observations in linear regression model. Some of these methods is to detect a single outlier and the other is to detect multiple outliers (single – row diagnostics). The single – row diagnostics can be extended to include subset of observations rather than a single observation (Belsley & Welsch 1980). In this paper, we will use some of widely-used measures that depend on the single – row diagnostics as the following :

2.1 Mahalanobis Distance : (McLachlan 1999)-(Mishra 1994)

Mahalanobis proposed this measure in (1936) to detect contaminated or outlier data points in linear regression model . His measure has played an important role in statistics and data analysis .
The generalized distance can defined as follows :

$$D_i = \sqrt{\left(y_i - E(y_i)\right)' S^{-1}(y_i - E(y_i))} \qquad \text{.......... (2.1)}$$

Where :
S : covariance matrix .
We will reject the null hypothesis; and the observation will be outlier when :

$$D_i^2 > \chi^2_{(n-p,\alpha)}$$

In the linear regression model can compute the distance as the following equation :

$$D_i = \sqrt{\left(\hat{y}_i - \bar{\hat{y}}\right)' S^{-1}(\hat{y}_i - \bar{\hat{y}})} \qquad \text{.......... (2.2)}$$

Where :
$\hat{y}_i$ **: forecasting value .**
$\bar{\hat{y}}$ **: forecasting values mean .**
We can compute Mahalanobis distance in many statistical packages like SPSS .

2.2 Cook's Distance : (Cook 1979)

In (1979) Cook presented a method to detect the influential observation in multiple linear regression which is based on the measure of the distance between ($\hat{\beta}$) and ($\hat{\beta}_i$) as follows :

$$D_i = \frac{(\hat{\beta}_i - \hat{\beta})' X'X(\hat{\beta}_i - \hat{\beta})}{pS^2} \le F_{(p,n-p,1-\alpha)} \qquad \text{........ (2.3)}$$

Where :

$\hat{\beta}$: denotes the least square estimate of (β) .

$\hat{\beta}_i$: denotes the least squares estimate of (β) with the (ith) point deleted.

$$S^2 = \frac{\hat{e}'\hat{e}}{n-p} \quad \text{........ (2.4)}$$

If the (D_i) > $F_{(p,n-p,1-\alpha)}$; then the (ith) single – row is an outliers .

2.3 Serbert, Montgomery and Rollier Procedure (1998) :

(Adnan et al, 2003)

They considered a procedure to identify the outliers in multiple linear regression by using the (OLS) method and the single linkage clustering method , where :

The cluster analysis is a method for detecting a natural groupings of items or variables where the items show a high internal homogeneity and low external homogeneity. It includes two groups: hierarchical and non-hierarchical,where the hierarchical method divided into two types :

i. Agglomerative hierarchical method :

It starts with (n) clusters and ends with one cluster which contains all of the data points . (This was conducted by Serbert and et al) .

ii. Divisive hierarchical method :

It starts with one cluster and ends up with (n) clusters with each cluster contains one data point .

The single linkage clustering method : It is a method that depends on the smallest distance between a data point in the first cluster and a point in the second cluster .

Serbert et al method depends on the following steps :

i. **Find the standardized predicted values (depending on the OLS) .**
ii. **Grouping the data set by using the single linkage clustering algorithm (Agglomerative hierarchical method) with Euclidean distance between pairs of standardized predicted values , and this can be graphically shown in the form of a dendogram or tree diagram .**
iii. **Number of the clusters depend on the height of the cut (stopping rule) ; which determine as the following equation :**

$$ch = \bar{h} + ks_h \quad \text{......... (2.5)}$$

Where :

$\bar{h}$: **Average height of the tree .**

k : constant .

s_h : **The standard deviation of the heights .**

iv. **The clean data set is the largest cluster formed. It includes the median , and the other clusters contains the outliers .**

2.4 Adnan , Mohamad and Setan Procedure : (Adnan et al 2003)

Adnan et al (2003) proposed a modified procedure of Serbert et al, where they used the robust fit (least trimmed of squares (LTS) instead of the ordinary least squares (OLS) fit; then they applied the backward steps of (Serbert and others) procedure, depending on the standardized predicted values or the residual values .

The (LTS) : It is a method of robust regression estimate proposed by Rousseeuw (1984). It minimizes the sum of squared residuals by selecting smallest (m) of residual and (n-m) residuals are deleted ; and then find the estimators, depending on the (m) observations which satisfy the objective function as the following :

$$Min._{\hat{\beta}} \sum_{i=1}^{m} e_i^2$$ **(Rousseeuw 1984)**

Where :

m = (n/2) + ((p+1)/2) ………. (2.6)

The (LTS) has a high breakdown point of up to (50%) ; which is the highest possible value . (Georgiev 2008)

Breakdown point : It is the smallest part of unusual data that can cause to false the estimator .

3. Application :

We will apply a aforementioned methods to detecting the outliers : Mahalanobis , Cook , Serbert and Adnan ; and we will use three measures for comparison : masking ; swamping and standard error estimate (which calculate after delete the outliers), we study one of an important disease that infects the people; which is called (Hepatitis Disease). It will represent the dependent variable . This disease depends on four tests to detecting it :

i. Glutamate Oxaloacetate Transaminase (G.O.T) .
ii. Glutamate Pyruvate Transaminase (G.P.T) .
iii. Total Serum Bilirubin (T.S.B) .
iv. Alkaline Phosphatase (Alk) .

They will represent the independent variables .

In this paper, we will study several sample sizes , small (n=25) ; medium (n=50) ; large (n=150) and for two cases : firstly real values of observations; and secondly, after adding (10%) of outliers to this observations .

i. (n = 25) :

We will apply the four detecting methods to the real observations which appear in (table 3.1), and to the observations after adding the (3) outliers in the No. of (5,10,15) . The results for Mahalanobis and Cook distances are shown in the table (3.2) .

The dendrograms for Serbert and Adnan for the real observations are shown in figures (3.1),(3.2), respectively; and the dendrograms for the observations after adding the outliers are shown in figures (3.3),(3.4), respectively .

Table (3.1) Independent variables (n=25)

No.	Diseased	G.O.T.	G.P.T	Alk	T.S.B
١	no	١٣	١٢	٦٣	٦.٨
2	yes	٤٠	٤٤	٥١٠	٧٣.٥
٣	no	١٦	١٤	١٠٨	٨.٩
٤	yes	٢٠٠	٣٦٠	١٨٩	١٥٣.٠
٥	yes	١٠٠	٣٥٦	١٨٣	٨٢.٠
٦	no	١٨	٢٠	٩٩	٦.٨
٧	yes	٧٥	٦٥	٢٤٣	٩٨.٠
٨	no	١٩	١٨	٧٨	٥.١
٩	yes	١٠٠	٢٣٢	٢٧٩	٢٠٢.٠
١٠	yes	٩٠	٩٥	٥١٠	٦١.٠
١١	yes	٩٦	٢٧٢	٢٥٢	٣٤٢.٠
١٢	no	١٥	١٤	٦٠	٥.٣
١٣	no	٦	٨	٩٠	٨.٠
١٤	yes	٨٠	٢١٦	٢٣٤	١٣٦.٠
١٥	yes	٣٩	٢٨٠	٢٤٦	٣٤.٢
١٦	no	١٨	١٦	١٣٧	٨.٥
١٧	no	٥	٧	١٥١	٣.٤
١٨	yes	١٤٨	٣٢٨	٢٦٧	٥١.٣
١٩	yes	٧٦	٢٨٠	٤٩٥	٧٦.٩
٢٠	no	٤	٧	٤٠	٦.٨
٢١	no	١٧	١٦	١٤٤	٥.١
٢٢	yes	١٠٠	١٧٢	٢٧٠	٣٤٢.٠
٢٣	yes	١٣٣	٣١٢	٢٢٨	٣٥.٠
٢٤	yes	٨٠	٣٤٤	٢٨٨	٤٩.٥
٢٥	yes	٣٥	٣٥	٢٤٥	١١٧.٩

Reference : Educational Babylon Hospital for Women and Children

Table (3.2) Mahalanobis and Cook distance (n=25)

No.	Before		After	
	Mahalanobis	Cook	Mahalanobis	Cook
١	١١.٩٩٨٥٧	٠.٠٣٠٦٢	١٢.٠٠٢٢٢	٠.٠٠٤١٠
٢	١.٢٧١٤١	٠.٠٨٦٦٨	١.٢٧٠٥٢	٠.٠٠٤٤٩
٣	١.٠١٦٣٣	٠.٠٥٣٥٣	١.٠٠٥١٤	٠.٠٠١١٦
٤	٦.٠٠٠٢٨	٠.٠٦٦٣٥	٦.٠٣٨٣٢	٠.٠٠٠٢٠
٥	٤.٩١٩٨١	٠.٠١٤٧٥	٥.١٤٥٩٣	٠.٣٣٨٩٧
٦	٠.٧٠٨٢٤	٠.٠٤٤٩٢	٠.٧٠٠٢٤	٠.٠٠٠١٥
٧	٢.٧٦٩٧٠	٠.٠٠٩٩٩	٢.٧٧٨٧٣	٠.٠٠٠٠٧
٨	١.١٤٧٨٨	٠.٠٢١٥٩	١.١٧٦٦٠	٠.٠٠٩٦٣
٩	٣.٦٩٣٢٥	٠.٠٠١٨٠	٣.٧٢١٠٤	٠.٠٠٣٨٧
١٠	٧.٤٨١٦٦	٠.٠٩٥٠٩	٧.٢٧١٦١	٠.٤٣٤٩٩
١١	٠.٧٢٧٤٨	٠.٠٣١٤١	٠.٧١٠٥٦	٠.٠٠١١٧
١٢	٢.٧٧١٧٣	٠.٠٠١٣٤	٢.٧٦٤٤١	٠.٠٠١٣٠
١٣	٢.٦٨٨٢٥	٠.٠١١٩٣	٢.٦٩٧٨٠	٠.٠١٨٧٦
١٤	٤.٣٣١٨٥	٠.٠٢٤٥٣	٤.٢٣٤٠٧	٠.٠٤١٢٦
١٥	١.٢٠٨٣٠	٠.٠١٥٨٦	١.١٦٢٨٥	٠.١٥٩٨٢
١٦	١٠.٦٨٥١٣	٠.١٤١٢٥	١٠.٧٣٧٣٥	٠.٠٢٧٤٧
١٧	١٣.٤٨٨١٢	٠.٠٠٠٠١	١٣.٥٣٢١٤	٠.٠٠١٩٣
١٨	١.٢٢٢٧٥	٠.٠١٦٠٥	١.٢٤٢٦٥	٠.٠٠٤٨١
١٩	١.٢١٧٦٦	٠.٠١٥١٧	١.٢٣١١٣	٠.٠٠٣٨٩
٢٠	٢.٠٩١٣٢	٠.٠١٠٠٠	٢.٠٩١٣٠	٠.٠٠٠٣٢
٢١	١.٢٥٠٧٨	٠.٠١٧٠١	١.٢٥٢٦٢	٠.٠٠٣٤٩
٢٢	٠.٨٣٥٣٢	٠.٠٠٨٥٦	٠.٨٧١٧٤	٠.٠٠١٤٣
٢٣	١.٤٠٢٩٥	٠.٠١٤٨١	١.٤٠٦٢٠	٠.٠٠١٩٥
٢٤	٩.٨٠١٧٣	٠.٠١٤٩٠	٩.٦٧٣٧٦	٠.٩٠٩٣٥
٢٥	١.٢٦٩٤٧	٠.٠١٤٥٩	١.٢٨١٠٦	٠.٠٠٣٤٧

