جدولة النشاطات الحرجة في أدارة المشاريع التصادفية

د.علاء الدين نوري احمد   د. نذير عباس ابراهيم
جامعة النهرين

المستخلص

تم في هذا البحث أعتماد مسألة تتعلق بشبكة المشاريع التصادفية، عندما يكون هنالك خللا ($interruption$) في بعض أو جميع تلك النشاطات. وعلى ضوء ذلك تم بناء أسلوب رياضي لجدولة ($scheduling$) النشاطات الحرجة من خلال بناء بعض العلاقات الرياضية متأسسة على مفهوم الكلف التأخيرية الناتجة من النشاطات ذات التوقفات المفاجئة ($interruption$)، وتم عرض مثال مأخوذ من [12] للتحقق من الاسلوب المتبع.

Scheduling Critical Activities of Stochastic Projects Management

Abstract

In this paper, we consider the problem of stochastic project network when some or all activities are interrupted. An approach has been built to schedule the critical activities, by constructing some expressions based on the project lateness costs due to the interruption activities. Two simple example are presented to validate our approach.

Key words: Project Management, Project scheduling, Stochastic activity duration, Stochastic PERT.

Introduction

Recently, Projects planning and optimal timing, under uncertainty are extremely critical for many organizations, see [19]. Having an effective mathematical model will give project managers a significant tool for replanning projects allocation decisions in response events and outcomes. As a result, the uncertainty associated with such risky projects should be reduced.

No one factor can account for, or prevent, failure in a project. It is common practice for a project team to develop a comprehensive risk assessment and risk management plan. Identifying the most critical activities with regard to schedule risk is a problem faced by all project managers.

The problem of identifying critical activities (CA) in a deterministic network is well understood, by standard Critical Path Method (CPM) known as Program Evaluation and Review Technique (PERT). Since a project could be delayed if these activities were not completed in the scheduled time. The classical PERT method is the probably is the best-known mechanism for considering the stochastic nature of activity durations, in which it is possible to develop estimates of the uncertainty of the project make span. For more details see [8] & [10].

مجلة العلوم الاقتصادية والإدارية
المجلد 18
العدد 68
الصفحات 1- 8

Numerous papers have been written about the PERT method, with stochastic activity durations. In [6], a recursive algorithm is developed for determining either the CDF or moments of the project makespan distribution. In [3] & [13] bounds are obtained for the makespan PDF, and developed the makespan distribution for project network with exponentially distributed activity times using Markov Pert Network (MPN), also, developing approximations using discritization of continuous density functions, simplifying the convolution of activity densities, which offer practical implementation. In [5] a Branch –and-Bound algorithm is presented for solving a discrete version of this problem when activity times must be either normal or crashed (i.e., a binary state).

Identifying critical activities in a stochastic project is problematic. Several methods had been proposed contain series draw backs which lead to identifying critical activities incorrectly, leaving project mangers without means to identifying and rank the most probable sources of project delays, and with activities represent the best opportunities for successfully addressing schedule risk.

In an important related paper [4], literature review is presented on determining the criticality of activities in stochastic project networks, evaluating a number of approaches for assessing criticality and sensitivity.

In several large construction projects ,it has been observed that firms have been earned significant returns by optimally managing the time- cost trade-of decisions needed to avoid penalties associated with delaying the successful completion of a project within due data. The overall importance of the stochastic project compression problem has been noted in much of the project management literatures [10].

In this paper, we are considered, the project consists of stochastic activities (SA), i.e, those that are interrupted for an uncertain amount of time. Also the initial processing time is considered to be known, however, the length of the interrupted and the final processing time are uncertain.

In this study, we deal with a series of two-stages decisions, where in the first stage all the available activities are schedule, and in the second stage we are scheduling the activities taking into consideration the cost of lateness due to the activities interruptions. Based on [12], who developed general expressions for determining activities late starting and ending time distributions, we built an approach for identifying and scheduling the critical activities, according to a weighted cost per unit time of the interruption, constructed by evaluating the total expected interruption time for an activity, and the total expected lateness, given that the total latest finishing time is greater than or equal to the given due date.