Figure (3.1) Serbert before (n=25)

Rescaled Distance Cluster Combine

```
  C A S E   0        5        10        15        20        25
  Label  Num  +---------+---------+---------+---------+---------+
         1   -+
        12   -+
         8   -+
        13   -+
         3   -+
         6   -+
        16   -+-----------------------------------------------+
        21   -+                                   |
        17   -+                                   |
        20   -+                                   |
         7   ---+-------+                         |
        25   ---+       |                         |
         5   -+         |                         |
        23   -+         |                         |
         9   -+         |                         |
        18   -+         +-------------------------------------+
        24   -+         |
         2   -+         |
        10   -+         |
        22   -+---+     |
         4   -+   |     |
        11   -+   +-----+
        14   -+   |
        15   -+   |
        19   -----+
```

Figure (3.2) Adnan before (n=25)

Rescaled Distance Cluster Combine

```
 C A S E      0    5       10       15       20      25
Label  Num  +-----+-------+-------+-------+-------+

       9   -+
      22   -+
       4   -+
      18   -+
      10   -+
      24   -+
       2   -+
       5   -+
      15   -+-+
      23   -+ +-------------------+
      14   -+ |             +-----------------+
      11   ---+             |            +-------+
      19   -----------------------+             |      |
       7   -+---------------------------------+      |
      25   -+                                  |
      16   -+                                  |
      17   -+                                  |
      21   -+                                  |
       3   -+--------------------------------+
       6   -+
       1   -+
      12   -+
       8   -+
      13   -+
      20   -+
```

Figure (3.3) Serbert after (n=25)

Rescaled Distance Cluster Combine

```
 C A S E       0        5       10       15        20        25
 Label  Num  +-------+-------+-------+--------+--------+

       4   -+
       7   -+
      17   -+
       1   -+
       6   -+
       8   -+
       3   -+
       9   -+
      11   -+
      22   -+
      12   -+-+
      21   -+ |
      25   -+ |
      15   -+ |
      19   -+ +---------------------------+
      16   -+ |                   |
      18   -+ |                   |
      23   -+ |                   +-----------------+
       2   -+-+                   |            |
      20   -+                     |            |
      13   ---+---------------------------+           |
      14   ---+                                |
       5   ---------------+-+                         |
      10   ---------------+ +------------------------------+
      24   -----------------+
```

Figure (3.4) Adnan after (n=25)

Rescaled Distance Cluster Combine

```
 C A S E      0        5       10       15        20       25
Label  Num  +-------+ -------+-------+--------+--------+

      9  -+
     16  -+
     12  -+-+
      1  -+ |
     22  -+ |
      4  -+ |
     11  -+ |
      7  -+ |
     17  -+-+-+
      3  -+ | |
      6  -+ | |
      8  ---+ |
     21  -+   +--------------------------+
     25  -+   |                          |
     19  -+   |                          |
     15  -+---+                           |
     18  -+   |                          +---------------+
      2  -+   |                          |               |
     23  -+   |                          |               |
     20  -----+                           |               |
     13  -+                              |               |
     14  -+------------------------------+                |
      5  -+                                              |
     10  -+----------------------------------------------+
     24  -+
```

Table (3.3) shows the summary for the methods and contains (7) columns :

i. (Case) : Represents before and after adding the outliers .

ii. (Method) : includes (non) i.e without depending on any method for detecting outliers , and the methods of detect .

iii. (ch) : calculate the (ch) value by the equation (2.4) for Serbert and Adnan methods .

iv. (Outliers) : The outliers detected by the methods ; where they are all compared with Mahalanobis distance ($\chi^2_{(20,0.05)}=31.4$) , and Cook distance ($F_{(5,20,0.05)}=2.71$) . Both methods can not detect any outliers in the two cases . Outliers are detected by Serbert and Adnan , depending on (ch) value . Differences are found between them for (before) case, and same results for the (after) case .

v. (Masking) : Mahalanobis and Cook have masking for all the adding outliers ; but Serbert and Adnan have only No. (15) .

vi. (Swamping) : Mahalanobis and Cook did not have swamping ; but Serbert and Adnan have the same swamping in the No. (13,14,24) .

vii. (Std. Error Est.) : Adnan has (0.09) ; which is less than others for (before) case , and for the (after) case Serbert and Adnan have (1.2) which is less than others .

Table (3.3) Summary (n=25)

case	method	ch	outliers	masking	swamping	Std. Error Est.
before	none	-	-	-	-	0.2
	Mah.	-	-	-	-	0.2
	Cook	-	-	-	-	0.2
	Serbert	8.6	(16,17,20,21)	-	-	0.2
	Adnan	11.1	(3,7,16,17,19, 21,25)	-	-	0.09
after	none	-	-	-	-	1.58
	Mah.	-	-	5,10,15	-	1.58
	Cook	-	-	5,10,15	-	1.58
	Serbert	9.9	(5,10,13,14,24)	15	(13,14,24)	1.2
	Adnan	9.9	(5,10,13,14,24)	15	(13,14,24)	1.2

ii. (n = 50) :

We will apply the detecting methods to the real observations which are shown (table 3.4) , and to the observations after adding the (5) outliers in the No. of (1,10,20,35,45) . The results for Mahalanobis and Cook distances are shown in the table (3.5) ; the dendrograms for Serbert and Adnan for the real observations are shown in figures (3.5),(3.6), respectively. The dendrograms for the observations after adding the outliers will are shown in figures (3.7),(3.8), respectively .

Table (3.4) Independent variables (n=50)

No.	Diseased	G.O.T.	G.P.T	Alk	T.S.B
1	yes	٥٥	١٢٩	٢٣٨	٣٤٢.٠
2	yes	٢٩	٨٠	٧٢	٢٠.٥
3	yes	٢٤	٢٥	١٨٠	١٢٠.٠
4	yes	٢٢	٢٤٨	١٩٨	٤٧.٨
5	yes	٥٠	٨٩	٣٧٠	٣١.٠
6	yes	٢٧	٣٧	١٦٠	٣٢.٥
7	yes	٥١	٢٧٠	٢٠٧	٧١.٥
8	no	٥	٤	١٣٦	٥.٨
9	yes	٦٢	٣١٠	٢٥٤	٢٤.٠
10	yes	٣٧	٢٦٢	٤٨١	٨١.٠
11	yes	٣١	٥٠	١٩٩	٥١.٣
12	yes	٩٦	٣٣٢	٢٥٨	٨٣.٣
13	yes	٧٨	٣٣١	٣٤٨	٨٥.٥
14	yes	١١٠	٣١٨	٣٥١	٦٣.٣
15	no	١٥	١١	٩٠	٣.٤
16	yes	١٠٤	٣١٢	٢٤٨	٣٤٢.٠
17	yes	١٥٢	٣١٠	١٩٩	٦٥.٠
18	no	٦	٨	١٠٠	٣.٤
19	no	٩	٨	٩٢	٥.١
20	no	٥	٥	٣٦	٨.٥
21	no	١٩	١٣	٨٨	٦.٨
22	yes	٣٣	١٦٨	٢٤٣	٨٥.٥
23	no	١٧	١٦	٦٩	٥.٣
24	yes	٣٧	٨٠	٤٩٠	١٠٦.٠
25	no	٩	٥	٨٧	٨.٣
26	no	١٣	١٢	٦٣	٦.٨
27	yes	٤٠	٤٤	٥١٠	٧٣.٥
28	no	١٦	١٤	١٠٨	٨.٩
29	yes	٢٠٠	٣٦٠	١٨٩	١٥٣.٠
30	yes	١٠٠	٣٥٦	١٨٣	٨٢.٠
31	no	١٨	٢٠	٩٩	٦.٨
32	yes	٤٠	٣٥	٢٤٣	٩٨.٠
33	no	١٩	١٨	٧٨	٥.١
34	yes	١٠٠	٢٣٢	٢٧٩	٢٠٢.٠
35	yes	٩٠	٩٥	٥١٠	٦١.٠

36	yes	٩٦	٢٧٢	٢٥٢	٣٤٢.٠
37	no	١٥	١٤	٦٠	٥.٣
38	no	٦	٨	٩٠	٨.٠
39	yes	٨٠	٢١٦	٢٣٤	١٣٦.٠
40	yes	٣٩	٢٨٠	٢٤٦	٣٤.٢
41	no	١٨	١٦	١٣٧	٨.٥
42	no	٥	٧	١٥١	٣.٤
43	yes	١٤٨	٣٢٨	٢٦٧	٥١.٣
44	yes	٧٦	٢٨٠	٤٩٥	٧٦.٩
45	no	٤	٧	٤٠	٦.٨
46	no	١٧	١٦	١٤٤	٥.١
47	yes	١٠٠	١٧٢	٢٧٠	٣٤٢.٠
48	yes	١٣٣	٣١٢	٢٢٨	٣٥.٠
49	yes	٨٠	٣٤٤	٢٨٨	٤٩.٥
50	yes	٣٥	٣٥	٢٤٥	١١٧.٩

Reference : Educational Babylon Hospital for Women and Children .

Table (3.5) Mahalanobis and Cook distance (n=50)

No.	Before		After	
	Mahalanobis	Cook	Mahalanobis	Cook
١	١١.٥٥٢٤٦	٠.٠٠٨٠٤	٢٢.٦٠٢٣١	٠.٨٢٨٩٤
٢	١.٢٦٦٠٥	٠.٠٧٦٠٥	١.٠٣٦٩٠	٠.٠٠٠٠٠
٣	٠.٨٣٩٩٧	٠.٠٤٥٠٣	٠.٨٣٨٢٧	٠.٠٠٠١٥
٤	٦.٠٤٧٩٨	٠.٠٣٩٨٤	٤.٧٩٧٩٠	٠.٠٠١١٩
٥	٣.٣٤١٣٠	٠.٠١٢١٨	٣.١٢٤٤٨	٠.٠٠٣٠٥
٦	٠.٦١٤٩١	٠.٠٣٩٦٢	٠.٦٢٢٥٠	٠.٠٠٠٠٦
٧	٣.١٨٦٤١	٠.٠٠٩٦٨	٢.٥٢٠٣٩	٠.٠٠٠٠٦
٨	١.١٦٣٧٧	٠.٠٠٧٤٩	١.١٣٤٨٩	٠.٠٠٠٧٤
٩	٤.٢٥٨٧٥	٠.٠٠٢٨٥	٤.٤٦٦٩٤	٠.٠٠٠٦٩
١٠	٨.٥٠٤٨٣	٠.٠٥٦٩١	٨.٣٦٩٤٧	٠.٩٤٠٣٨
١١	٠.٥٠٥٩٣	٠.٠٢٥٨٥	٠.٥٣٥٨٤	٠.٠٠٠٠٥
١٢	٢.٤٧٣٦٨	٠.٠٠٠١٥	٢.٤١٤٢١	٠.٠٠٠٧٨
١٣	٣.٤٩١١١	٠.٠٠٨٤٢	٣.٨٥١٣٨	٠.٠٠٠٥٦
١٤	٣.٠٢٢٦٠	٠.٠٠٦٢١	٣.٠٧٢٠٧	٠.٠٠٤٥٦
١٥	١.٣٠٣٠٧	٠.٠٠٤٣٢	١.٢٥٢٢٦	٠.٠٠٠٥٩
١٦	٩.٩١٩٧٩	٠.٠٥١١٦	٩.٣٥٧١٦	٠.٠١٤٧٧
١٧	٧.٩٠٧٩٦	٠.٠٠٥٥٩	٦.٠٥٢٥٦	٠.٠١٨٢٢
١٨	١.٢٩٣٦٧	٠.٠٠٤٦٥	١.٢١٠٨٩	٠.٠٠٠٨٨
١٩	١.٣٠٨٠٣	٠.٠٠٤١٨	١.٢٣٥١٧	٠.٠٠٠٨٣
٢٠	٢.١٢٥٣٢	٠.٠٠١٠٩	٢.٤٩٣٤٨	٠.١١٢٢٢
٢١	١.٣٢٦٨٧	٠.٠٠٤٦٢	١.٢٨٢٢٤	٠.٠٠٠٥٢
٢٢	١.٢٦٧١٠	٠.٠٠٩٨٩	٠.٨٩٧٣١	٠.٠٠٠٠٨
٢٣	١.٥٠٠٤٦	٠.٠٠٣٤٦	١.٤٢٢٢٩	٠.٠٠٠٧١
٢٤	٧.٤٦٧٦١	٠.٠٠١٤٨	٧.٢٧٥٣١	٠.٠٠١٥٧
٢٥	١.٣٦٢٤٤	٠.٠٠٣٨٢	١.٢٨٩١٢	٠.٠٠٠٩١
٢٦	١.٥٨٧٣٠	٠.٠٠٢٨٣	١.٤٨٩٠٥	٠.٠٠٠٩٥
٢٧	٩.٧٦٧١٧	٠.٠٠٠٠٠	٩.٠٩٦٠٨	٠.٠٠٨٢٦
٢٨	١.٠٩٦٨٥	٠.٠٠٥٨٩	١.٠٦٤٥٦	٠.٠٠٠٥٢
٢٩	١٥.٧٢٣٣٨	٠.٠١٣١٤	١٣.٦٢٩٩٤	٠.٠٦٣٩٥
٣٠	٤.١٤٦٨٩	٠.٠٠١٥٤	٣.٦٠٣٢٤	٠.٠٠٠٦٣
٣١	١.١٣٤٢٥	٠.٠٠٥٦٠	١.٠٨٥٧٧	٠.٠٠٠٥٠
٣٢	١.٥٢٦٠١	٠.٠٣١٠١	١.٥٨٣١٦	٠.٠٠٠١١
٣٣	١.٣٩٣١٢	٠.٠٠٤١٩	١.٣٢٦٧٩	٠.٠٠٠٥٧
٣٤	٢.٣٠٤٥٩	٠.٠٠٠٢٣	٢.٦٨٨٤٦	٠.٠٠٠٠٢
٣٥	١١.٩٩٣٢١	٠.٠١١٤٨	٩.٥٣٢٧٠	٠.٥٨٣٤٢
٣٦	٩.٦٢٢٩٤	٠.٠٢٨١٥	٩.٣٠٦٢٤	٠.٠١٥٤٧
٣٧	١.٦٢٧٤٦	٠.٠٠٢٧٨	١.٥٣٢٣٣	٠.٠٠٠٨٧
٣٨	١.٣٦١٥٣	٠.٠٠٤١٦	١.٢٥١٨٧	٠.٠٠١٠٤
٣٩	٠.٦٦٥٤٠	٠.٠٠١٩١	٠.٧٠٦٣٠	٠.٠٠٠٠٢
٤٠	٥.١٦٣٣٨	٠.٠٠٩٩٤	٤.٩٠٦٨٨	٠.٠٠٠٠٢
٤١	٠.٩٥٨٠٩	٠.٠٠٨٤٥	٠.٩٤٦٤٨	٠.٠٠٠٣٤
٤٢	١.١٥٢١٧	٠.٠٠٩٢٤	١.١٤٥٩١	٠.٠٠٠٦٦
٤٣	٦.٨٤٤٤٤	٠.٠٠٠٨٩	٥.١٣٦٥٦	٠.٠١٧٢٨

٤٤	٥.٧٩٣٤٢	٠.٠٥٦٧٦	٦.٧٣٧١٣	٠.٠٠٥٢١
٤٥	٢.٠٧٦٤٩	٠.٠٠١٣٠	٢.٤٨٢٥٩	٠.١١١٣٩
٤٦	٠.٩٦٥٥١	٠.٠٠٩٠٠	٠.٩٥٨٣٢	٠.٠٠٠٣٢
٤٧	١٠.٣١٩٤٥	٠.٠٠٤٢١	١١.٩١٢٠٧	٠.٠٠٩٩٤
٤٨	٥.٥٦٠٨٧	٠.٠٠٢٧٢	٤.٢٤٢٠١	٠.٠١١١٦
٤٩	٤.٠١٦٦٩	٠.٠٠٠٩٣	٤.٢٨٥٨٥	٠.٠٠١١٨
٥٠	٢.١٤٧٣٠	٠.٠٢٧٠٢	٢.١٩٤٣٩	٠.٠٠٠٠٠

Figure (3.5) Serbert before (n=50)

```
Rescaled Distance Cluster Combine

 C A S E      0         5        10        15        20        25
Label   Num   +---------+---------+---------+---------+---------+

        24    -+
        49    -+
        34    -+
        29    -+
        35    -+
        12    -+-+
        43    -+ |
         1    -+ +-+
        27    -+ | |
        13    -+-+ |
        14    -+ | |
        47    ---+ +---+
         4    -+---+   |
        22    -+   |   |
        39    -+   |   |
        40    -+   |   |
         9    -+-+ |   |
        48    -+ | |   +---------------------------------------+
       17   -+ | |   |                                       |
        30    -+ +-+   |                                       |
         7    ---+     |                                       |
         5    ---+     |                                       |
        10    -+-+     |                                       |
        16    -+ +-+   |                                       |
        36    ---+ +---+                                       |
        44    -----+                                           |
        20    -+---+
|
        45    -+   |                                           |
        41    -+   |                                           |
        42    -+   |                                           |
         2    -+   +---------------+                           |
        46    -+   |               |                           |
        26    -+   |               |                           |
        37    -+   |               |                           |
        18    -+---+               |                           |
        21    -+                   |                           |
        19    -+                   |                           |
        38    -+                   +-------+                   |
        15    -+                   |       |                   |
        33    -+                   |       |                   |
        25    -+                   |       |                   |
        23    -+                   |       |                   |
        28    -+                   |       +-------------------+
        31    -+                   |       |
         8    -+                   |       |
         6    ---------------------+       |
         3    -+---------------------+     |
        11    -+                     +---+
        32    -+---------------------+
        50    -+
```

Figure (3.6) Adnan before (n=50)

```
Rescaled Distance Cluster Combine

   C A S E    0       5      10       15     20       25
  Label  Num  +--------+--------+--------+--------+---------+

          15   -+
          19   -+
          38   -+
          21   -+
          25   -+
          33   -+
          18   -+
          23   -+
          26   -+
          37   -+
          28   -+
          31   -+-+
           2   -+ |
          46   -+ |
          41   -+ +-----+
          42   -+ |     |
           8   -+ |     +-+
          20   -+-+     | |
          45   -+       | |
           6   ---------+ |
           3   -+---------+-------------------------------------+
          11   -+         |                                     |
          32   -+---------+                                     |
          50   -+                                               |
          10   ---+-------+                                     |
          44   ---+       |                                     |
          14   -+         |                                     |
          36   -+---+     +------------------------------
-------+
          13   -+   |     |
          16   -+   |     |
           4   -+   |     |
           5   -+---+-----+
          22   -+   |
           7   -+   |
          39   -+   |
           9   -+   |
          30   -+   |
          27   -+   |
          17   -+   |
          40   -+   |
           1   -+---+
          48   -+
          34   -+
          47   -+
          24   -+
          43   -+
          29   -+
          12   -+
          35   -+
          49   -+
```

Figure (3.7) Serbert after (n=50)

```
Rescaled Distance Cluster Combine

   C A S E     0         5        10        15        20        25
  Label   Num  +---------+---------+---------+---------+---------+

           25   -+
           50   -+
           18   -+
           19   -+
           26   -+
            2   -+
            8   -+
           37   -+
           34   -+
           42   -+
           23   -+
           31   -+
           33   -+
           15   -+
           28   -+
           39   -+
           21   -+
            6   -+
           32   -+
           41   -+
           11   -+
           46   -+
           38   -+
           40   -+
            9   -+
           13   -+
           30   -+
           24   -+
            3   -+
           12   -+
           49   -+
           20   -+
           22   -+
           45   -+
            7   -+
            4   -+
           27   -+
           44   -+
            5   -+
           14   -+
           16   -+
           36   -+
           10   -+-----------------------------------------------+
           47   -+                                               |
           29   -+                                               |
           43   -+                                               |
           35   -+                                               |
           17   -+                                               |
           48   -+                                               |
            1   -------------------------------------------------+
```