The Problem

Assume that we have a set of N activities, each has the duration $(t_i), (i = 1, \ldots, N)$ can be simply denoted by:

$t_i = t_i^n \equiv uninterrupted = estimated\ processing\ time$

and if activity is interrupted, then it is simply given by:

$$t_i = t_i^p + t_i^r \qquad (1)$$

Where (t_i^p) is the processing time before the interruption, (t_i^r) is the processing time during the interruption.

In a stochastic project network, the activities times (durations) are random variables. Activity starting and ending times, as well as activity slack times, are therefore random variables. A new direction for identifying critical path activities (CPA) in stochastic project network (SPN) is based on different philosophy, than in deterministic project network (DPN), where each critical activity must correspond to zero time slack activity, while such condition need not to be necessary in (SPN), in which the critically index, is defined as the probability that an activity will lie on a critical path may not introduced risk of project delay (i.e., schedule risk) into the project network. , see [2].

Now, if we define the following notation :

$\gamma_i = The\ set\ of\ immediate\ predecessors\ of\ activity\ i$

$\Gamma_i = The\ set\ of\ immediate\ successors\ of\ activity\ i$

$p_i(t) = the\ activity\ time\ probability\ density\ function\ PDF\ activity\ i$

$P_i(t) = The\ activity\ time\ cumulative\ distribution\ function\ CDF$ activity i

$p_{E,i}(t) = The\ earliest\ start\ time\ PDF\ for$ activity i

$p_{L,i}(t) = The\ latest\ start\ time\ PDF\ for$ activity i

$P_{E,i}(t) = The\ earliest\ start\ time\ CDF\ for$ activity i

$P_{L,i}(t) = The\ latest\ start\ time\ CDF\ for$ activity i

$f_{E,i}(t) = The\ earliest\ finish\ time\ PDF\ for$ activity i

$f_{L,i}(t) = The\ latest\ finish\ time\ PDF\ for$ activity i

$F_{E,i}(t) = The\ earliest\ finish\ time\ CDF\ for$ activity i

$F_{L,i}(t) = The\ latest\ finish\ time\ CDF\ for$ activity i

By assuming the early start schedule distributions to be continuous distributions, we can define the following:

$$P_{E,i}(t) = \prod_{j\epsilon\gamma_i} F_{E,j}(t) \quad (2)$$

$$p_{E,i}(t) = \frac{dP_{E,i}(t)}{dt} \quad (3)$$

$$F_{E,i}(t) = \int_0^t P_i(t-t_1)p_{E,i}(t_1)dt_1 \quad (4)$$

$$f_{E,i}(t) = \frac{dF_{E,i}(t)}{dt} \quad (5)$$

$$P_{L,i}(t) = \int_{t_1=0}^{t}\int_{t_2=t}^{\infty} p_i(t_2-t_1)f_{L,i}(t_2)dt_2dt_1 \quad (6)$$

$$p_{L,i}(t) = \frac{dP_{L,i}(t)}{dt} \quad (7)$$

$$F_{L,i}(t) = \prod_{j\epsilon\Gamma_i}\{1-P_{L,i}(t)\} \quad (8)$$

$$f_{L,i}(t) = \frac{dF_{L,i}(t)}{dt} \quad (9)$$

The early starting and ending time distribution for every activity is determined by proceeding sequentially forward through the network, using equations (2 – 5), while, by setting $F_{L,n} = F_{E,n}$ and processing sequentially backwards through the network, beginning with activity n and terminating with activity 1, the late start schedule distributions can be calculated using equations (6 – 9). In [12], the sources of schedule risk in a stochastic project network is identified, and the general expression for determining an activity's late starting and ending time distributions are developed to identify the critical activities using the activity critically index to those found using stochastic activity metrics. The early and late starting time distributions are used to calculate expected total slack for activity i as follows:

$$E[TS_i] = \int_0^{\infty}\int_0^{z_1+t}(z_2-z_1)p_{E,i}(z_1)p_{L,i}(z_2)dz_2dz_1 \cong \sum_{z_1}\sum_{z_1}(z_2-z_1)p_{E,i}(z_1)p_{L,i}(z_2) \quad (10)$$

and then a ranking of activities can be identified, whether are most (least) likely to introduce a delay into a project, having lowest (largest) E(TS), respectively. Also, claiming that considering $E(TS|L)$ and $E(L|TS)$ yield different ranking criticality index, and poor performance due to ignoring many factors that we are constructed in this paper.

In this paper, we are simplifying the method presented in [12], by constructing an expression for determining the total expected interruption time and the total expected lateness, given that the total latest finishing time is greater than or equal to the known due date D, as follows:

$$E[Tt_i^r] = \int_0^{\infty}\int_0^{z_1+t_i^r} (z_2 - z_1)p_{E,i}(z_1)p_{L,i}(z_2)dz_2dz_1 \qquad (11)$$
$$\cong \sum_{z_1}\sum_{z_2}(z_{2,i} - z_{1,i})p_{E,i}(z_{1,i})p_{L,i}(z_{2,i})$$

$$E(L|F_{L,n} \geq D, Tt_i^r) = \int_D^{\infty}(t - D)p_{L,n}(t|Tt_i^r)dt \qquad (12)$$
$$\cong \sum_{t_i \geq D}(t_i - D)p_{L,n}(t|Tt_i^r)$$

then, we compute the total interruption cost of the project resulting from activity j as follows:

$$r_j = \frac{c_j^r}{E[Tt_j^r]} * E(L|F_{L,n} \geq D, Tt_i^r) \qquad (13)$$

Where, $L = F_{L,n} - D$, represents project lateness, and c_j^r is the interrupted cost of activity j.

Now, our approach, consist of the following steps:
Step 1: Given the mean & critical index for each activity.
Step 2: Calculate $F_{L,n}$ based on classical PERT.
Step 3: Compute the following: $E[Tt_j^r], E(L|F_{L,n} \geq D, Tt_i^r)$ and r_j
Step 4: Calculate r_j , and choose the activity j with the maximum value of r_j first.
Step 5: Repeat step 4 for the remaining activities.

Tested Problem

Problem (1)

As we believe that, no theoretical difficulties are raised, we perform our method, by considering the same example of project network in table (1) and Figure (1) below, presented in [12], to validate our approach. Given the project due date D calculate $F_{L,n}$ and then $L = F_{L,n} - D$. in which according to their critical indices, and their expected total lateness $E(L|F_{L,i} \geq D, Tt_i^r)$ the ranking of critical activities is 4 , 5 , 2 , 3, which is the same result but in less computations due to more factors are needed to be calculated in [12].

Task	1	2	3	4	5	6
Distribution	Det	Beta	Beta	Beta	Beta	Det
Mean	-	51.00	43.00	52.00	19.00	-
SD	-	18.55	10.33	18.49	8.94	-
Critically Index	-	0.227	0.085	0.687	0.555	-
$E(L\|F_L \geq D, Tt_i^r)$	-	10.5	7.00	9.00	4.00	-
$E\left[Tt_j^r\right]$	-	0.501	0.498	0.785	0.682	-

Table (1): Sample project task activity time distributions

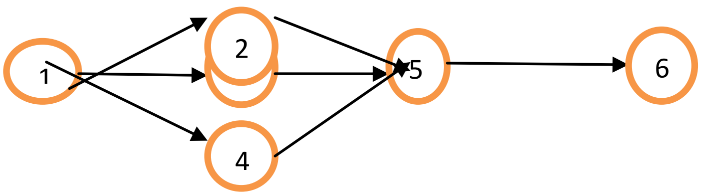

Figure (1)
Problem (2)