Figure (3.8) Adnan after (n=50)

```
    Rescaled Distance Cluster Combine

   C A S E     0       5       10       15      20       25
  Label   Num  +-------+-------+-------+-------+-------+

          4   -+
         12   -+
         49   -+
          7   -+
         22   -+
         40   -+
         30   -+
         14   -+
         27   -+
          9   -+
         50   -+
         10   -+
         39   -+-+
         13   -+ |
         44   -+ |
         34   -+ |
         24   -+ |
          8   -+ |
         42   -+ |
          2   -+ |
          3   -+ |
         41   -+ |
         46   -+ |
         38   -+ +---------------------------------------------+
         19   -+ |                                             |
         25   -+ |                                             |
         28   -+ |                                             |
         31   -+ |                                             |
         18   -+ |                                             |
         20   -+ |
|
         45   -+-+                                             |
         23   -+ |                                             |
         33   -+ |                                             |
         37   -+ |                                             |
         15   -+ |                                             |
         26   -+ |                                             |
         21   -+ |                                             |
          6   -+ |                                             |
         32   -+ |                                             |
         35   -+ |                                             |
         29   -+ |                                             |
         17   -+-+                                             |
         48   -+                                               |
         11   -+                                               |
          5   -+                                               |
         43   -+                                               |
         16   -+---------+                                     |
         36   -+         +-----------------------------------+ |
         47   -----------+                                   +-+
          1   -----------------------------------------------+
```

Table (3.6) below shows the summary for the methods where they are all compared with Mahalanobis distance ($\chi^2_{(٤٥,0.05)}$=٦1.4) , and Cook distance ($F_{(5,٤٥,0.05)}$=2.٤٢) . Both methods can not detect any outliers in the two cases . Outliers are detected by Serbert and Adnan , depending on (ch) value . Differences are found between them for (before) case, and same results for the (after) case .

Mahalanobis and Cook have masking for all the adding outliers ; but Serbert has only two in the No. (20,45) ; and Adnan has in the No. (10,20,35,45)

Mahalanobis and Cook did not have swamping ; but Serbert has in the No. (17,29,43,47,48) ; and Adnan has in the No. (5,11,16,36,43,47,48) .

Serbert has (0.23),(2.14) Std. Error Est. before and after cases, respectively ; which are less than others .

Table (3.6) Summary (n=50)

case	method	ch	outliers	masking	swamping	Std. Error Est.
Before	none	-	-	-	-	0.26
	Mah.	-	none	-	-	0.26
	Cook	-	none	-	-	0.26
	Serbert	7.7	(3,6,8,11,15,19,21,23,25,28, 31,32,33, 38)	-	-	0.23
	Adnan	6.3	(2,3,6,8,11,15,18,19, 20,21,23,25,26,28, 31, 32,33,37, 38,41,42,45, 46)	-	-	-
After	none	-	-	-	-	3.32
	Mah.	-	-	1,10,20,35,45	-	3.32
	Cook	-	-	1,10,20,35,45	-	3.32
	Serbert	11.6	(1,10,17,29,35, 43,47, 48)	20,45	17,29,43, 47,48	2.14
	Adnan	8.4	(1,5,11,16,36, 43,47, 48)	10,20,35, 45	5,11,16,36,43,47 ,48	3.46

iii. (n = 150) :

The observations appear in (table 3.7) ; and adding (15) outliers for the No. of (10,20,35,48,65,70,85,90,100,108,115,125,133,140,148) .

The results for Mahalanobis and Cook distances are shown in table (3.8) ; the dendrograms for Serbert and Adnan for the real observations are shown in figures (3.9),(3.10), respectively ; and the dendrograms for the observations after adding the outliers are shown in figures (3.11),(3.12), respectively .

Table (3.7) Independent variables (n=150)

No.	Diseased	G.O.T.	G.P.T	Alk	T.S.B
١	yes	٥٥	١٢٩	٢٣٨	٣٤٢.٠
٢	yes	٢٩	٨٠	٧٢	٢٠.٥
٣	yes	٢٤	٢٥	١٨٠	٢١.٠
٤	yes	٢٢	٢٤٨	١٩٨	٤٧.٨
٥	yes	٥٠	٨٩	٣٧٠	٣١.٠
٦	yes	٢٧	٣٧	١٦٠	٣٢.٥
٧	yes	٥١	٢٧٠	٢٠٧	٧١.٥
٨	no	٥	٤	١٣٦	٥.٨
٩	yes	٦٢	٣١٠	٢٥٤	٢٤.٠
١٠	yes	٣٧	٢٦٢	٤٨١	٨١.٠
١١	yes	٣١	٥٠	١٩٩	٥١.٣
١٢	yes	٩٦	٣٣٢	٢٥٨	٨٣.٣
١٣	yes	٧٨	٣٣١	٣٤٨	٨٥.٥
١٤	yes	١١٠	٣١٨	٣٥١	٦٣.٣
١٥	no	١٥	١١	٩٠	٣.٤
١٦	yes	١٠٤	٣١٢	٢٤٨	٣٤٢.٠
١٧	yes	١٥٢	٣١٠	١٩٩	٦٥.٠
١٨	no	٦	٨	١٠٠	٣.٤
١٩	no	٩	٨	٩٢	٥.١
٢٠	no	٥	٥	٣٦	٨.٥
٢١	no	١٩	١٣	٨٨	٦.٨
٢٢	yes	٣٣	١٦٨	٢٤٣	٨٥.٥
٢٣	no	١٧	١٦	٦٩	٥.٣
٢٤	yes	٣٧	٨٠	٤٩٠	١٠٦.٠
٢٥	no	٩	٥	٨٧	٨.٣
٢٦	no	١٣	١٢	٦٣	٦.٨
٢٧	yes	٤٠	٤٤	٥١٠	٧٣.٥
٢٨	no	١٦	١٤	١٠٨	٨.٩
٢٩	yes	٢٠٠	٣٦٠	١٨٩	١٥٣.٠
٣٠	yes	١٠٠	٣٥٦	١٨٣	٨٢.٠
٣١	no	١٨	٢٠	٩٩	٦.٨
٣٢	yes	٤٠	٣٥	٢٤٣	٩٠.٠
٣٣	no	١٩	١٨	٧٨	٥.١
٣٤	yes	١٠٠	٢٣٢	٢٧٩	٢٠٢.٠
٣٥	yes	٩٠	٩٥	٥١٠	٦١.٠
٣٦	yes	٩٦	٢٧٢	٢٥٢	٣٤٢.٠
٣٧	no	١٥	١٤	٦٠	٥.٣
٣٨	no	٦	٨	٩٠	٨.٠
٣٩	yes	٨٠	٢١٦	٢٣٤	١٣٦.٠
٤٠	yes	٣٩	٢٨٠	٢٤٦	٣٤.٢
٤١	no	١٨	١٦	١٣٧	٨.٥
٤٢	no	٥	٧	١٥١	٣.٤
٤٣	yes	١٤٨	٣٢٨	٢٦٧	٥١.٣
٤٤	yes	٧٦	٢٨٠	٤٩٥	٧٦.٩
٤٥	no	٤	٧	٤٠	٦.٨
٤٦	no	١٧	١٦	١٤٤	٥.١
٤٧	yes	١٠٠	١٧٢	٢٧٠	٣٤٢.٠
٤٨	yes	١٣٣	٣١٢	٢٢٨	٣٥.٠
٤٩	yes	٨٠	٣٤٤	٢٨٨	٤٩.٥
٥٠	yes	٣٥	٣٥	٢٧٦	١١٧.٩
٥١	yes	٩٦	٣٠٦	٢٨٧	٣٧.٦
52	yes	٣١	٣٩	١٩١	٢٨.٠
53	yes	٢٥	٢٠	١٤٧	٤٨.٠
٥٤	yes	٢٩	٩٤	٣٦٧	٢٥.٠
٥٥	no	٥	٤	١١٥	٣.٥
٥٦	no	١٧	١٠	١٢٢	٤.٠
٥٧	no	١١	٤	٩٦	٣.٣
٥٨	no	٥	٥	١١٤	٣.٤
٥٩	yes	٦٥	٥٠	٣٩٨	١٠٩.٤
٦٠	yes	٢٥	١٧٦	٢٩٧	٧٧.٠
٦١	no	١١٠	٤	١٠٠	٣.٣
٦٢	no	٦	٩	١١٧	٨.٥
٦٣	yes	٥٠	٤٨	١٩٦	٢٠٣.٠
٦٤	yes	٤٨	٤٨	١٨٦	٢٠٣.٠