We generating another simple example numerical example, by convention tasks 1 & 9 have zero activity duration with probability 1.0, illustrated in table (2) and in figure (2). Therefore, any delay introduced into the project schedule will be one of the remaining tasks 2-8, depend on both their locations within the project network and their activity distributions given in the table below:

Task	1	2	3	4	5	6	7	8	9
Distribution	Det	Beta	Beta	Beta	Beta	Beta	Beta	Beta	Det
Mean	-	35.0	43.0	52.0	19.0	20.0	32.0	40.0	-
SD	-	10.5	1-.3	18.4	8.0	12.0	15.4	8.9	-
Critically Index	-	0.44	0.37	0.59	0.63	0.61	0.65	0.73	-
$E(L\|F_L \geq D, Tt_i^r)$	-	9	10	6	7	5	10	3	-
$E\left[Tt_j^r\right]$	-	0.33	0.31	0.62	0.66	0.58	0.69	0.71	-

Table (2): Sample project task activity time distributions

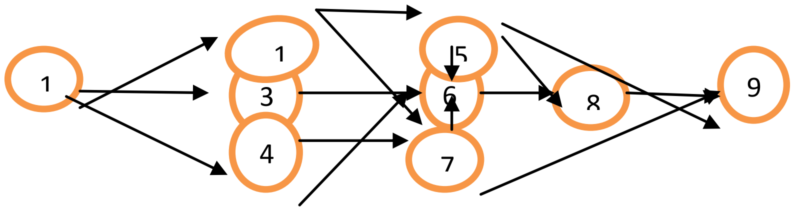

Figure (1)

Conclusion

In this study, we addressing an important problem faced by many project managers, specifically, we introduced the concepts of stochastic project, and sensitivity in stochastic activity network, in which we are believe that the approach presented in this paper is the first one to test which tasks introduced in the schedule by developing new efficient pointers.

Many applications of such problem can be found in the deterministic project planning and scheduling literature. Therefore, more studies have been required to test the efficiency of the proposed method compared with other stochastic methods, considering different activities distributions.

References

[1] Cho, J. G. & B. J.Yum, "An uncertainty importance measure activities in PERT networks". Int. J. Prod. Res., 35:2737-2757. (1997).
[2] Demeulemeester, E. & W. Herroelen, "Project Scheduling :A Research Handbook." 1st Edn., Kluwer Academic Puplishers,Boston,MA. (2002).
[3] Dobin, B., " Bounding the project completion time distribution in PERT networks". Operat. Res., 33:862-881. (1985b).
[4] Elmaghraby, S. E. " On critically and sensitivity in activity networks". Eur. Jr. Operat. Res., 127:307-313. (2000).
[5] Gutjahr, W. J., Strauss, C. and Wagner. "A stochastic branch and –bound approach to activity crashing in project management". INFORMS Journal on Computing , 12(2), 125-135. (2000),
[6] Hagstrom, J., "Commuting the probability distribution of project duration in a PERT network". Networks, 20:231-244. (1990).
[7]Hereoelen, W. and, Leus, R. "Project scheduling under uncertinity:survey and research potentials", European Journal of Operational Research , 165,289-306. (2005).
[8]Herroelen, W., "Generating robust project baseline schedule". TutORials in Operations Research.INFORMS National Meeting Seattle, WA., ISBN: 13: 978-1-877640-22-3, PP:124-144. (2007).
[9] Johnson, G. A., and Schou, C. D. "Expediting projects in PERT with stochastic time estimates .Project Management Journal, 21(2), 29-34. (1990).
[10] Klastorin, T. D. "Project Management: Tools and Trade –offs", Wiley New York, NY. (2004).
[11] Mitchell, G. & T. Klastorin, "An effective methodology for the stochastic project compression problem". IIE Trans., 39:957-969. (2007).
[12] Mitchell, G., "On Calculating Activity Slack in Stochastic Project Network". American Jr. of Econ. & Busns. Admin. 2(1), 83-90. (2010).
[13] Santiago, I. and Vakili , P. "On the value of flexibility in R & D projects . O. R. 24(1), 177-182. (2005).