٦٥	yes	٧٠	٨٨	٩٠٠	٦٠.٠
٦٦	no	١٣	٤	١٢٧	٨.٥
٦٧	yes	١٢٤	٣٦٨	١٩٨	٨٣.٠
٦٨	yes	٥١	٢٦٤	٢١٩	١٠٧.٠
٦٩	yes	٢٩	٢٦	١٥٠	٥٨.٠
٧٠	no	٩	٧	٩٤	٨.٣
٧١	yes	٢٥	٣٤	٥٢٥	٩٨.٠
٧٢	yes	٧٤	٢٥١	٥١٧	٢٠٥.٥
٧٣	yes	٥٠	٣٥٠	٤٩٦	٢٢٥.٧
٧٤	yes	١٢١	٢٨٢	٤٠٥	٨٧.٥
٧٥	no	٨	٨	٩٤	٠.٥
٧٦	yes	٢٥٠	٣٥٠	٤٩٦	٢٠٠.٠
٧٧	yes	٢٢٤	٣٠٤	٢٩١	٢٠٥.٢
٧٨	no	٩	٨	٨١	٠.٤
٧٩	no	١١	٧	٦٩	٠.٨
٨٠	yes	٤٤	١٣٦	٣٠٦	١٣.٧
٨١	no	٨	١٦	١١٤	٧.٧
٨٢	no	٢٥	٩	٥٥	٠.٥
83	no	٨	١٠	١٥٦	٠.٦
٨٤	no	٢٧	٩	١٢٦	٠.٩
٨٥	yes	١٤٤	١٨٤	١٠٥	١٣٨.٠
٨٦	no	١٥	٤	١٣٢	٥.٩
٨٧	no	٩	٩	١٠٥	٨.٥
٨٨	yes	١٣٥	٢٤٠	٢٨٠	٥١.٣
٨٩	no	٨	١٢	١٢٣	٦.٨
٩٠	yes	١٢٨	٤٤	٢٠٠	٢٩.٠
٩١	yes	١٤٨	٢٥	١٨٩	٢٠.٥
٩٢	no	١٠	١٤	١١٤	١٣٦.٠
٩٣	yes	٣٤	٣٤	٤٢	١٨.١
٩٤	yes	٢٥٠	٤٢٨	٢٤٦	٢٧.٣
٩٥	yes	١١٤	٢٦٤	٢٢٢	٤٨.٥
٩٦	no	١٧	٨	٥٨	٥.٢
٩٧	no	١٥	١٥	٧٨	٥.٣
٩٨	no	١٥	٤	١٢٠	٣.٥
٩٩	yes	٥٠	١٧٦	٢٩١	٣٨.٥
١٠٠	no	٥	٥	١٠٥	٣.٥
١٠١	no	١٧	٧٤	٩٣	٨.٦
١٠٢	no	١٥	٥	١٢٩	٥.٨
١٠٣	no	١٨	١٩	١٠٥	٣.٨
١٠٤	no	١٣	٧	١٢٠	٨.٥
١٠٥	no	٨	٩	٨٤	٨.٥
١٠٦	no	١٩	٥	١٠٢	٥.١
١٠٧	no	١٠	٢٠	١١٧	٨.٥
١٠٨	yes	٦٠	٧٢	٩٦	٣٢.٥
١٠٩	yes	٨٢	١٣٢	١١٢	٣٨.٧
١١٠	yes	٤١	٧٢	١٠٥	١٩.٣
١١١	yes	١٠٤	٢٧٠	٣٠٠	٧١.٨
١١٢	no	١٥	٥	١٢٠	٨.٥
١١٣	no	٨	١٠	١٢٩	٥.٨
١١٤	no	٩	٤	١٤٠	١٥.٤
١١٥	no	٦	٥	٧٢	١٤.٣
١١٦	no	١٣	١٨	١٢٠	١١.٠
١١٧	yes	١٢٠	٣٣٤	٢٥٠	٥١.٥
١١٨	yes	٦٨	٨٨	٢٦٤	٢٠.١
١١٩	No	١٧	١٨	٩٠	٥.٣
١٢٠	yes	١٢٥	٢٥٦	٤٢٠	٢٨.٨
١٢١	no	٨	٨	١٢٣	٩.٨
١٢٢	no	٥	١٢	٧٢	١٣.٣
١٢٣	yes	١١٢	٣٠٤	٣٥٩	٢٥.٥
١٢٤	yes	٧٠	٣١٢	٣٦٠	٤٥.٥
١٢٥	yes	٤٧	٣٥٢	٢٧٦	١٧٠.٥
١٢٦	no	٨	٨	٥١	٣.٥
١٢٧	yes	٤١	٤١٦	٢٢٨	٢٢٢.٠
١٢٨	no	٦٠	٢٦	٢٩٨	٨.٥
١٢٩	no	٥	٥	١٢٩	٨.٥

١٣٠	yes	٨٥	٤٢٤	٣٩٠	١١٧.٠
١٣١	yes	٣٥	١٢٠	٢٩٠	٣٨.٥
١٣٢	no	١٠	١٨	١٢٠	٨.٥
١٣٣	yes	٣٥	٨٨	١٩٠	٣٨.٥
١٣٤	yes	١٠٤	٤٠٠	٢١٠	٣٦.٨
١٣٥	yes	١٢٥	٤٣٢	٢٠٧	٥٣.٥
١٣٦	yes	٣٣	٣٢٨	٢٦٤	١٨.٥
١٣٧	no	٤	٤	٩٠	٨.٥
١٣٨	no	١٩	٩	١٥٠	١٠.٢
١٣٩	no	١٠	١٠	٥٠	١٣.٣
١٤٠	no	٨	٦	٨٥	١٣.٦
١٤١	no	٩	٥	٦٦	٦.٨
١٤٢	no	٨	٧	١٠٢	٥.١
١٤٣	yes	٣٥	٣٤	١٢٠	٢٩.١
١٤٤	no	١١	٥	٩٠	٥.١
١٤٥	yes	٨٠	٢٣٢	٢٢٥	٦٦.٣
١٤٦	no	٨	٩	١١١	٦.٨
١٤٧	yes	٣٥	٣٠٤	١٥٠	٢٩.٤
١٤٨	yes	٥٢	٣٠	٩٠	٥٣.٠
١٤٩	yes	١٠٠	٢٨٠	٤١٩	١٨.٩
١٥٠	yes	٤٧	٢٠٠	٤٣٠	٢٠.٥

Reference : Educational Babylon Hospital for Women and Children .

Table (3.8) Mahalanobis and Cook distance (n=150)

No.	Before		After	
	Mahalanobis	Cook	Mahalanobis	Cook
١	٢١.٨٩٧٦٨	٠.٠٠١٠٦	١٧.٢٠٧٨٤	٠.٠٠٠٠٣
٢	١.٠٠٩٦٢	٠.٠١٦٠٩	٠.٩٦٢١٠	٠.٠٠٠٠٩
٣	٠.٥٥٣١٩	٠.٠١٠٦٨	٠.٥٧٢٣٧	٠.٠٠٠٠٢
٤	٤.٥١٢٠٩	٠.٠١٠١٩	٤.٢٣٩١٢	٠.٠٠٠٠١
٥	٣.٠٣١٥٥	٠.٠٠٦٠٩	٣.٥٦٥٨٠	٠.٠٠٠٠٠
٦	٠.٣٥٠٢٦	٠.٠٠٨٩٨	٠.٣٢٣٤٧	٠.٠٠٠٠٢
٧	٢.٨٧٨٠٩	٠.٠٠٣٠٦	٣.١٦٣٤٢	٠.٠٠٠٠٥
٨	٠.٨٨٨٦٥	٠.٠٠١١٩	٠.٨١٠١٧	٠.٠٠٠١٣
٩	٤.٥٠٩١٢	٠.٠٠٢٠٩	٤.٧١٩٨٣	٠.٠٠٠٠٩
١٠	٧.٥٤١٢٣	٠.٠٠٠٩١	٢١.٧٣٤٢٨	٠.٥٠٣٦٤
١١	٠.٣٨٩١٦	٠.٠٠٦٨٧	٠.٤٣٢٩٠	٠.٠٠٠٠١
١٢	٢.٨٩٤٠١	٠.٠٠٠٠٠	٣.٣٦٨٧١	٠.٠٠٠١٦
١٣	٣.٥٠٧١٣	٠.٠٠٠٤٩	٤.٠٢١٩٨	٠.٠٠٠٠١
١٤	٢.٩٥٩٧٦	٠.٠٠٠٦٠	٣.٣٣٩١٨	٠.٠٠٠١١
١٥	٠.٨٨٦٤٤	٠.٠٠٠٨٥	٠.٧٨٠٦٩	٠.٠٠٠٠٧
١٦	١٨.٧٧٠٢٤	٠.٠٣٠٩٠	١٥.١٤٣٠٤	٠.٠٠٠١٤
١٧	٥.٧٥٧٨٣	٠.٠٠٠٠٥	٣.٦٧٠٤١	٠.٠٠٠٨٧
١٨	٠.٩٣٧٩٩	٠.٠٠٠٨٠	٠.٨٥١١٩	٠.٠٠٠١٠
١٩	٠.٩٢٥٤٦	٠.٠٠٠٧٧	٠.٨٣٧٧٢	٠.٠٠٠٠٨
٢٠	١.٥٩٦١٥	٠.٠٠٠٣٢	٢٥.٦٧٥٥٤	٠.٣٦٨٩٠
٢١	٠.٨٦٥٧٨	٠.٠٠٠٩٥	٠.٧٣٣٥٠	٠.٠٠٠٠٦
٢٢	١.١٦٢٤١	٠.٠٠٣٥١	١.١٤١٦٣	٠.٠٠٠٠٠
٢٣	١.٠٤١٢١	٠.٠٠٠٧٩	٠.٩١٢٢٣	٠.٠٠٠٠٦
٢٤	٧.٨٨٠٨١	٠.٠٠١٩٢	٨.٩١٢٤٧	٠.٠٠٠٣٩
٢٥	٠.٩٥٧٩٢	٠.٠٠٠٧٣	٠.٨٦٣٣٨	٠.٠٠٠٠٨
٢٦	١.١٣٠٨٢	٠.٠٠٠٦٥	١.٠١٦٨٧	٠.٠٠٠٠٦
٢٧	٩.٦٦٩١١	٠.٠٠٥٢١	١١.١٢٩٥٠	٠.٠٠٠٤٣
٢٨	٠.٧٢٢٧٤	٠.٠٠١١٠	٠.٦٣٦١٩	٠.٠٠٠٠٧
٢٩	١٢.٠٩٨٥٥	٠.٠١٠٧٤	٦.٨٣٠٦١	٠.٠٠٢٣٦
٣٠	٤.٧٩٠١٣	٠.٠٠٠٢٢	٥.٢٧٨١٦	٠.٠٠٠٤٧
٣١	٠.٧٤٨١٧	٠.٠٠١١٠	٠.٦٥٠٢٨	٠.٠٠٠٠٦
٣٢	١.٦٣٩٤٤	٠.٠٠٩٣٢	١.٦٨٢٦٨	٠.٠٠٠٠١
٣٣	٠.٩٤٠٠٠	٠.٠٠٠٩١	٠.٨٠٧٥١	٠.٠٠٠٠٥
٣٤	٤.٦٢١٧١	٠.٠٠٠٢٠	٣.٧٧٥٤٩	٠.٠٠٠٠٨
٣٥	٩.٣١٥٠٥	٠.٠٠٠٢٠	٨.٣٠٢٧٥	٠.١٩٠٨١

٣٦	١٨٫٧٨٤٦٨	٠٫٠١٩١٦	١٤٫٩٤٩١٦	٠٫٠٠٠٠٧
٣٧	١٫١٥٧٧٨	٠٫٠٠٠٦٧	١٫٠٣٢٨٣	٠٫٠٠٠٠٦
٣٨	٠٫٩٧٤٤٢	٠٫٠٠٠٧٥	٠٫٨٨٤٩٨	٠٫٠٠٠٠٩
٣٩	١٫٥٦٩٤٠	٠٫٠٠٠٧٥	١٫٤١٥٣١	٠٫٠٠٠٠٥
٤٠	٤٫٤٨٦٣٤	٠٫٠٠٤٦٠	٤٫٣٨٥٧٢	٠٫٠٠٠٠٢
٤١	٠٫٦٢٩٤٤	٠٫٠٠١٤٩	٠٫٥٧٣٦١	٠٫٠٠٠٠٨
٤٢	٠٫٩٢٦٦٤	٠٫٠٠١٤٦	٠٫٨٤٦٩٧	٠٫٠٠٠١٤
٤٣	٥٫١٣٨٨٦	٠٫٠٠٠٣٧	٣٫٨٧٣٢١	٠٫٠٠٠٦٤
٤٤	٥٫٧٣٣٥٦	٠٫٠٠٣٥٦	٦٫٤٣٢٥٩	٠٫٠٠٠٠٥
٤٥	١٫٥٥٥٠٩	٠٫٠٠٠٣٤	١٫٤٦٤٢٧	٠٫٠٠٠٠٨
٤٦	٠٫٦٦٧٦١	٠٫٠٠١٥٦	٠٫٦١٨٥٤	٠٫٠٠٠٠٩
٤٧	٢٠٫١٩٣٤٩	٠٫٠٠٦٣٤	١٥٫٦٣٢٨٩	٠٫٠٠٠١٠
٤٨	٤٫٦٤٢٤٧	٠٫٠٠٠٢٣	٢٨٫٧٤١٩١	٠٫٦٠٢٣٦
٤٩	٤٫٣٢٣٣١	٠٫٠٠٠٠١	٤٫٧٨١٣٣	٠٫٠٠٠١٠
٥٠	٢٫٩١٢٩٠	٠٫٠١٠٦٠	٢٫٩٤٩٩٧	٠٫٠٠٠٠٠
٥١	٢٫٩٤٨٨٤	٠٫٠٠٠١٤	٣٫٦٥٤٩٣	٠٫٠٠٠٦١
52	٠٫٤٣٢٦٨	٠٫٠٠٨٤٠	٤٫٧٨١٣٣	٠٫٠٠٠١٠
53	٠٫٦٤٣٦٤	٠٫٠١١٧٥	٠٫٥٣٩٥٣	٠٫٠٠٠٠٢
٥٤	٣٫٢٥٧٧٦	٠٫٠٠٨٢٠	٣٫٤٥١٤٤	٠٫٠٠٠٠٢
٥٥	٠٫٩١٦٢٠	٠٫٠٠٠٩١	٠٫٨٢٩٦٨	٠٫٠٠٠١١
٥٦	٠٫٧٣٤٧٥	٠٫٠٠١١٩	٠٫٦٥٣٥٤	٠٫٠٠٠٠٨
٥٧	٠٫٩٠٨٤٧	٠٫٠٠٠٧٨	٠٫٨١٥٢٠	٠٫٠٠٠٠٨
٥٨	٠٫٩١٥٣٤	٠٫٠٠٠٩٠	٠٫٨٢٨٢٦	٠٫٠٠٠١١
٥٩	٥٫٤٧٤٠١	٠٫٠٠٤٦٦	٦٫٣٢٩١٣	٠٫٠٠٠٠٠
٦٠	٢٫١٥٢٧٨	٠٫٠٠٣٦٩	١٫٩٢٨٥٢	٠٫٠٠٠٠١
٦١	٨٫٧٣٠٢١	٠٫٠١٨٥٣	٤٫٥٣٢٠٦	٠٫٠٠٠٠٤
٦٢	٠٫٨٤٣٢٩	٠٫٠٠١٠٤	٠٫٧٦١٣٩	٠٫٠٠٠١١
٦٣	٧٫٧١٠١١	٠٫٠٢٠٠٧	٥٫٩٨٩٩١	٠٫٠٠٠٠٤
٦٤	٧٫٧٥٦٥٥	٠٫٠٢١٨٩	٥٫٩٧٩٢٠	٠٫٠٠٠٠٤
٦٥	٤٠٫٨٠١٠٦	٠٫١٨٠٨٧	١٫٦٦٩٢٤	٠٫٠٤٠٩٩
٦٦	٠٫٧٥٩٣١	٠٫٠٠١١٨	٠٫٦٩٤٤٣	٠٫٠٠٠٠٩
٦٧	٥٫٠٤٥٧٤	٠٫٠٠٠٠٧	٠٫٦٩٤٤٣	٠٫٠٠٠٠٩
٦٨	٢٫٨٥٩٣٠	٠٫٠٠١٨٩	٣٫٠١٢٢٦	٠٫٠٠٠٠٣
٦٩	٠٫٧٠٦٧٨	٠٫٠١١١٨	٠٫٥٥٩٨٦	٠٫٠٠٠٠٣
٧٠	٠٫٩٠١٩٢	٠٫٠٠٠٨١	١٨٫٤٣٩٤٤	٠٫٤٣٩٨٢
٧١	١١٫٢٨٠٨٤	٠٫٠٠٥٥٥	١٢٫٥١٣١٠	٠٫٠٠٠٩٦
٧٢	٨٫٤٨٠٩٧	٠٫٠١٥٩٧	٩٫١١٤٥٥	٠٫٠٠٠٣٣
٧٣	١٢٫٦٨٨٦١	٠٫٠٣٩٤٧	١١٫٥٠٦٦٢	٠٫٠٠٠٨٨
٧٤	٣٫١٧٩٥٤	٠٫٠٠١٧٥	٣٫٥٨١٧٨	٠٫٠٠٠٠٨
٧٥	٠٫٩٥٧٥٤	٠٫٠٠٠٧٤	٠٫٨٧٠١٣	٠٫٠٠٠٠٩
٧٦	٢٠٫٥٢٣٩٢	٠٫٢١٨١٢	١٤٫٧٨٠٩٣	٠٫٠٠٢٧٣
٧٧	١٦٫٩٩٢٨٤	٠٫٠٤٦١٥	٩٫٢٥٣٠١	٠٫٠٠٢٧٢
٧٨	١٫٠٣٤٨٢	٠٫٠٠٠٦٤	٠٫٩٤٢٢١	٠٫٠٠٠٠٨
٧٩	١٫١٢٣٢٤	٠٫٠٠٠٥٧	١٫٠١٦٦١	٠٫٠٠٠٠٧
٨٠	١٫٦٠٣٧٩	٠٫٠٠٥٠٩	١٫٧٢٢٨٦	٠٫٠٠٠٠١
٨١	٠٫٧٩٥٣٦	٠٫٠٠١٠٩	٠٫٧١٨٨٧	٠٫٠٠٠١٠
٨٢	١٫٣٤٣٧٣	٠٫٠٠٠٧٢	١٫٠٦٨٩٩	٠٫٠٠٠٠٤
83	٠٫٨٩٦٥٠	٠٫٠٠١٥٩	٠٫٨٣٠٦٤	٠٫٠٠٠١٤
٨٤	٠٫٨٤٦٧٨	٠٫٠٠١٤٨	٠٫٦٨٦١٣	٠٫٠٠٠٠٦
٨٥	٨٫٥٤٦٩٨	٠٫٠٠٨٠٤	٣٠٫٥٥٦٧٨	٠٫٤٩٢٣٦
٨٦	٠٫٧٦٤٣٩	٠٫٠٠١٢٥	٠٫٧٠٠٦٢	٠٫٠٠٠٠٩
٨٧	٠٫٨٣٢٧٧	٠٫٠٠٠٩٣	٠٫٧٥٢٢٠	٠٫٠٠٠٠٩
٨٨	٣٫٧٤٠٨٥	٠٫٠٠٠٢٥	٢٫٣٦٣٩٣	٠٫٠٠٠٣٤
٨٩	٠٫٧٨٩٧٩	٠٫٠٠١١٤	٠٫٧١٦٢٥	٠٫٠٠٠١٠
٩٠	٨٫٩٠٦٥٧	٠٫٠٣٣٠١	١٣٫٩٢٢١٨	٠٫١٩١٧٨
٩١	١٤٫٠٧٢٤٢	٠٫٠٥٧٥٨	٧٫٩٤٦١٣	٠٫٠٠٢٢٣
٩٢	٤٫٤٧٧١٨	٠٫٠١٠٧٥	٣٫٤٦٥٩١	٠٫٠٠٠٤٢
٩٣	١٫٤٩١٤٦	٠٫٠٢٥٣٣	١٫١١٧٦٢	٠٫٠٠٠١٥
٩٤	٢٢٫٧٧٠٣٤	٠٫٠٤٧٢٥	١٣٫٨٢٢٨٣	٠٫٠٠٨٤٠
٩٥	٢٫٤٨٥٦٠	٠٫٠٠٠٩٩	١٫٩٠٩٦٣	٠٫٠٠٠٢٧
٩٦	١٫٢٠٤٥٥	٠٫٠٠٠٦٢	١٫٠٤١٩٨	٠٫٠٠٠٠٥
٩٧	٠٫٩٥٨٠٠	٠٫٠٠٠٨١	٠٫٨٤٩٥٠	٠٫٠٠٠٠٦
٩٨	٠٫٧٩٦٠٠	٠٫٠٠١٠٨	٠٫٧١٤٣٥	٠٫٠٠٠٠٨
٩٩	٠٫٩٩٧١٨	٠٫٠٠٢٤٦	١٫٢٠٣٨١	٠٫٠٠٠٠١

١٠٠	٠٫٩٤٣٠٦	٠٫٠٠٠٨١	٢٫١٤١٧٥	٠٫٠٣٩٢٥
١٠١	٠٫٩٧٣٣١	٠٫٠٠٢١٢	٠٫٩٦١٢٩	٠٫٠٠٠٠٨
١٠٢	٠٫٧٥٧١٥	٠٫٠٠١٢٢	٠٫٦٨٩٧٨	٠٫٠٠٠٠٩
١٠٣	٠٫٧٤٠٤٤	٠٫٠٠١١٢	٠٫٦٤٥٥٤	٠٫٠٠٠٠٧
١٠٤	٠٫٧٤٧٢٠	٠٫٠٠١١٢	٠٫٦٧٥٧٨	٠٫٠٠٠٠٩
١٠٥	٠٫٩٨٠٦١	٠٫٠٠٠٧٤	٠٫٨٩٠١٦	٠٫٠٠٠٠٩
١٠٦	٠٫٨٤٤٧٥	٠٫٠٠٠٩٨	٠٫٧١٦٠١	٠٫٠٠٠٠٦
١٠٧	٠٫٧٣٣١٤	٠٫٠٠١٢٠	٠٫٦٦٣٤٤	٠٫٠٠٠٠٩
١٠٨	١٫٢٠٣٥٨	٠٫٠١٣٨١	٠٫٨٩١٢٦	٠٫٠٣٢١٢
١٠٩	١٫٦٧٤٥٤	٠٫٠١٠٣١	٠٫٧٢٢٧٠	٠٫٠٠٠١٨
١١٠	٠٫٥٨٣٦٢	٠٫٠١٠٩٨	٠٫٣٩٠١١	٠٫٠٠٠٠٧
١١١	١٫٦٠١٥١	٠٫٠٠٠٠٢	١٫٧٥٠٠١	٠٫٠٠٠١٠
١١٢	٠٫٧٥٢٠٠	٠٫٠٠١١٣	٠٫٦٧٤٣٥	٠٫٠٠٠٠٨
١١٣	٠٫٧٩٨٣٠	٠٫٠٠١١٨	٠٫٧٢٧٣٢	٠٫٠٠٠١١
١١٤	٠٫٧٧٥٠٢	٠٫٠٠١٣٨	٠٫٧٢٠٨٠	٠٫٠٠٠١٢
١١٥	١٫١٢٧١٤	٠٫٠٠٠٦٤	١١٫١٣٤٤٨	٠٫١٥٧١٩
١١٦	٠٫٦٦٩٧٢	٠٫٠٠١٢٥	٠٫٦٠٤٨٢	٠٫٠٠٠٠٩
١١٧	٣٫٧٧٥٢٢	٠٫٠٠٠٠٠	٠٫٦٠٤٨٢	٠٫٠٠٠٠٩
١١٨	١٫٤٣٦١٠	٠٫٠٠٦٦٠	١٫٣٣٥٢٨	٠٫٠٠٠٠٦
١١٩	٠٫٨٣٤٩٩	٠٫٠٠٠٩٧	٠٫٧٣٠٢٤	٠٫٠٠٠٠٦
١٢٠	٥٫٥٢١١٤	٠٫٠٠٠٧٩	٥٫٣٩٤٦٨	٠٫٠٠٠١٩
١٢١	٠٫٧٩٠٩٥	٠٫٠٠١١٢	٠٫٧١٨٩٦	٠٫٠٠٠١١
١٢٢	١٫١٣٦٤٦	٠٫٠٠٠٧٠	١٫٠٤١١٧	٠٫٠٠٠٠٩
١٢٣	٤٫١٦٢٤٦	٠٫٠٠٠٢٠	١٫٠٤١١٧	٠٫٠٠٠٠٩
١٢٤	٤٫١٧٨٦٧	٠٫٠٠٠٠١	٤٫١٩٦٨٣	٠٫٠٠٠١٧
١٢٥	٨٫٤٨٩٥١	٠٫٠٠٠٨١	٣٫٣١٥٢٩	٠٫٠٦٨٠٧
١٢٦	١٫٣٤٥٧٧	٠٫٠٠٠٤٣	١٫٢٤٧٨٦	٠٫٠٠٠٠٧
١٢٧	١٦٫٨٩٩٣٣	٠٫٠٠٧٢٤	١٫٢٤٧٨٦	٠٫٠٠٠٠٧
١٢٨	٣٫٤١٨٢٠	٠٫٠١٨٩٢	٣٫٤٩٥٠٧	٠٫٠٠٠٢٩
١٢٩	٠٫٨٦٤٥٨	٠٫٠٠١١٣	٠٫٧٨٣٩٦	٠٫٠٠٠١٢
١٣٠	٧٫٢٨٨٦٣	٠٫٠١٢٤٩	٧٫٨٥٧٠٩	٠٫٠٠٠٠٠
١٣١	١٫٠٠٦٦٦	٠٫٠٠٤٤٨	١٫١٦٤٥٠	٠٫٠٠٠٠٠
١٣٢	٠٫٧٢٩٢٢	٠٫٠٠١٢٠	٠٫٦٦٠٤٢	٠٫٠٠٠١٠
١٣٣	٠٫٠٨٨٣١	٠٫٠٠٤٨١	٧٫٤٦٨٦٧	٠٫٠٤٧٨٠
١٣٤	٧٫٣١١٨٢	٠٫٠٠٠٠٠	٧٫٩٠١٢٦	٠٫٠٠٠٧٣
١٣٥	٨٫٣١٢٥٩	٠٫٠٠١١٧	٨٫٧٩٤٥١	٠٫٠٠١١٧
١٣٦	٨٫٢١٦٤٨	٠٫٠٠٤٧٢	٧٫٦٠٦٧٩	٠٫٠٠٠٠١
١٣٧	١٫٠٢٢٢٩	٠٫٠٠٠٧٠	٠٫٩٢٦٩٨	٠٫٠٠٠١٠
١٣٨	٠٫٦٩٢٣٤	٠٫٠٠١٦٨	٠٫٦٥٢١٩	٠٫٠٠٠٠٩
١٣٩	١٫٣٢٢١٦	٠٫٠٠٠٥٧	١٫٢٠٤٨١	٠٫٠٠٠٠٧
١٤٠	٠٫٩٧٤٣٥	٠٫٠٠٠٧٧	٥٫٧٧٢٨٥	٠٫٠٢٩١٧
١٤١	١٫١٥١٢٢	٠٫٠٠٠٥٥	١٫٠٤٥٨٢	٠٫٠٠٠٠٨
١٤٢	٠٫٨٨٧١٥	٠٫٠٠٠٨٤	٠٫٨٠٣٥٤	٠٫٠٠٠٠٩
١٤٣	٠٫٥٥٩٨٦	٠٫٠١١٧٧	٠٫٣٥١٦٥	٠٫٠٠٠٠٥
١٤٤	٠٫٩٢٩٦٧	٠٫٠٠٠٧٥	٠٫٨٣٣١٦	٠٫٠٠٠٠٨
١٤٥	٠٫٨٤٨٧٩	٠٫٠٠١٢٣	١٫٠٠٣٢٢	٠٫٠٠٠٠٧
١٤٦	٠٫٨٣٢٦١	٠٫٠٠٠٩٧	٠٫٧٥٣٩٧	٠٫٠٠٠١٠
١٤٧	٧٫٠٥٧٤٠	٠٫٠١٤١٣	٧٫٢٧٣٦٩	٠٫٠٠٠١٦
١٤٨	١٫٦٣٤٧٥	٠٫٠١٩٣٤	٠٫٨٣٠٣١	٠٫٠٣١٠٣
١٤٩	٥٫٠٥٣٩٠	٠٫٠٠٠٣٨	٥٫٢٨٦٠٢	٠٫٠٠٠٠٦
١٥٠	٥٫١٦٤٥٠	٠٫٠٠١٢٩	٥٫١٤٥٥٤	٠٫٠٠٠٠٣

Figure (3.9) Serbert before (n=150)

```
Rescaled Distance Cluster Combine
    C A S E      0         5        10        15        20        25
  Label   Num   +---------+---------+---------+---------+---------+

           55    -+
           58    -+
           33    -+
           15    -+
          142    -+
           25    -+
          105    -+
          140    -+
          144    -+
           23    -+
           38    -+
           75    -+
           18    -+
           97    -+
          100    -+
           19    -+
           57    -+
           70    -+
           79    -+
          141    -+
           96    -+
          139    -+
           26    -+
           82    -+
           37    -+
          115    -+
           78    -+
          122    -+
          137    -+
           42    -+
          116    -+
           84    -+
          114    -+
           83    -+
           56    -+
          107    -+
          102    -+
          132    -+
           86    -+
           81    -+
          129    -+
           98    -+
           28    -+
          104    -+
          103    -+
          112    -+
           89    -+
           31    -+
          121    -+
            8    -+
           66    -+
          113    -+
           62    -+
           93    -+
           21    -+
           87    -+
          119    -+
          146    -+
          106    -+
           41    -+
           46    -+
          138    -+
          101    -+
          126    -+
            2    -+
```

```
  20   -+
  45   -+
  53   -+
 148   -+
   3   -+
  61   -+
 110   -+
  69   -+
  92   -+
   6   -+
 143   -+
 108   -+
  52   -+
  59   -+
 145   -+
   7   -+
  85   -+
  40   -+
  24   -+
 150   -+
  39   -+
  95   -+
  68   -+
 136   -+
  27   -+
   9   -+
  71   -+
  11   -+
 133   -+
  32   -+
 128   -+
 109   -+
  50   -+
 118   -+
  90   -+
  91   -+
  60   -+
  99   -+
  64   -+
 131   -+
  63   -+
  22   -+
 147   -+
  54   -+
  80   -+
   4   -+
   5   -+
  29   -+
  36   -+
  44   -+
  74   -+
 127   -+
  34   -+
 123   -+
 120   -+
 135   -+
  13   -+-----------------------------------------------+
  10   -+                                               |
  43   -+                                               |
 149   -+                                               |
 125   -+                                               |
  14   -+                                               |
  1   -+                                                |
  51   -+                                               |
  30   -+
```

```
                                                   |
   48   -+                                                      |
   88   -+                                                      |
   12   -+                                                      |
  117   -+                                                      |
   49   -+                                                      |
  134   -+                                                      |
  124   -+                                                      |
   17   -+                                                      |
  111   -+                                                      |
   35   -+                                                      |
   67   -+                                                      |
   47   -+                                                      |
   94   -+                                                      |
  130   -+                                                      |
   72   -+                                                      |
   16   -+                                                      |
   65   -+                                                      |
   73   -+                                                      |
   77   -+                                                      |
   76   -------------------------------------------------------+
```

Figure (3.10) Adnan before (n=150)

```
Rescaled Distance Cluster Combine

   C A S E     0        5        10        15        20        25
  Label   Num  +--------+--------+--------+--------+--------+

           57   -+
           70   -+
           15   -+
           18   -+
           21   -+
          142   -+
          119   -+
          100   -+
           75   -+
          144   -+
           19   -+
           33   -+
          105   -+
          137   -+
           25   -+
          140   -+
           97   -+
           93   -+
           38   -+
           23   -+
           78   -+
           58   -+
           87   -+
           55   -+
          106   -+
          146   -+
            8   -+
           56   -+
           66   -+
          113   -+
          116   -+
          104   -+
          129   -+
           89   -+
          107   -+
          121   -+
          112   -+
           98   -+
          132   -+
            2   -+
          101   -+
           86   -+
          102   -+
```

```
 81   -+
103   -+
 28   -+
 31   -+
 62   -+
 96   -+
141   -+
 37   -+
 82   -+
 26   -+
 79   -+
115   -+
122   -+
 84   -+
114   -+
 42   -+
 41   -+
 83   -+
 46   -+
143   -+
148   -+
138   -+
110   -+
  6   -+
 69   -+
  3   -+
 53   -+
108   -+
 61   -+
 92   -+
126   -+
139   -+
 20   -+---+
 45   -+   |
 11   -+   |
109   -+   |
133   -+   |
 52   -+   |
 72   -+-+ |
 73   -+ | |
 50   -+ | +-------------------------+
118   -+ | |                                           |
 64   -+ | |                                           |
 91   -+ | |                                           |
  4   -+ | |                                           |
 90   -+ | |                                           |
 63   -+ | |                                           |
128   -+ | |                                           |
 32   -+ +-+                                           |
147   -+ |                                             |
 22   -+ |                                             |
131   -+ |                                             |
 85   -+ |                                             |
 39   -+ |                                             |
 95   -+ |                                             |
  5   -+ |                                             |
  9   -+ |                                             |
 30   -+ |                                             |
136   -+ |                                             |
  7   -+-+                                             |
 80   -+                                               |
 60   -+                                               |
 99   -+                                               |
 54   -+                                               |
 68   -+                                               |
145   -+                                               |
 40   -+                                               |
 44   -+                                               |
130   -+                                               |
 77   -+                                               |
 74   -+                                               |
 94   -+                                               |
 16   -+                                               |
```

```
       35   -+                                                         |
       10   -+                                                         |
      120   -+                                                         |
      149   -+                                                         |
       48   -+                                                         |
      134   -+                                                         |
        1   -+                                                         |
       17   -+                                                         |
       59   -+                                                         |
       67   -+                                                         |
       12   -+                                                         |
       88   -+                                                         |
      117   -+                                                         |
       49   -+                                                         |
       51   -+                                                         |
       14   -+                                                         |
       36   -+                                                         |
       29   -+                                                         |
      111   -+                                                         |
      135   -+                                                         |
       13   -+                                                         |
      123   -+                                                         |
       27   -+                                                         |
      124   -+                                                         |
       43   -+                                                         |
       24   -+                                                         |
       71   -+                                                         |
       47   -+                                                         |
      127   -+                                                         |
      150   -+                                                         |
      125   -+                                                         |
       34   -+                                                         |
       65   -+-------------------+
       76   -+
```

Figure (3.11) Serbert after (n=150)

```
  Rescaled Distance Cluster Combine

 C A S E      0         5        10        15        20        25
Label   Num  +---------+---------+---------+---------+---------+

        126   -+
        127   -+
        143   -+
        145   -+
        111   -+
        139   -+
         12   -+
         26   -+
         79   -+
         97   -+
        124   -+
         45   -+
         21   -+
         84   -+
        119   -+
         15   -+
        106   -+
        141   -+
         31   -+
        101   -+
        103   -+
        120   -+
         78   -+
         23   -+
         33   -+
         37   -+
        114   -+
        129   -+
          3   -+
          8   -+
```

```
 81   -+
132   -+
 53   -+
138   -+
  6   -+
 66   -+
 67   -+
 49   -+
 52   -+
 34   -+
 69   -+
146   -+
107   -+
 46   -+
116   -+
117   -+
 86   -+
 41   -+
 74   -+
104   -+
137   -+
 18   -+
102   -+
147   -+
 87   -+
142   -+
 14   -+
144   -+
118   -+
 38   -+
112   -+
 39   -+
 98   -+
 56   -+
 75   -+
 28   -+
 57   -+
 25   -+
105   -+
122   -+
123   -+
 19   -+
 62   -+
121   -+
 55   -+
 89   -+
 58   -+
128   -+
113   -+
  9   -+
  7   -+
 11   -+
 83   -+
149   -+
 16   -+
 42   -+
 68   -+
 92   -+
115   -+
 36   -+
 32   -+
 99   -+
 40   -+
 47   -+
 63   -+
 64   -+
  4   -+
 80   -+
 13   -+
100   -+
110   -+
  2   -+
 96   -+
```

```
  82   -+
 109   -+
 135   -+
  88   -+
  95   -+
 108   -+
 148   -+
 125   -+
  65   -+
  30   -+-+
  93   -+ |
 134   -+ |
 133   -+ |
   1   -+ |
  60   -+ |
  44   -+ |
 150   -+ |
  54   -+ +-+
  22   -+ | |
 136   -+ | |
 131   -+ | |
   5   -+ | |
  50   -+ | |
 130   -+ | |
  59   -+ | +-+
  43   -+ | | |
  51   -+-+ | |
  10   -+   | |
  76   -+   | +-----------------------+
  77   -+-+ | |                       |
  91   -+ | | |                       |
  61   ---+-+ |                       |
  17   ---+   |                       +---------+
  29   -+-----+                       |         |
 140   -+                             |         |
  71   -+-----------+                 |         |
  73   -+           +-----------------+         +-------+
  27   -+           |                           |       |
  72   -+-----------+                           |       |
  24   -+                                       |       |
  48   -+-------+                               |       |
  90   -+       +-------------------------------+       |
  35   -----+---+                                       |
  94   -----+                                           |
  20   -+-----+                                         |
  70   -+     +-----------------------------------------+
  85   -------+
```

Figure (3.12) Adnan after (n=150)

```
  Rescaled Distance Cluster Combine

 C A S E 0      5       10     15    20       25
Label   Num  +-------+-------+------+-------+-------+

        75   -+
       137   -+
        57   -+
        19   -+
        38   -+
        90   -+
       119   -+
       106   -+
        21   -+
       105   -+
        15   -+
       144   -+
        25   -+
         8   -+
        91   -+
        41   -+
        46   -+
       114   -+
       108   -+
        31   -+
       142   -+
        18   -+
        85   -+
       148   -+
         2   -+
       103   -+
        87   -+
        28   -+
        55   -+
        98   -+
        58   -+
       146   -+
        84   -+
        62   -+
       133   -+
        81   -+
       104   -+
        56   -+
       112   -+
       113   -+
       129   -+
       143   -+
        86   -+
       110   -+
       116   -+
       117   -+
       121   -+
        89   -+
       132   -+
        66   -+
        67   -+
       102   -+
       101   -+
       107   -+
        83   -+
       109   -+
        42   -+
       138   -+
       122   -+
       123   -+
        78   -+
        97   -+
        33   -+
        96   -+
       139   -+
```

```
 20   -+
 35   -+
126   -+
127   -+
 93   -+-+
 61   -+ |
 82   -+ |
 45   -+ |
 79   -+ |
141   -+ |
 26   -+ |
 37   -+ +---+
 23   -+ |   |
 48   -+ |   |
 69   -+ |   |
  6   -+ |   |
 92   -+ |   +-----+
  3   -+ |   |     |
 53   -+ |   |     |
 11   -+-+   |     |
115   -+     |     |
 65   -+-+   |     |
100   -+ +---+     |
125   ---+         |
 44   -+           |
 76   -+-------+   |
130   -+       |   |
 54   -+       |   |
 60   -+       |   |
 12   -+       |   +-------+
111   -+       |   |       |
136   -+       |   |       |
  5   -+       |   |       |
  1   -+       |   |       |
 34   -+       |   |       |
 49   -+       |   |       |
 52   -+       |   |       |
 59   -+       |   |       |
 77   -+       |   |       |
  9   -+       |   |       |
135   -+       |   |       |
 40   -+       |   |       |
 43   -+       +---+       |
 68   -+-+     |           |
134   -+ |     |           |
 99   -+ |     |           |
 39   -+ |     |           |
 50   -+ |     |           |
 51   -+ |     |           |
  4   -+ |     |           |
145   -+ |     |           |
 80   -+ |     |           |
 88   -+ |     |           |
 30   -+ |     |           |
131   -+ |     |           |
  7   -+ |     |           +---------------------------+
 22   -+ |     |           |                           |
 94   -+ |     |           |                           |
 29   -+ |     |           |                           |
 32   -+ +-----+           |                           |
128   -+ |                 |                           |
118   -+ |                 |                           |
 64   -+ |                 |                           |
 63   -+ |                 |                           |
147   -+ |                 |                           |
 17   -+ |                 |                           |
 95   -+ |                 |                           |
 10   -+ |                 |                           |
124   -+ |                 |                           |
 47   -+ |                 |                           |
 14   -+ |                 |                           |
 13   -+ |                 |                           |
150   -+ |                 |                           |
```

```
149   -+ |                  |                            |
 16   -+ |                  |                            |
 36   -+-+                  |                            |
 74   -+                    |                            |
120   -+                    |                            |
 24   -+                    |                            |
 71   -+                    |                            |
 27   -+                    |                            |
 72   -------------+--------+                            |
 73   -------------+                                     |
 70   ----------------------------+----------------------+
140   ----------------------------+
```

Table (3.9) below shows the summary for the methods , where they are all compared with Mahalanobis distance ($\chi^2_{(145,0.05)}$=67.2), and Cook distance ($F_{(5,45,0.05)}$=2.21) . Both of them can not detect any outliers in the two cases. Outliers are detected by Serbert and Adnan , depending on (ch) value . Differences are found between them for before and after cases .

Mahalanobis and Cook have masking for all the adding outliers ; but Serbert has in the No. (10,65,100,108,115,125,133,148) ; and Adnan has in the No. (20,35,48,65,85,90,100,108,115,125,133,148)

Mahalanobis and Cook didn't have swamping ; but Serbert has in the No. (24,27,71,72,73,94) ; and Adnan has in the No.(13,14,16,17,24,27,36,47,63,64,71,72,73,74,95,118,120,124,128,147,149,150) .

Mahalanobis , Cook and Serbert have (0.3) Std. Error Est. before case , and Serbert has (2.51) after case , which are less than others .

Table (3.9) Summary (n=150)

case	method	ch	outliers	masking	swamping	Std. Error Est.
Before	none	-	-	-	-	0.3
	Mah.	-	none	-	-	0.3
	Cook	-	none	-	-	0.3
	Serbert	12.5	(1,10,12,13,14,16,17, 30,35,43,47,48,49,51,65,67,72,73,76,77,88,94,111,117,124,125, 130,134,149)	-	-	0.3
	Adnan	3.6	(1,10,12,13,14,16,17, 24,27,29,34,35,36,40,43,44,47,48,49,51,54,59,60,65,67,68, 71,74,77,80,88,94,99,111, 117,120,123,124,125,127,130,134,135,145,149,150)	-	-	0.33
After	none	-	-	-	-	3.21
	Mah.	-	-	10,20,35, 48,65,70, 85,90,100,108,115, 125,133, 140,148	-	3.21
	Cook	-	-	10,20,35, 48,65,70, 85,90,100,108,115, 125,133, 140,148	-	3.21
	Serbert	5	(20,24,27,35,48,70,71,72,73,85,90,94,140)	10,65,100,108,115, 125,133, 148	24,27,71, 72,73,94	2.51
	Adnan	4.3	(10,13,14,16,17,24,27,36,47,63,64,70,71,72,73,74,95,118,120,124,128,140,147,149,150)	20,35,48, 65,85,90, 100,108, 115,125, 133,148	13,14,16, 17,24,27, 36,47,63, 64,71,72, 73,74,95, 118,120, 124,128, 147,149, 150	3.05

4. Conclusions :

- **Mahalanobis and Cook Methods could not detect any outlier for all cases, although many of outliers are inserted in the observations. This is because both of them depend on detecting a single outlier, and if the outliers are grouping, they may have detected them .**
- **When (n=25), Adnan's method has the smallest (Std. Error Est.) for before case . It has the same results of Serbert method's after adding the outliers and both of them reduced (Std. Error Est.) .**
- **When (n=50), Serbert method's has the smallest (Std. Error Est.) , masking and swamping .**
- **When (n=150), Serbert method's has the smallest (Std. Error Est.) , masking and swamping .**

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